• Wave Packets and Group Velocity
• $\lambda$: Order of Magnitude
• Evidence for Wave Behavior of Particles
• The "old Quantum Theory"
**I₃ Particle Waves**

**I₃,1 De Broglie Hypothesis:**

\[
\begin{align*}
\text{Particle: Energy}\ E & \quad \Leftrightarrow \quad \text{associated wave} \\
\text{Momentum}\ P & \quad \lambda = \frac{h}{P} \\
\end{align*}
\]

\[
\nu = \frac{E}{\hbar}
\]

**I₃,3 Superposition of Particle Waves**

\[
f(x,t) = \Re\left\{ e^{i[k_0 x - \omega(k_0) t]} \int \phi(s) e^{i(s - \frac{d\omega}{dk}(k_0) t)} ds \right\}
\]

- **infinite plane wave:** 
  - Crests move at 
  - \( V_{\text{phase}} = \frac{\omega}{k} = \frac{c^2}{k} \)

- **envelope function:** 
  - travels at group velocity 
  - \( V_{\text{group}} = \frac{d\omega(k)}{dk} = V_{\text{particle}} \)

\[
f(x,t) = \text{wave packet}
\]

\[
x = \sum \delta(x - \Delta x)
\]

\[
V_{\text{phase}} \quad \Rightarrow \quad V_{\text{group}}
\]
Conclusion:

- wave packet associated with "localized" particle:
  - position of envelope function (max. wave amplitude) matters
  - group velocity matters, not phase velocity
  - localized wave packet $\Rightarrow$ momentum of particle is not well defined (uncertainty)
  - $\Delta x \sim \hbar \Delta p_x$

=) free particle in Schrödinger's QM
Example: \( v_{\text{group}} = v_{\text{particle}} = \frac{c}{2} \quad v_{\text{phase}} = \frac{c^2}{v} = 2c \)
Example: $v_{\text{group}} = v_{\text{particle}} = c/2 \quad v_{\text{phase}} = c^2/v = 2c$
The envelope function of the wave packet associated with a localized particle should be related to…

A. The size of the particle (smaller size -> shorter envelope function)

B. The range of space in which the particle might be found if its position would be measured

C. Something else
Group and Phase Velocity for de Broglie’s Particle Waves:

**Phase Velocity:** \( V_{ph} = \frac{\omega}{k} = \frac{\lambda}{2\pi} \quad \nu = \frac{c^2}{\sqrt{\nu} - \text{speed of particle}} \)

**Group Velocity:** \( V_{group} = \frac{d\omega(k)}{dk} \)

\[
\omega = 2\pi \nu = 2\pi \frac{E}{\hbar} = \frac{2\pi}{\hbar} \sqrt{\frac{\hbar^2}{4\pi^2} k^2 c^2 + m_0^2 c^4} = \frac{2\pi}{\hbar} \sqrt{\frac{\hbar^2}{4\pi^2} k^2 c^2 + m_0^2 c^4}
\]

\[
k = \frac{2\pi}{\hbar} \sqrt{\frac{\hbar^2}{4\pi^2} k^2 c^2 + m_0^2 c^4} = \omega(k)
\]

\[
= \frac{2\pi}{\hbar} \sqrt{\frac{\hbar^2}{4\pi^2} k^2 c^2 + m_0^2 c^4} = \omega(k)
\]

\[
\Rightarrow \frac{d\omega}{dk} = \frac{2\pi}{\hbar} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{\frac{\hbar^2}{4\pi^2} k^2 c^2 + m_0^2 c^4}} \cdot \frac{\hbar^2}{4\pi^2} c^2 \gamma = \frac{1}{E} \cdot \frac{\hbar}{2\pi} c^2 k
\]

\[
= \frac{1}{E} \cdot \frac{\hbar}{2\pi} c^2 \frac{2\pi}{\lambda} = \frac{c^2}{E} \cdot \frac{\lambda}{m_0 c^2} = \frac{\gamma m_0 c}{E} = V = \text{particle speed} \quad \Rightarrow \text{group velocity} \quad \Rightarrow
\]

\[
\Rightarrow \text{group velocity} = \text{speed of an envelope function} = \text{particle speed}
\]

**Good!**
**I₃,₅ λ=h/p: Order of Magnitude Estimate**

**Or: Why wasn’t this noticed before?**

*thermal neutrons (300K)* → \( \lambda = 1.5 \, \text{Å} \)

*electrons at 100 eV* → \( \lambda = 1.2 \, \text{Å} \)

*neutrons at 10 MeV* → \( \lambda = 7 \times 10^{-15} \, \text{m} \)

\( m = 19 \) at 12/15

comparable to visible light

\( \lambda = 7 \times 10^{-31} \, \text{m} \)

\( \lambda = 400 - 700 \, \text{nm} \)

\( \lambda = 4 \times 7 \times 10^{-9} \, \text{m} \)

\( \Rightarrow \) recall 2-slit exp.: maxima for \( \sin \theta = \frac{m \lambda}{d} < 1 \)

\( \Rightarrow \) need \( \lambda \ll d \)

\( \Rightarrow \) for particle: need "slit" spacing / diffraction grid on Å scale (or less)

\( \Rightarrow \) use crystals!
Evidence for de Broglie’s Particle Waves:

Davisson-Germer Experiment (1925): Scattering of low energy electrons by a crystal surface

\[ \lambda \approx 1 \text{ Å} \]

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\[ \lambda \approx 1 \text{ Å} \]
G. P. Thompson’s Experiment: Diffraction of 10 – 40 keV electrons by a thin polycrystalline foil

\[ \lambda \approx 0.1 \, \text{Å} \]

polycrystalline film \(\Rightarrow\) Bragg condition satisfied for any given reflecting plane \(\Rightarrow\) concentric circles
Diffraction pattern of X-ray beam passing through Al foil

Diffraction pattern of electron beam passing through Al foil
Electron diffraction by polycrystalline aluminum

Laue pattern of electron diffraction by a single crystal

(Courtesy of Prof. Y. Soejima, Dept. of Physics, Kyushu Univ.)
2-slit Interference of Electrons

(a) Moving electrons

(b) After 100 electrons

(c) After 3000 electrons

(d) After 70000 electrons
Diffraction of Neutrons

\[ \lambda = \text{several Å down to } <10^{-14} \text{ m} \]

Diffraction of fast neutrons from Al, Cu, and Pb nuclei. [from French, after A Bratenahl, Phys Rev 77, 597 (1950)]
The Spallation Neutron Source (SNS) in Oak Ridge, TN
Why Neutrons?

Neutrons are **NEUTRAL** particles. They
- are highly penetrating,
- can be used as nondestructive probes, and
- can be used to study samples in severe environments.

Neutrons have a **MAGNETIC** moment. They can be used to
- study microscopic magnetic structure,
- study magnetic fluctuations, and
- develop magnetic materials.

Neutrons have **SPIN**. They can be
- formed into polarized neutron beams,
- used to study nuclear (atomic) orientation, and
- used for coherent and incoherent scattering.

The **ENERGIES** of thermal neutrons are similar to the energies of elementary excitations in solids. Both have similar
- molecular vibrations,
- lattice modes, and
- dynamics of atomic motion.

The **WAVELENGTHS** of neutrons are similar to atomic spacings. They can determine
- structural sensitivity,
- structural information from $10^{-13}$ to $10^{-9}$ cm, and
- crystal structures and atomic spacings.

Neutrons "see" **NUCLEI**. They
- are sensitive to light atoms,
- can exploit isotopic substitution, and
- can use contrast variation to differentiate complex molecular structures.
Scattering of Alpha Particles

Angular distribution of 40 MeV alpha particles scattered from niobium nuclei. [from French after G. Igo et al., Phys Rev 101, 1508 (1956)]
Crystal Diffraction of Neutral Helium (1930)

$\lambda \approx 1 \, \text{Å}$

Fig. 2-16  (a) Experimental arrangement used by Stern et al. to investigate crystal diffraction of neutral helium atoms.  (b) Experimental results showing central reflection peak ($\phi = 0^\circ$), plus first-order diffraction peaks ($\phi = 11^\circ$).  In the experiment, $\theta = 18.5^\circ$.  

from French after Estermann and Stern, Z Phys 61, 95 (1930)
Fullerene molecule C60, consisting of 60 carbon atoms.
I_4 The “Old Quantum Theory”

I_{4,1} Key Ideas / Concepts / Postulates:

1) Photons, all particles have both particle-like and wave-like properties

2) Precisely-defined trajectories do not exist at the quantum level

3) The exact behavior of a given particle cannot be predicted → only its probable behavior

   ⇒ statistical interpretation

4) The probability that a single particle is observed in a given region is proportional to intensity of its associated wave field: \( I \propto |A|^2 \)

   ⇒ \( P \propto |A|^2 \)
\[ \pi \text{ (call it 2-slit experiment)} \]

wave amplitude on screen:
- one slit open: \( A_0 \)
- both slits open: interference

\[ A_{\text{total}} = 2 A_0 \cos \left( \frac{\pi d}{\lambda} \sin \theta \right) \]

\( \Rightarrow \) intensity on screen \( \propto \) probability for a particle to arrive at a given region along the screen \( \propto \) statistical distribution of large number of particles on the screen

\[ I(\theta) \propto P(\theta) \propto |A_{\text{total}}|^2 = 4 |A_0|^2 \cos^2 \left( \frac{\pi d}{\lambda} \sin \theta \right) \]

\( \Rightarrow \) probability

\[ \sqrt{P} \propto |A| \text{ also called probability amplitude (quantum amplitude)} \]

later: wavefunctions \( \Psi \) (complex) \( \Rightarrow \) \( P \propto |\Psi|^2 \)
2-slit experiment with particles: Assume that only one slit is open, and that the probability of a particle to arrive at a small section $\Delta x$ of the screen is $F$. What is the maximum probability of finding a particle in that section $\Delta x$ of the screen if both slits are open simultaneously?

A. $\sqrt{2} F$
B. $2 F$
C. $4 F$
D. $2 \sqrt{F}$
E. $F^2$

- one slit open: prob. $F \Rightarrow$ amplitude $= \sqrt{F}$, complex phase factor
- both slits open: add probability amplitudes $\Rightarrow$ max. probability amplitude $= 2 \sqrt{F}$
  $\Rightarrow$ max. probability $= (2 \sqrt{F})^2 = 4F$
5) If a particle is confined into a small volume, its energy is quantized =) "energy levels" "energy states"

6) de Broglie- Einstein postulates:
\[ \lambda = \frac{h}{p} \quad (p = \hbar k) \quad \hbar = \text{wave number} \]
\[ \nu = \frac{E}{\hbar} \quad (E = \hbar \omega) \quad \omega = \text{angular freq.} \]

7) Superposition principle