## Lecture 20

## 5. RF Systems and Particle Acceleration

### 5.2 Accelerating RF Cavities

5.2.3 Standing wave cavities
5.2.4 The pillbox cavity
5.2.5 Higher-Order-Modes
5.2.6 SRF primer

## Parallel circuit model

A resonant cavity can be modeled as a series of parallel circuits representing the cavity eigenmodes:
dissipated power

$$
P_{c}=\frac{V_{c}^{2}}{2 R}
$$

shunt impedance

$$
R_{s h}=2 R
$$

quality factor

$$
Q_{0}=\omega_{0} C R=\frac{R}{\omega_{0} L}=R \sqrt{\frac{C}{L}}
$$

impedance

$$
Z=\frac{R}{1+i Q\left(\frac{\omega}{\omega_{0}}-\frac{\omega_{0}}{\omega}\right)} \approx \frac{R}{1+2 i Q\left(\frac{\omega-\omega_{0}}{\omega_{0}}\right)}
$$

## Connecting to a power source

- Consider a cavity connected to an RF power source

- The input coupler can be modeled as an ideal transformer:



## External \& loaded $Q$ factors

- If RF is turned off, stored energy will be dissipated now not only in $R$, but also in $Z_{0} \cdot n^{2}$, thus

$$
\begin{gathered}
P_{t o t}=P_{0}+P_{e x t} \\
P_{0}=P_{c}=\frac{V_{c}^{2}}{2 R}=\frac{V_{c}^{2}}{R / Q \cdot Q_{0}} \quad P_{e x t}=\frac{V_{c}^{2}}{2 Z_{0} \cdot n^{2}}=\frac{V_{c}^{2}}{R / Q \cdot Q_{e x t}}
\end{gathered}
$$

- Where we have defined an external quality factor associated with an input coupler. Such $Q$ factors can be identified with all external ports on the cavity: input coupler, RF probe, HOM couplers, beam pipes, etc.
- Then the total power loss can be associated with the loaded $Q$ factor, which is

$$
\frac{1}{Q_{L}}=\frac{1}{Q_{0}}+\frac{1}{Q_{e x t 1}}+\frac{1}{Q_{e x t} 2}+\ldots
$$

## Coupling parameter $\beta$

- For each port a coupling parameter can be defined as

$$
\text { so } \begin{aligned}
\beta & \equiv \frac{Q_{0}}{Q_{e x t}} \\
\frac{1}{Q_{L}} & =\frac{1+\beta}{Q_{0}}
\end{aligned}
$$

- It tells us how strongly the couplers interact with the cavity. Large $\beta$ implies that the power leaking out of the coupler is large compared to the power dissipated in the cavity walls:

$$
P_{e x t}=\frac{V_{c}^{2}}{R / Q \cdot Q_{e x t}}=\frac{V_{c}^{2}}{R / Q \cdot Q_{0}} \cdot \beta=\beta P_{0}
$$

- And the total power from an RF power source is

$$
P_{\text {tot }}=P_{\text {forw }}=(\beta+1) P_{0}
$$

| Multicell Cavities: Why? |
| :---: |
|  |  |
|  |  |

## Multicell Cavities: Why?

Example: 500 GeV Linear Collider


21,024 9-cell cavities: 27.8 km (17.3 miles)
189,216 1-cell cavities: 75.4 km ( 46.8 miles)

## Multicell cavities

- Several cells can be connected together to form a multicell cavity.
- Coupling of $\mathrm{TM}_{010}$ modes of the individual cells via the iris (primarily electric field) causes them to split into a passband of closely spaced modes equal in number to the number of cells.


- The width of the passband is determined by the strength of the cell-to-cell coupling $k$ and the frequency of the $n$-th mode can be calculated from the dispersion formula

$$
\left(\frac{f_{n}}{f_{0}}\right)^{2}=1+2 k\left[1-\cos \left(\frac{n \pi}{N}\right)\right]
$$

where $N$ is the number of cells, $n=1 \ldots N$ is the mode number.


## Two Coupled Cells: TM010 Modes



## Multicell cavities








cell \#



- Figure shows an example of calculated eigenmodes amplitudes in a 9-cell TESLA cavity compared to the measured amplitude profiles. Also shown are the calculated and measured eigenfrequencies.
- A longer cavity with more cells has more modes in the same frequency range, hence the reduction in frequency difference between adjacent modes. The number of cells is usually a result of the accelerating structure optimization.
- The accelerating mode for SC cavities is usually the $\pi$-mode, which has the highest frequency for electrically coupled structures.
- The same considerations are true for HOMs.


## Accelerating $\pi$-mode:



- cell-to-cell phase advance
- bunch takes $1 / 2$ RF period pass cell -> energy gain in each cell!


## Simulation Example:

TM010 Eigenmode-Spectrum of a 9-Cell Cavity


## Dispersion Relation



Mode Beating during Cavity Filling

- Modeling of the transient state (mode beating)

Example: 7-cells, $k_{c c}=1.85 \%, Q_{L}=3.410^{6}$

5.2.4 The pillbox cavity

accelerating mode TMO1O

$$
\begin{aligned}
& E_{z}=E_{0} J_{0}\left(\frac{2.405 r}{R}\right) e^{\text {int }} \\
& H_{\rho}=-i E_{0} \sqrt{\frac{\varepsilon_{0}}{\mu_{1}}} y_{1}\left(\frac{2.405 r}{R}\right) e^{\text {int }} \\
& \omega_{010}=\frac{2.405}{R} c, \text { ie. indef. of } L
\end{aligned}
$$

- Tramit-tim_factor.


$$
\Rightarrow \text { for } L=\beta \frac{\lambda}{2} \text { (transit-tim }=\frac{1}{2} \text { RF period) } E_{0} L / 2 \quad \text { wit } \xi=\frac{\omega}{V} \frac{L}{2}
$$

- Quality factor $Q_{0}$ :

$$
\begin{aligned}
& Q_{1}=\omega \frac{\text { stored energy }}{\text { Power dissipated in wall }} \\
& \text { Pour dissipated inwal6 } P<2 \pi \downarrow \left\lvert\,=E_{0} J_{0}\left(\frac{\omega}{c} \nu\right)\right. \\
& \rightarrow \text { st ore energy }=\mathbb{U}=\frac{1}{2} \varepsilon_{0} \int_{0} \iint_{0} E^{2} d \rho r d r d z \int_{E=E}^{\text {with }} \text { at } t=t_{0} \\
& =\frac{1}{2} \varepsilon_{0} L 2 \pi E_{0}^{2} \int_{0}^{R} J_{0}^{2}\left(\frac{\omega}{c} r\right) r d r \\
& \begin{array}{l}
=\frac{1}{2} \gamma_{0} C 2 \pi E_{0}^{2}\left(\frac{C}{w}\right)^{2} \int_{0}^{c} y_{0}^{2}(u) u d u \quad \text { with } u=\frac{\omega}{c} \tau \\
=2.405
\end{array} \\
& z_{01}=2.405 \\
& \text { use } \int_{0} y_{0}^{2}(u) u d u=\frac{1}{2}(z_{01} \underbrace{y_{1}\left(z_{01}\right.}_{0.52}))^{2}
\end{aligned}
$$

$$
\begin{aligned}
\Rightarrow U & =\frac{1}{2} \varepsilon_{0}\{\underbrace{L 2 \pi \frac{1}{2}\left(\frac{c}{w}\right)^{2}\left(\frac{w}{c} R\right)^{2}}_{L \pi R^{2}=V_{0} 1}\} E_{0}^{2} J_{1}^{2}(2.405) \\
& =\frac{1}{2} \varepsilon_{0} \underbrace{}_{\text {average field }}{ }^{2}=E^{2}, \sin u \quad E=E(v)
\end{aligned}
$$

$\rightarrow$ Power dissipated in wall:
$E_{11}=0$ at wall, but wall currents are induced by $B_{11}(t)$

$\Rightarrow$ surface currents wits shin depth: $\delta=\sqrt{\frac{2}{\mu_{0} w \sigma}}$ $\sigma=$ conductivity
Example: cope: $\sigma=5.96 .10^{7} \frac{1}{\Omega \mathrm{~m}}$

$$
\Rightarrow \delta=3 \mu \mathrm{~m} \text { at } f=0.5 \mathrm{JH} / 2
$$

$$
\left.\frac{\vec{j}}{\frac{\sum_{\vec{\beta}}}{0 \mathcal{O}_{0}}}\right\} \delta=\text { shin depth }
$$

for small aquas of surface:


$$
\Rightarrow \quad B_{11} \cdot h=\mu_{0} j h \delta \quad \Rightarrow j=\frac{1}{\mu_{0}} \frac{B_{11}}{\delta}
$$

surface mistanc:

$$
\begin{aligned}
& R_{S}=\rho \frac{\rho \text { length of conductor }}{x \text {-sectional ara }}=\rho \frac{h}{h \delta}=\frac{1}{\sim \delta} \\
& \text { ristivity }=\frac{1}{a}
\end{aligned}
$$

example: copper at 3004: $f=0.5 \mathrm{gHz} \rightarrow R_{S}=5.7 \mathrm{~m} \Omega$
$\Rightarrow$ dissipated power per surface area:

$$
\begin{aligned}
& \frac{P}{A}=\frac{\frac{1}{2} \frac{1}{R_{S} T^{2}}}{A}=\frac{1}{2} R_{s} \frac{(j h \delta)^{2}}{h^{2}} \\
& \Rightarrow \\
& \frac{P}{A}=\frac{1}{2} R_{J} \frac{1}{\mu_{0}^{2}} B_{11}^{2}=\frac{1}{2} R_{s} H_{11}^{2}
\end{aligned}
$$

$\Rightarrow$ for total power dissipated in wall, integrate $P / A$ over entix surface of pillbox cavil:
with $\left|B_{11}\right|=\frac{\varepsilon_{0}}{c} y,\left(\frac{w}{c} r\right)$

$$
\begin{aligned}
& P_{\text {tube }}=\frac{1}{2} R_{S} \frac{1}{\mu_{0}^{2}} \int_{A} \frac{E_{0}^{2}}{c^{2}} J_{1}^{2}\left(\frac{w}{c} R\right) d a \\
&=\frac{1}{2} R_{S} \frac{\varepsilon_{0}}{\mu_{0}} E_{0}^{2} y_{1}^{2}\left(\frac{w}{c} R\right)(2 \pi R L) \\
& \text { Pend walls }=2 \cdot \frac{1}{c} R_{S} \frac{\varepsilon_{0}}{\mu_{0}} E_{0}^{2} \int_{0}^{2 \pi} \int_{0}^{R} y_{1}^{2}\left(\frac{w}{c} r\right) v d v d \rho \\
& \text { two plats } \\
&=\frac{1}{2} R_{S} \frac{\varepsilon_{0}}{\mu_{0}} E_{0}^{2} 4 \pi\left(\frac{c}{w}\right)^{2} \int_{0}^{2} y_{1}^{2}(u) u d u
\end{aligned}
$$

$$
\begin{aligned}
\Rightarrow P_{\text {lad wall }} & =\frac{1}{2} R_{J} \frac{\varepsilon_{0}}{\mu_{0}} E_{0}^{2} J_{1}^{2}\left(\frac{w}{c} R\right)(2 \pi R L) \frac{R^{u=z_{01}}}{L} \\
\Rightarrow P_{\text {wall }} & =P_{\text {tube }}+P_{\text {end wall }} \\
& =\frac{1}{2} R_{S} \frac{\varepsilon_{0}}{\mu_{0}} E_{0}^{2} J_{1}^{2}\left(\frac{w}{c} R\right) 2 \pi P L\left(1+\frac{P}{L}\right)
\end{aligned}
$$

$\Rightarrow$ quality factor:

$$
\begin{aligned}
Q_{0}=w \frac{U}{P_{\text {wall }}}=\frac{\mu_{0} c 2.405}{2 R_{s}(1+R / L)} \quad \begin{aligned}
& \text { example corves at } 3004, \\
& f=0.5 \mathrm{JHz}, L=\lambda / 2 \\
& \Rightarrow Q \approx 30,000
\end{aligned}
\end{aligned}
$$

- geomitry factor $G$

$$
G=Q_{0} \cdot R_{s}=\frac{\mu_{0} c \cdot 2.405}{2(1+R / L)}
$$

$$
\left.\begin{array}{l}
\text { Cavity radius } \propto \frac{1}{f} \\
\text { cavity lugtc } \propto \frac{1}{f}
\end{array}\right\} \Rightarrow \begin{aligned}
& G \text { is inderedat of } \\
& \text { material proper bis and } \\
& \text { frequency i defined } b y
\end{aligned}
$$ frepucscy; defined by

$\Rightarrow$ for pillbox cavity wit $C L=\frac{\lambda}{2}$ : share only!

$$
\begin{aligned}
& R=\frac{2.405}{\omega} \cdot c \quad L=\frac{\lambda}{2}=\frac{c}{\omega} \pi \\
& \text { give : } G=\frac{\mu_{0} c 2.405}{2\left(1+\frac{2.405}{\pi}\right)}=257 \Omega
\end{aligned}
$$

- Shan impedance for $L=\lambda / 2 \rightarrow T=\left(\frac{2}{\pi}\right)$

$$
\begin{aligned}
P_{s h}=\frac{V^{2}}{P_{\text {walls }}} & =\frac{\left(\frac{2}{\pi}\right)^{2} E_{0}^{2} L^{2}}{\frac{1}{2} R_{s} \frac{\varepsilon_{0}}{\mu_{0}} E_{0}^{2} y_{1}^{2}\left(\frac{\omega}{C} R\right) 2 \pi R L\left(1+\frac{R}{L}\right)} \\
& =\frac{4 L}{\pi^{3} R_{3} \frac{\varepsilon_{0}}{\mu_{0}} J_{1}^{2}\left(\frac{\omega}{c} R\right) R\left(1+\frac{R}{L}\right)}
\end{aligned}
$$

- $R / Q$-factor for pill - $60 \times$ cavity with $L=\lambda / 2$

$$
\begin{aligned}
\frac{R_{s h}}{Q_{0}} & =\frac{4 L 2 R_{J}(1+R / L)}{\pi^{3} R_{s} \frac{\varepsilon_{0}}{\mu_{0}} J_{1}^{2}\left(\frac{\omega}{c} R\right) R(1+R / C) \mu_{0} C \cdot 2.405} \\
L_{R} & =\frac{\pi}{2.405}
\end{aligned}
$$

$$
\begin{array}{r}
\Rightarrow \frac{R_{s h}}{Q_{0}}=\frac{8}{\pi^{2} \varepsilon_{0} y_{1}^{2}(2.405) c \cdot 2.405^{2}}=196 \Omega \\
\quad \text { i.e. inder. of } R_{s}, f ; \text { defied by share only }
\end{array}
$$

### 5.2.5 Higher-Order-Modes

## - Higher order modes

- Introduction: HOMs
- HOM excitation by a beam
- HOM damping schemes
- HOM damping examples and results


## HOM Excitation by a Bunch



The bunched beam excites higher-order-modes (HOMs) = wakefields $=$ electromagnetic fields in the cavity.

## Beam-Cavity Interaction

- Bunch traverses a cavity
- $\Rightarrow$ deposits electromagnetic energy, which is described as wakefields (time domain) or higher-order modes (HOMs, frequency domain)
- Subsequent bunches are affected by these fields and at high beam current one must consider instabilities

from S. Belomestnykh


## Single Bunch Monopole Losses: Wake Potential of a Point-Charge



- When a charge passes through a cavity, it excites HOMs.
- If it passes exactly an axis, it will only excite monopole modes.
- For a point charge, the HOM excitation depends only on the bunch charge and the cavity shape.
- The excited field can be described by the wake potential.



## Monopole, Dipole and Quadrupole Modes...



Figure 5: One mid-cell of a TESLA cavity. The electric field of the 1.3 GHz accelerating $\pi$-mode is shown. The left graph shows the electric field in a plane perpendicular to the cavity axis.



Dipole


Figure 6: One mid cell of a TESLA cavity. The electric field of the 1.79 GHz $\pi$-mode of the first dipole passband is shown. The left graph shows the electric field in a plane perpendicular to the cavity axis.

$$
\left.\begin{array}{rlr}
\widetilde{\boldsymbol{E}}(r, \phi, z)=\sum_{m} & \left(\widetilde{E_{r}^{(m)}}(r, z) \cos (m \phi)\right. & e_{r} \\
& +\widetilde{E_{\phi}^{(m)}}(r, z) \sin (m \phi) & e_{\phi} \\
& +\widetilde{E_{z}^{(m)}}(r, z) \cos (m \phi) & e_{z}
\end{array}\right) .
$$

## Methods of HOM Calculations: Frequency Domain

Complex eigenvalue solution (becoming available, SLAC codes, ANSYS beta, HFSS) gives real and imaginary parts of impedance directly, hence $R$ and $Q$.


HFSS 3D complex Eigenvalue solution, 5-cell cavity with enlarged beam-pipes.

## Time-Domain Method (I)

Time domain (FFT) method (developed at SLAC, widely used, ABCI, MAFIA etc.)


3D MAFIA model of PEP-II cavity.




Short-, medium- and long-range wakes*.

## Time-Domain Method (II)



Calculation vs bead-pull measurements.


Measured vs calculated HOM spectrum.

Method uses open boundaries on ports. FFT of long-range wake gives broad-band impedance spectrum in one run. Works best for strong coupling ( $\beta>1$ ). Frequency resolution set by wake length, max frequency set by mesh size (typ. $\sim 10 \mathrm{GHz}$ ).

## HOMs

## - Higher order modes

- Introduction: HOMs
- HOM excitation by a beam
- HOM damping schemes
- HOM damping examples and results


## HOM Excitation

The excited HOM power of a single bunch depends on:
$>$ the HOMs of the cavity (i.e. their shunt impedance),
$>$ the bunch charge $\left(\mathrm{P}_{\mathrm{HOM}} \propto \mathrm{q}_{\mathrm{b}}{ }^{2}\right)$,
$>$ the bunch length (i.e. the spectrum of a bunch).

$\Rightarrow$ Short bunches excite
very high frequency modes!

## Single Bunch Monopole Losses: The Bunch

Longitudinal charge distribution for a 600 um bunch:


Spectrum of a 600 um bunch:


## Beam-cavity interaction: Wave Function



Figure 8: A point charge $q_{1}$ traversing a cavity with an offset $r_{1}$ followed by a test charge $q_{2}$ with offset $r_{2}$.
Lorentz-Forces on test charge: $\quad \boldsymbol{F}=\frac{d \boldsymbol{p}}{d t}=q_{2}\left(\boldsymbol{E}+c \boldsymbol{e}_{\boldsymbol{z}} \times \boldsymbol{B}\right)$.

The integrated field seen by a test particle traveling on the same path at a constant distance $s$ behind a point charge $q$ is the longitudinal wake (Green) function $w(s)$.



Single Bunch Monopole Losses:
Wake Potential of a Point Charge after a TESLA Cavity

The fft of the wake function gives the cavity impedance $\mathrm{Z}(\omega)$ :


Single Bunch Monopole Losses:
Wake Potential of a Bunch after a TESLA Cavity


The wake potential $W$ is a convolution of the linear bunch charge density distribution $q(s)$ and the wake function $w$

## Single Bunch Monopole Losses:

Loss Factor

Once the longitudinal wake potential is known, the longitudinal loss factor, which tells us how much electromagnetic energy a bunch leaves behind in a structure can be defined as:

$$
k=\frac{\Delta U}{q^{2}} \quad k_{\|}=\int_{-\infty}^{\infty} \boldsymbol{q}(s) W(s) d s
$$

Average power loss:

$$
P_{\| \|}=k_{\| \|} Q_{\text {bunch }} I_{\text {beam }}
$$

> This is the total energy lost by a bunch divided by the time separation of two consecutive bunches.

- This does not include any interaction between bunches (i.e. resonant mode excitation)!!!


## Single Bunch Monopole Losses:

HOM Power Frequency Distribution

The frequency distribution of the HOM losses is determined by the bunch spectrum and the cavity impedance $\mathrm{Z}(\omega)$ :

$$
P(\omega) \propto Z(\omega)[\widetilde{q}(\omega)]^{2}
$$



## High current and short bunches



## Average HOM Power Examples



## Bunch Trains


> The HOMs excited by a bunch are decaying due to losses,
> but: still significant field present in the cavity when the next bunch enters the cavity!
$\nu \Rightarrow$ Resonant excitation of a HOM, if

$$
f_{H O M} \approx N \frac{1}{T_{b}}
$$

## HOM Excitation

The excited HOM power of a bunch train depends on:
the HOM losses of a single bunch,
$>$ the beam harmonic frequencies and the HOM frequencies (resonant excitation is possible!),
$>$ the bunch charge and the beam current $\left(\mathrm{P}_{\mathrm{HOM}} \propto \mathrm{QI}\right)$,
and the external quality factor, $\mathrm{Q}_{\mathrm{ext}}$ of the modes.
Lower $\mathrm{Q}_{\text {ext }}$ means less energy deposited by the beam:

$$
\mathrm{P}_{\mathrm{HOM}} \propto \mathrm{Q}_{\mathrm{ext}}
$$

## Bunch Trains and HOM Power

In average the total HOM losses per cavity are given by the single bunch losses ( 77 pC bunch charge, 2.6 GHz bunch repetition rate, $\sigma_{\mathrm{b}}=600 \mu \mathrm{~m}$ ):

$$
P_{\| l}=k_{\|} Q_{\text {bunch }} I_{\text {beam }}=10.4 \mathrm{~V} / \mathrm{pC} \cdot 77 \mathrm{pC} \cdot 0.2 \mathrm{~A}=160 \mathrm{~W}
$$

But: If a monopole mode is excited on resonance, the loss for this mode can be much higher:

$$
P=\left(\frac{R}{Q}\right) Q I_{\text {beam }}^{2}
$$

Example: To stay below 200 W :

- achieve (R/Q)Q < 5000,
- or avoid resonant excitation of the mode.


## Bunch Trains and Beam Harmonics

## Example: Cornell ERL:

$$
\begin{aligned}
f_{\text {HOM }} & =N \cdot 1.3 \mathrm{GHz} \text { in the injector } \\
f_{H O M} & =N \cdot 2.6 \mathrm{GHz} \text { in the main linac }
\end{aligned}
$$

... so most of the monopole modes in the ERL will not be excited resonantly.


## Bunch Trains: HOM Frequencies Spread

Can one design the HOM frequencies such, that non of the modes are excited resonantly?
> The higher the frequency, the more sensitive is the frequency of a HOM to small perturbations in the cavity shape:

$$
\text { Simple approximation: } \quad \frac{\Delta f_{\text {HOM }}}{f_{H O M}}=\text { const } \text {. }
$$

> How large is "const"? Example: 2.4 GHz modes at TTF

$\sigma_{f}=10 \mathrm{MHz}$
$\Rightarrow$ const $=0.4 \%$
i.e. $\sigma_{f}=20 \mathrm{MHz}$ at 5.2 GHz
$\sigma_{f}=31 \mathrm{MHz}$ at 7.8 GHz
$\sigma_{f}=42 \mathrm{MHz}$ at 10.4 GHz

Matthias Liepe, P4456/7656, Spring 2010, Cornell University

Bunch Trains:
A Simple Model: 10000 Monopoles with random f's



Bunch Trains:
A Simple Model: 10000 Monopoles with random f's



Bunch Trains:
A Simple Model: 10000 Monopoles with random f's


A Simple Model: 1000 Monopoles with random f's Total HOM Monopole Power for random Sets of Frequencies


## Higher-Order-Modes (HOMs)

Parasitic modes excited by the accelerated beam may lead to:
$>$ degradation of the beam quality (transverse emittance growth due to dipole modes, BBU, energy spread),
$>$ additional cryo-losses (wall losses, heating of cables and feedthroughs), mostly due to monopole modes.

$$
\Rightarrow \frac{\text { Requirements on the external quality factor, }}{\underline{Q_{\text {ext }}} \text { of the modes. }}
$$

Without additional damping the HOMs can have very high quality factors $\left(Q>10^{10}\right)$ !

