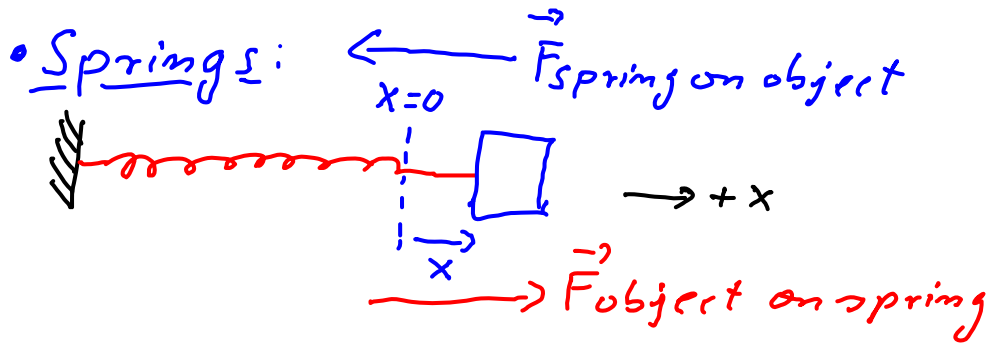


## Recap

## Lecture 12



Hooke's "law" for an ideal spring:

$$\vec{F}_{\text{spring on obj.}} = -\vec{k}x$$

restoring force

- Parallel springs:  $k_{\text{eff}} = k_1 + k_2 + k_3 + \dots$   $F_{\text{obj. on spring}} = +Kx$

- Series of springs:  $\frac{1}{k_{\text{eff}}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \dots$

### • Solid on Solid Friction:

- Force  $\parallel$  to interface between two surfaces that opposes relative motion

Case ①: Static friction force  $\vec{f}_s$ :

- no relative motion of surfaces in contact

- self-adjusts to prevent relative motion, up

to maximum:  $(f_s)_{\text{max}} = \mu_s N$

## Recap

⇒ static friction:

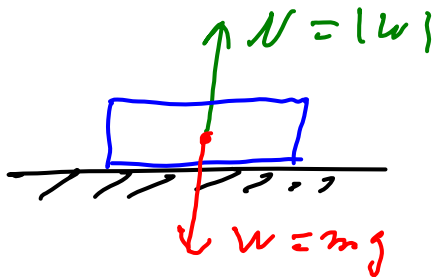
$$f_s \leq (f_s)_{\max} = \mu_s N$$

$\uparrow$  self-adjust                       $\uparrow$  coefficient of static friction

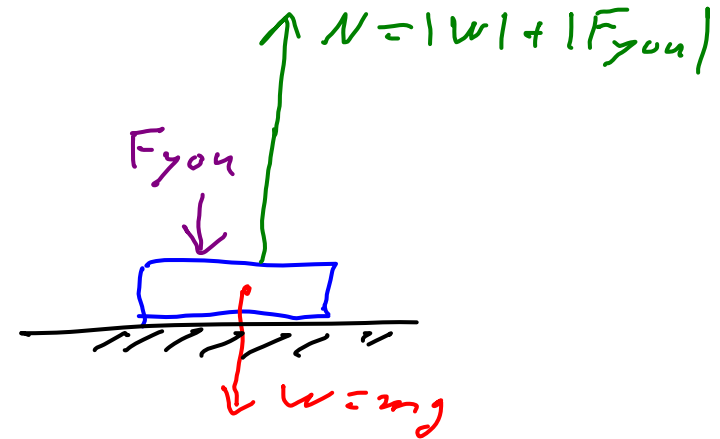
← normal force, = force  $\perp$  to surface, pressing surfaces together

Note:  $|N|$  is not necessarily equal to  $|W|$ !

Example:



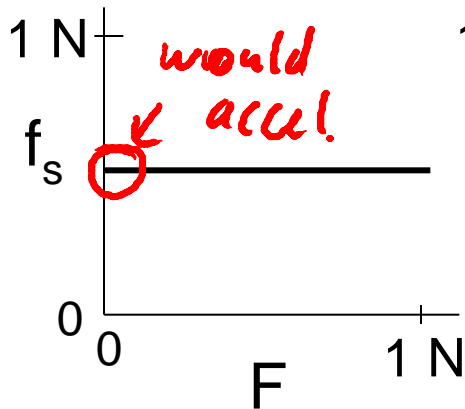
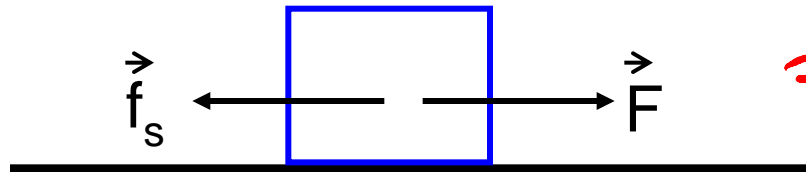
$$\leftarrow \sum \vec{F} = 0 \rightarrow$$



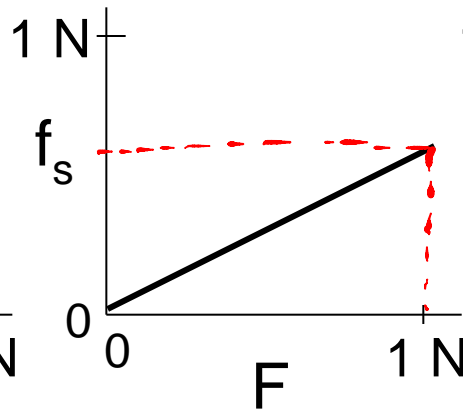
As the applied force  $F$  is increased, how does the static friction force  $f_s$  vary? (Assume  $F$  is less than that needed to make the block move.)

$\vec{a} = 0 \Rightarrow \sum \vec{F} = 0$  here  
 $\Rightarrow F = f_s$   
 self-adjusts!

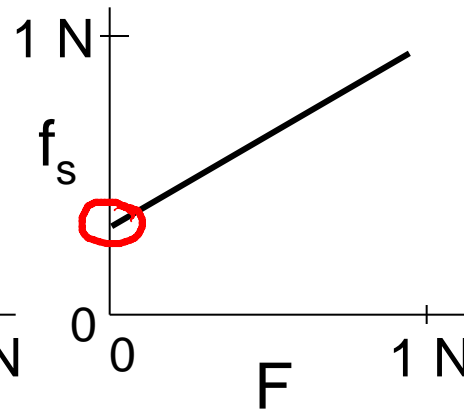
Note: If  $F > (f_s)_{max}$ , surfaces will begin to slip!



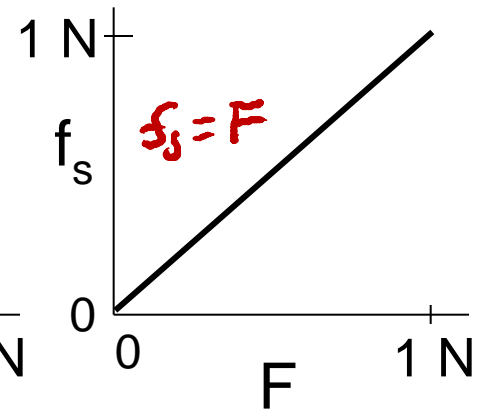
~~(A)~~



~~(B)~~



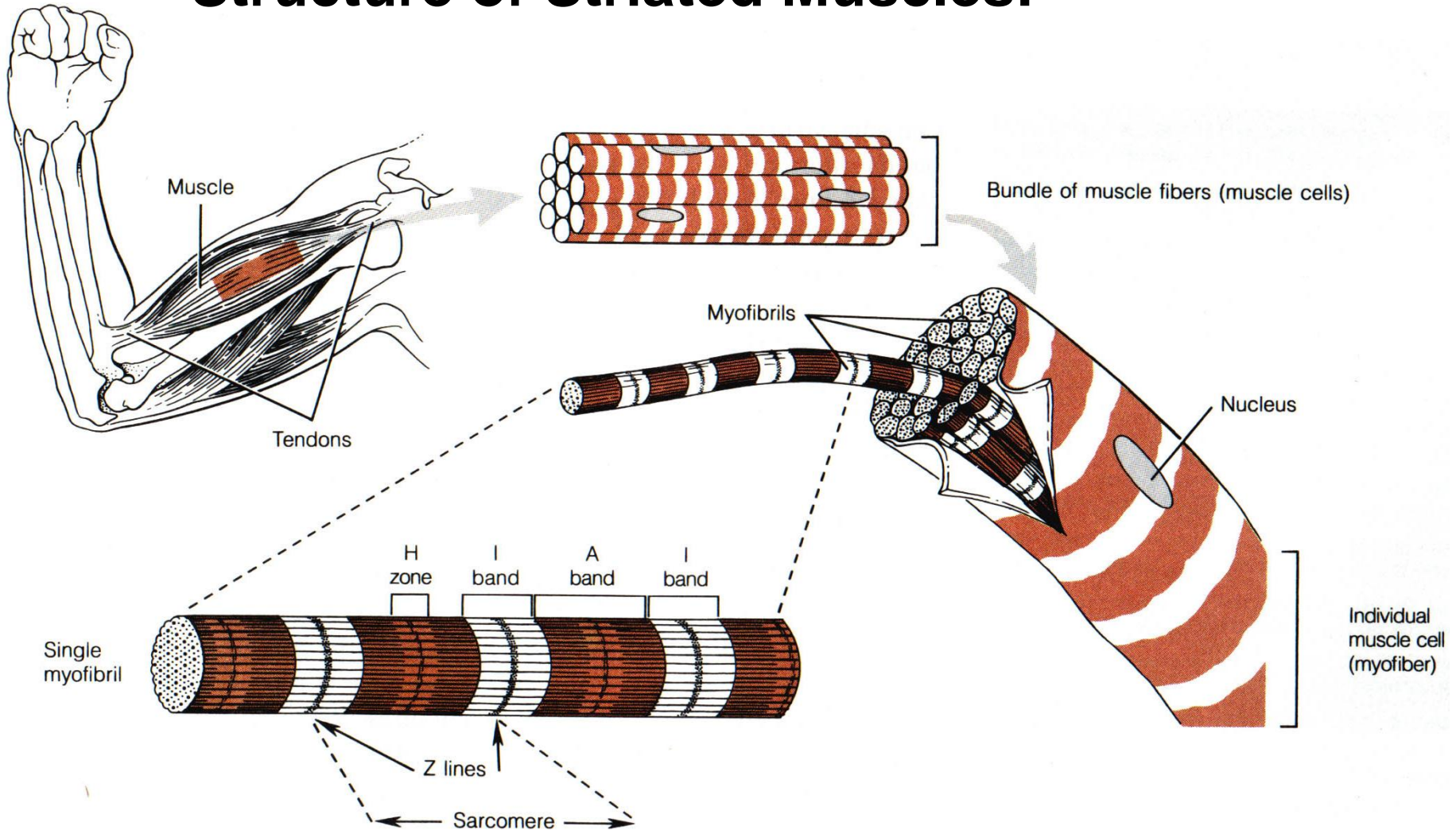
~~(C)~~



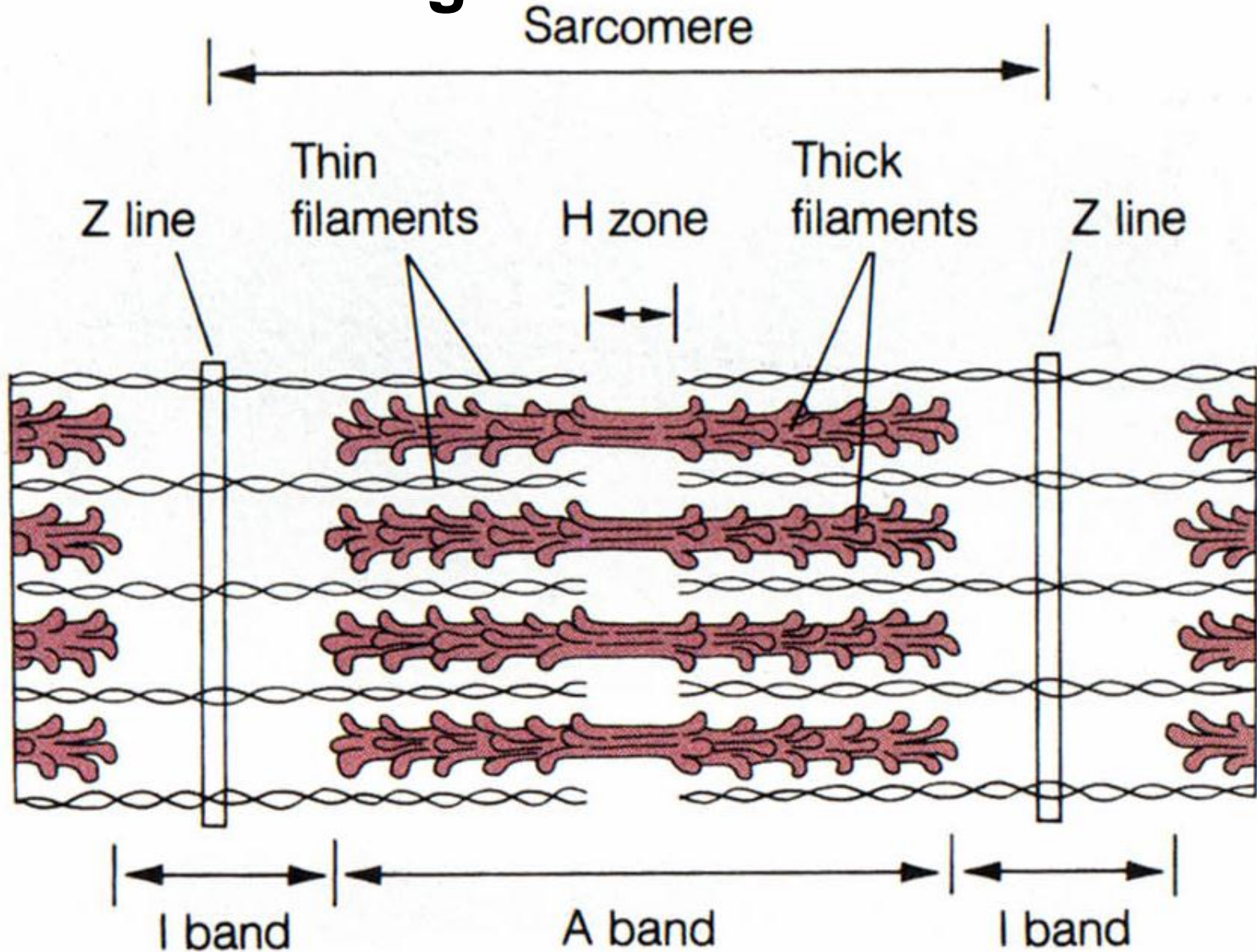
(D)

# Series and Parallel Combinations in Muscle

## Structure of Striated Muscles:

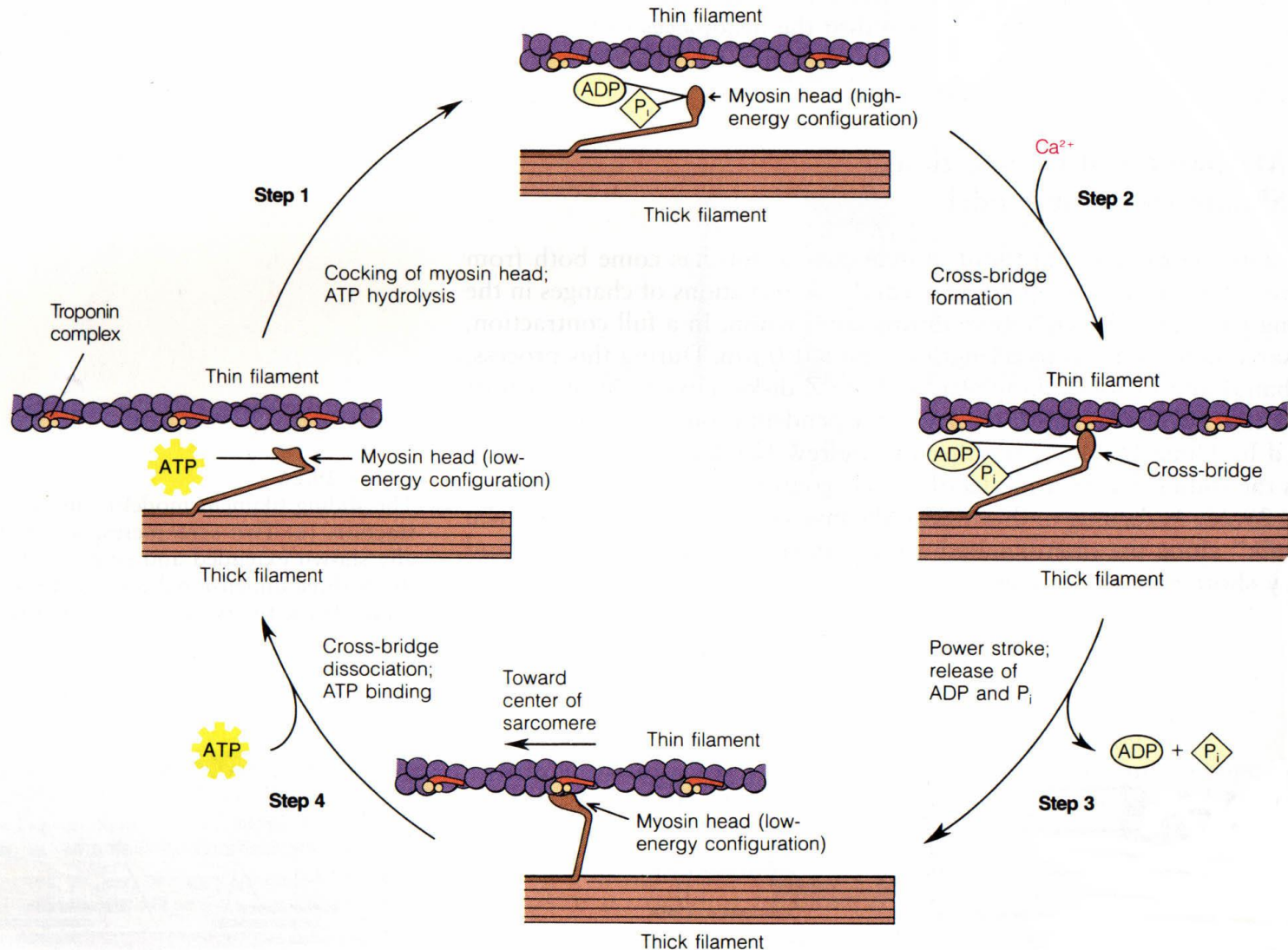


# Basic Building Block: The Sarcomere





# Microscopic mechanism of muscle force production and contraction:



## Some Numbers:

### Sarcomere length:

fully extended: 2.3  $\mu\text{m}$

fully contracted: 1.0  $\mu\text{m}$

**Force per myosin head:**  $\sim 5 \text{ pN} = 5 \times 10^{-12} \text{ N}$ .

# How to get a big motion/contraction?

- Connect M sarcomeres together *in series*:

$$\Delta x_{\text{tot}} = M \times (\Delta x \text{ of one sarcomere})$$

**Muscle contracts by ~40% of its length.**



## How to get a big motion/contraction?

- Connect M sarcomeres together *in series*:

$$\Delta x_{\text{tot}} = M \times (\Delta x \text{ of one sarcomere})$$

Muscle contracts by ~40% of its length.

## How to get a big force?

- Connect N actin filaments together *in parallel*:

$$F_{\text{tot}} = N \times \# \text{ of parallel filaments}$$

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## How to get a big force?

- Connect N actin filaments together *in parallel*:

$$F_{\text{tot}} = N \times \# \text{ of parallel filaments}$$

For typical striated muscle,

$F_{\text{max}} \approx 30 \text{ N}$  for each  $\text{cm}^2$  of muscle cross-sectional area.

# Today:

- Kinetic friction
- What force acting on a car produces its acceleration?



case ②: Kinetic friction force  $\vec{F}_k$  ("sliding" friction)

- relative motion of surfaces in contact (sliding)
- Kinetic friction  $F_k \sim$  independent of  $F$ ,  $a$ , relative velocity of surfaces
- good model:

$$F_k = \mu_k \cdot N$$

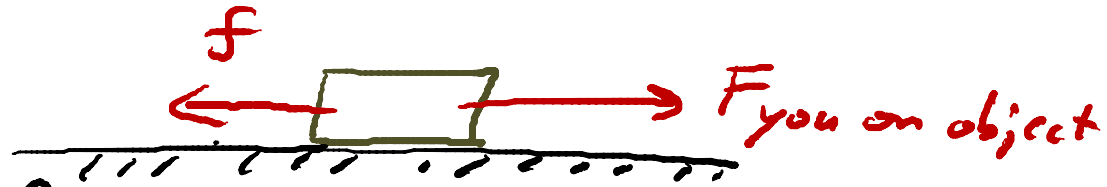
↑  
coefficient of  
kinetic friction

← magnitude of  
normal force,  
pressing  
surfaces together

in general:  $\mu_k < \mu_s$

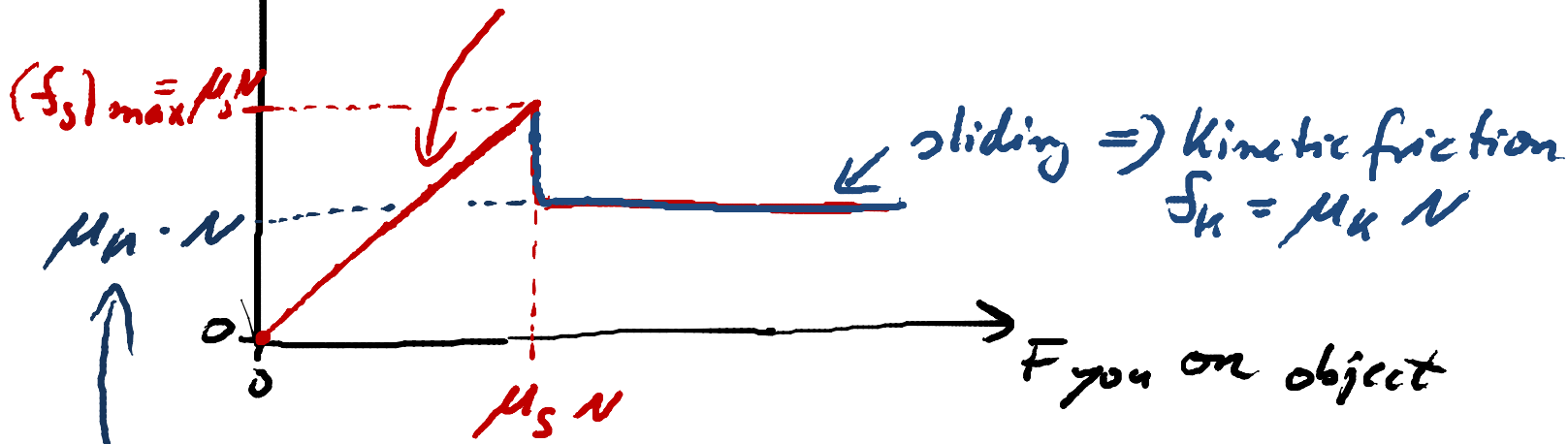
- friction force always opposes relative motion, but can act opposite to the direction of motion or in direction of motion!

# Model for friction force $f$ :



friction force  $f$

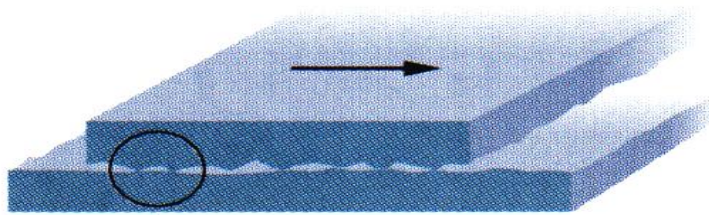
no relative motion  $\rightarrow$  static friction:  $f_s = F_{\text{you}}$



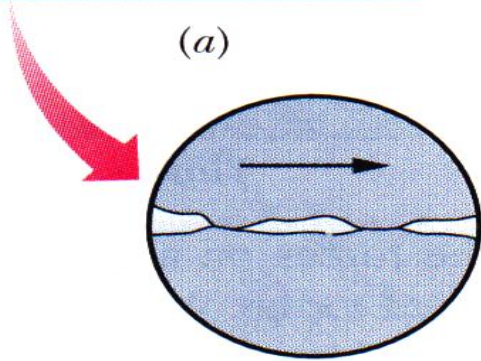
in general:  $\mu_k < \mu_s$

# Microscopic Origin of Friction:

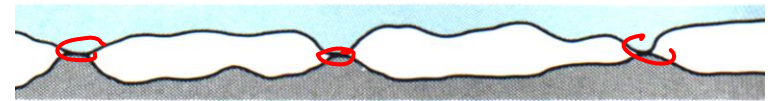
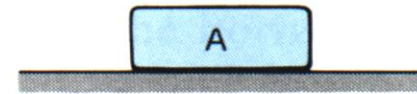
$$\left. \begin{aligned} (F_s)_{\max} &= \mu_s N \\ F_k &= \mu_k N \end{aligned} \right\} \begin{array}{l} \text{indep. of} \\ \text{surface area} \end{array}$$



(a)

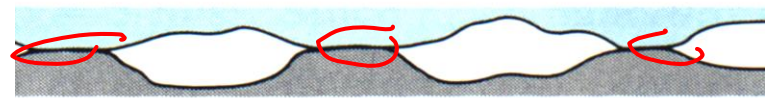
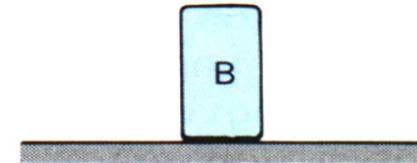


(b)



(a)

*Same friction force!*



(b)




## Coefficients of Friction<sup>a</sup>

	$\mu_s$	$\mu_k$
Steel on steel	0.74	0.57
Aluminum on steel	0.61	0.47
Copper on steel	0.53	0.36
Rubber on concrete	1.0	0.8
Wood on wood	0.25–0.5	0.2
Glass on glass	0.94	0.4
Waxed wood on wet snow	0.14	0.1
Waxed wood on dry snow	—	0.04
Metal on metal (lubricated)	0.15	0.06
Ice on ice	0.1	0.03
Teflon on Teflon	0.04	0.04
Synovial joints in humans	0.01	0.003

<sup>a</sup> All values are approximate.

general:  $\mu_s > \mu_k$

A car accelerates on a level road. What force acting on the car produces its acceleration?

$$\vec{F}_{\text{acting on car}} = m \vec{a}_{\text{car}}$$


in direction of motion!  
internal force



no relative motion  
of road-tire surfaces  
in contact  
 $\Rightarrow$  static friction!

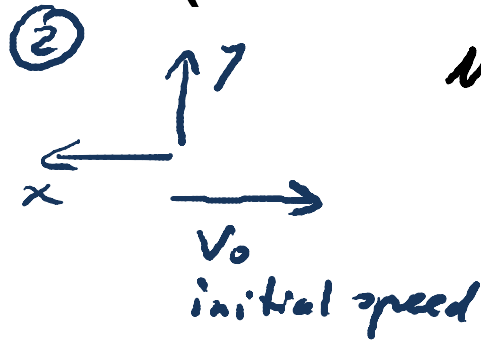
need force  
on car

- A. The force of the engine on the wheels
- B. The static friction force of the tires on the road
- C. The static friction force of the road on the tires
- D. The kinetic friction force of the tires on the road
- E. The kinetic friction force of the road on the tires

The coefficients of static and kinetic friction between tires and the road don't vary significantly with the mass of a car.

How does the stopping distance  $\Delta x$  of a car with mass  $m$  compare with that of a car with mass  $2m$ ?

(Assume the cars have the same initial speed.)



$$\begin{aligned} \text{NII: } \Sigma F_y &= m a_y = 0 \\ &= N - W = m g \\ \Rightarrow N &= W = m g \end{aligned}$$

$$\Sigma F_x = m a_x = f_s \leq (f_s)_{\max}$$

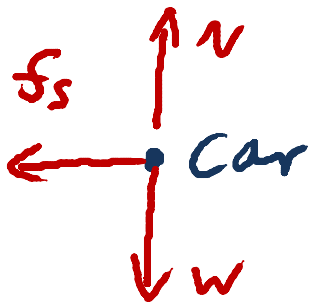
$$\Rightarrow (a_x)_{\max} = \frac{(f_s)_{\max}}{m}$$

$$= \frac{\mu_s N}{m} = \frac{\mu_s m g}{m}$$

$$\Rightarrow (a_x)_{\max} = \mu_s g \quad \text{indep. of mass}$$

$\Rightarrow \Delta x$  indep. of  $m_{\text{car}}$ !

③ FBD:



$\Delta x (2m) / \Delta x (m) = ?$

A. 1/4

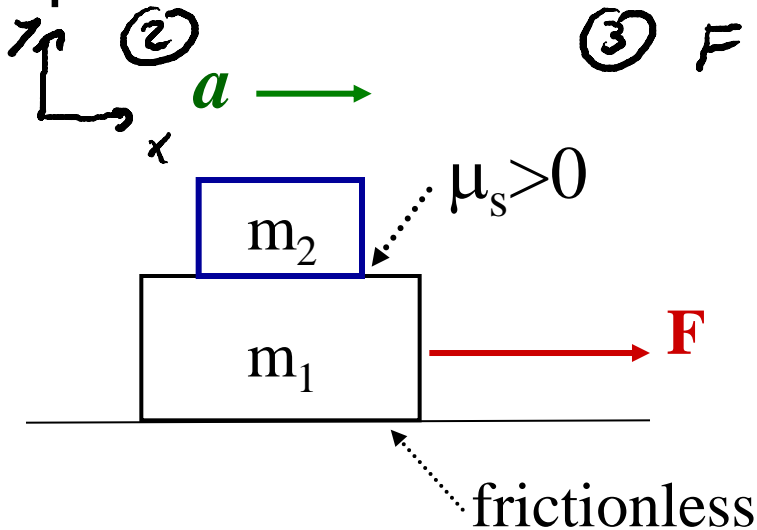
B. 1/2

**C. 1**

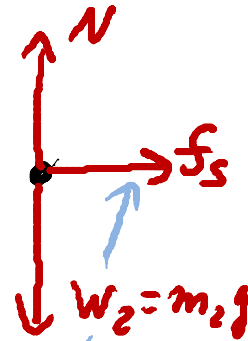
~~D. 2~~

E. 4

What is the maximum acceleration  $a$  that  $F$  can produce and still have the two blocks move together?



③ FBD of  $m_2$ :



friction force in direction of motion!

$a_{\max} = ?$

- A.  $g$
- ~~B.  $\mu_s m_2 g$  units!~~
- ~~C.  $\mu_s (m_1 + m_2) g$~~
- D.  $\mu_s g$**
- E. Insufficient information

$\Sigma F_y = m_2 a_y$

$= 0 = N - W_2 \Rightarrow N = W_2 = m_2 g$

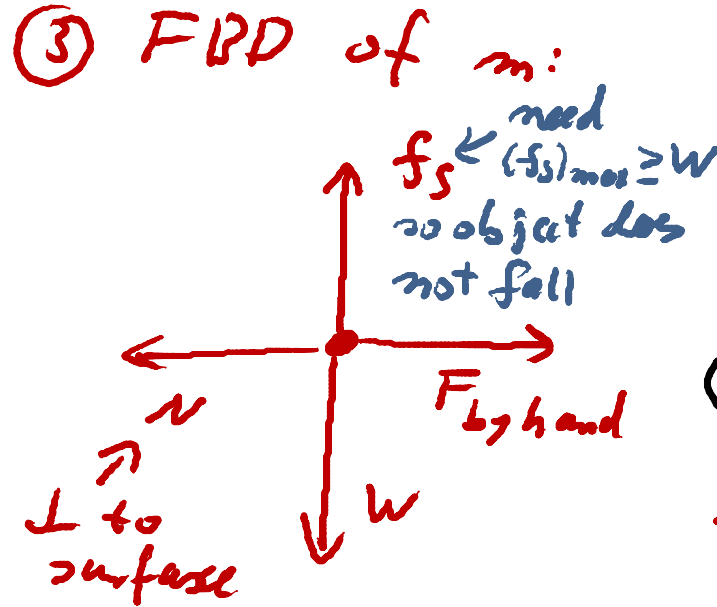
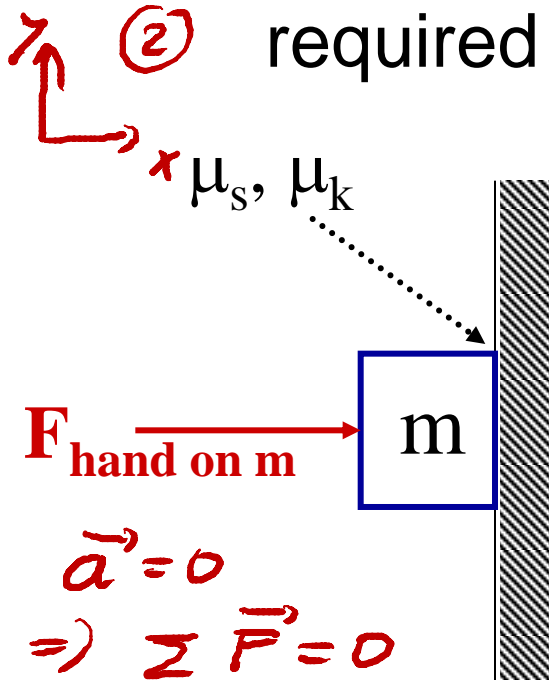
$\Sigma F_x = m_2 a_x = m_2 a = f_s \leq (f_s)_{\max}$

$\Rightarrow m_2 (a)_{\max} = (f_s)_{\max} = \mu_s N = \mu_s m_2 g$

$\Rightarrow \boxed{(a)_{\max} = \mu_s g}$

static friction force on  $m_2$  generates accel of  $m_2$

What is the minimum magnitude of  $F_{hand\ on\ m}$  required so that the block doesn't fall?



- $F_{min} = ?$
- A.  $mg$
  - B.  $\mu_s mg$
  - C.  $mg / \mu_s$**
  - ~~D.  $\mu_k mg$~~
  - ~~E.  $mg / \mu_k$~~

$$\sum F_x = m a_x = 0 = F_{hand} - N$$

$$\Rightarrow N = F_{hand\ on\ m}$$

$$\sum F_y = m a_y = 0 = f_s - W = f_s - mg$$

$$\Rightarrow mg = f_s \leq (f_s)_{max} = \mu_s N = \mu_s \cdot F_{by\ hand}$$

$$\Rightarrow F_{min} = mg / \mu_s$$