**Recap**

- **Forces in uniform circular motion:**
  \[ \vec{F}_{\text{on object}} = m \vec{a}^2 \quad \text{with} \quad 1 \vec{a}^2 = \frac{v^2}{r} \]
  and \( \vec{F}_{\text{on object}} \) and \( \vec{a}^2 \) point to center of circle, \( \perp \) to path.

- **Kinetic Energy:** \( K = \frac{1}{2} m v^2 \)

- **Work:** energy transferred to or from an object by force.
  For a constant force: \( W = \vec{F} \cdot \vec{d} = F d \cos \phi = F_{\parallel} d = F d_{\parallel} \),
  "dot" product of \( \vec{F}_{\parallel} \) to \( \vec{d} \) component of \( \vec{d} \parallel \) to \( \vec{F} \).
Today:

• Work and Energy:
  – Work done by a single force
  – Work-kinetic energy theorem
  – Work done by gravity, friction…
A ball of mass \( m \) is swung at the end of the rope in a horizontal circle. Its speed \( v \) is constant and the length of the rope is \( L \). Note: \( v = \text{const} \Rightarrow J^2 = \text{const} \Rightarrow \text{no work done} \)

What work \( W \) is done by the tension \( T \) in the rope when the mass moves a small distance \( s \) along the circle?

\[ W = ? \]

A. \( mg \, s \)
B. \( T \, s \)
C. \( T \, L \)
D. 0
Work done by a single, constant force on an object:

\[ \vec{F} \] acts on \[ \vec{d} \]

\( \vec{d} \): Displacement vector
\( \phi \): Angle between \( \vec{F} \) and \( \vec{d} \)

\[ \vec{d} = \vec{r}_2 - \vec{r}_1 \]

Work \( W \) is given by:

\[ W = \vec{F} \cdot \vec{d} \cos \phi = F_{\parallel \vec{d}} \cdot d = F \cdot d_{\parallel \vec{F}} \]

Note: if \( \phi = 90^\circ \), then \( W = 0 \) by force on object.
A 10 kg crate is pulled 10 m along a frictionless horizontal floor by a force $F_{\text{pull}} = 10 \text{ N}$ applied at an angle of 10° with respect to the horizontal, as shown. What is the net work done on the crate by all the forces that act on it?

$W_{\text{net}} = \ ?$

A. 100 $\cos 10^\circ$ J
B. 100 $\sin 10^\circ$ J
C. 1000 J + 100 $\sin 10^\circ$ J
D. 1000 J + 100 $\sin 10^\circ$ J

**Solution:**

1. **W = F d cos θ**

2. For $F_{\text{pull}}$:
   
   $W_{\text{pull}} = 10 \text{ N} \cdot 10 \text{ m} \cdot \cos 10^\circ = 100 \text{ J} \cdot \cos 10^\circ$

3. For $N$:
   
   $W_N = N d \cos 90^\circ = 0$

4. For weight $W_w$:
   
   $W_w = 0$

5. **Sum:** $W_{\text{net}} = 100 \text{ J} \cdot \cos 10^\circ + 0 + 0$

   - $W_{\text{net}} = 100 \cos 10^\circ \text{ J}$

   Circle A.
• Work done by a single constant force on an object

\[ W_{\text{by force on object}} = F \cdot d \cos \phi = F_{\parallel} d = F d_{\parallel} \]

Mathematical shorthand:

\[ W = \overrightarrow{F} \cdot \overrightarrow{d} \]

"dot" product of two vectors

\[ = F d \cos \phi \]

\[ \angle \text{between the two vectors} \]

Use this if you know components of vectors

\[ = F_x d_x + F_y d_y \]

\[ \text{Note: components can be < 0!} \]

Work for any coordinate system
• Work done by multiple forces (constant) acting on an object:

\[ W_{\text{net}} = W_1 + W_2 + \ldots = F_1 \cdot d + F_2 \cdot d + \ldots \]

\[ = (\sum \vec{F}) \cdot \vec{d} = \vec{F}_{\text{net}} \cdot \vec{d} = F_{\text{net}} \cdot d \cos \phi \]

[Diagram: \( \vec{F}_{\text{net}} \) and \( \vec{d} \) with angle \( \phi \).]
**Work - Kinetic Energy Theorem:**

- Work: Energy transferred to or from an object by force

- Kinetic energy: \( J_k = \frac{1}{2} m v^2 \)

\[ \Delta J_k \text{ of object} = J_{k_f} - J_{k_i} = \text{Work net on object by all forces that act on the object while it moves from some initial to some final position} \]

\[ \left( \frac{\text{change in kinetic energy of object}}{\text{energy of object}} \right) = \left( \frac{\text{net work done on object by forces}}{\text{on object by forces}} \right) = \Delta J_k \]

Note: \( \Delta J_k < 0 \) or \( \Delta J_k > 0 \) possible!
Check Work-Kinetic Energy theorem:

\[ \Delta K = K_f - K_i = \frac{1}{2} m v_{f,x}^2 - \frac{1}{2} m v_{i,x}^2 \]

\[ = W_{net} = W_N + W_{weight} + W_{pull} \]

\[ = 0 \quad \text{here} \]

\[ = \rightarrow F_{pull} \cdot \rightarrow d = F_{pull,x} d \]

\[ \rightarrow d \parallel \text{along} \, x \]

Use NEW: \[ \sum F_x = m a_x = F_{pull,x} \]

\[ \Rightarrow \frac{1}{2} m (v_{f,x}^2 - v_{i,x}^2) = F_{pull,x} d = m a_x d \]

\[ \Rightarrow v_{f,x}^2 - v_{i,x}^2 = 2 a_x d = 2 a_x \Delta x \]

\[ \int \text{nothing new...} \]

\[ \Rightarrow \text{same as Newton's laws, but work/kinetic} \]

\[ \text{helps to solve some problems much easier than} \]

\[ \text{Newton's Laws!} \]
A car of mass $m$ traveling at a speed $v_i$ is braked to a stop by a constant force $F$.

What is the stopping distance $d$ of the car?

(Use energy/work concepts to solve, not NII.)

\[ d = \frac{mv_i^2}{2F} \]

A. $\frac{F}{v_i}$  
B. $\frac{mv_i}{F}$  
C. $\frac{mv_i^2}{2F}$  
D. $\frac{mv_i^2}{F}$  
E. $\frac{2F}{mv_i^2}$
Work done by specific forces:

1. Work done by gravity:
   - \(\uparrow + Y < \text{important?}\)
   - \(\Delta Y < 0\)
   - \(W = m\vec{g}\)
   - \(-m g \Delta y < 0 \text{ for down motion}\)
   - \(W_{\text{by gravity}} = F \cdot d \cos \phi = F d\)
   - \(-m g (y_f - y_i)\)
   - \(-m g (\text{vertical displacement})\)
   - \(\uparrow \text{ for } +Y \text{ up!}\)

\(\downarrow + Y > 0\)
\(\downarrow + Y < 0\)