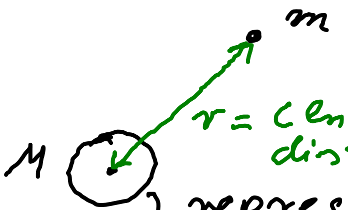
Recap: Gravity — Lecture 22

- Newton's Law of Gravitation:

$$|F_g(r)| = G\,\frac{Mm}{r^2}$$



r = center-to-center distance

represent uniform, spherical objects by point mass at objects center

$$\Rightarrow U_g(r) = -G\,\frac{Mm}{r} \quad \text{gravitational potential energy}$$

$$U_g(r\to\infty) = 0$$

- acceleration due to gravity:

$$g(r) = G\,\frac{M}{r^2} = \frac{F_g(r)}{m} \quad \text{assuming that there is no other force acting on obj.}$$

$\Rightarrow$ at Earth's surface: $g(r_{earth}) = G\,\dfrac{M_E}{r_E^2} = 9.8\ \mathrm{m/s^2}$

- satellite motion/orbital motion:

$$v_{orb} = \sqrt{\frac{GM}{r_{orb}}}$$

uniform circular motion under influence of gravity $\rightarrow F_g(r) = m\,\dfrac{v^2}{r}$

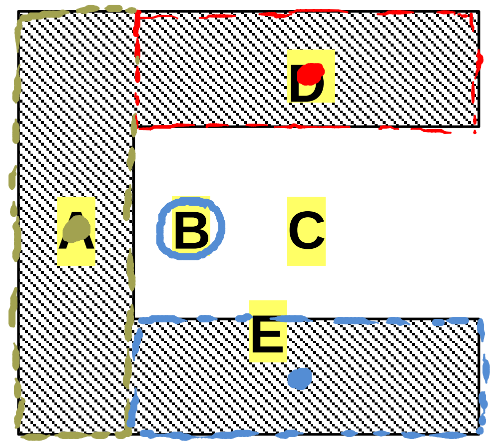
- escape speed:

$$v_{esc} = \sqrt{\frac{2GM}{r_p}}$$

minimum speed object needs to have at planet's surface ($r = r_p$) to reach $r \to \infty$ (so that $E_{mech} = 0$)

- Note: Use $F_g = mg$ and $\Delta U_g = mg\,\Delta y$ only for objects near Earth's surface!

Where is the center of mass?



→ break into small pieces whose COM you can "guess"

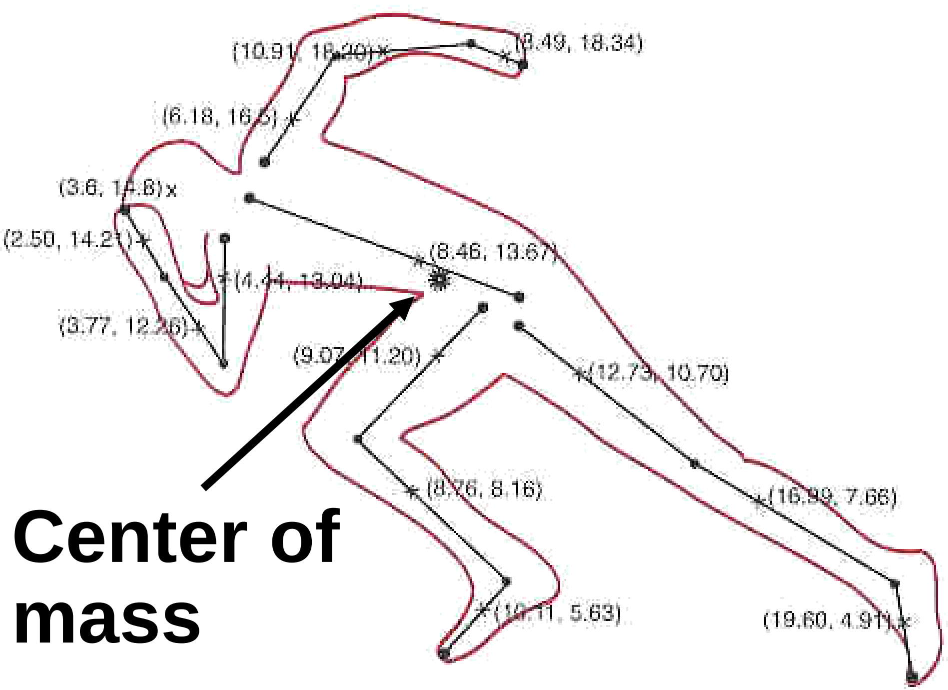
→ replace each piece by point mass at COM locations

→ evaluate COM for collection of point mass

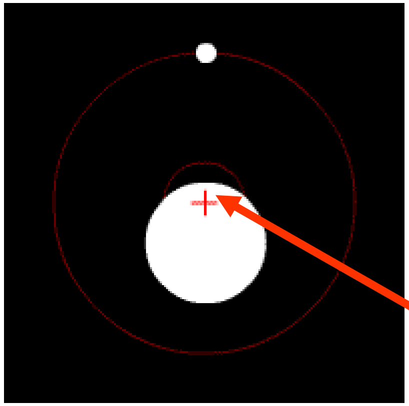
Note: COM point need not to be inside the object!

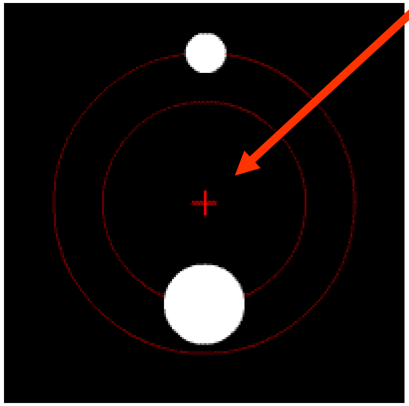
Today:

- Center of Mass
- Momentum



Center of mass

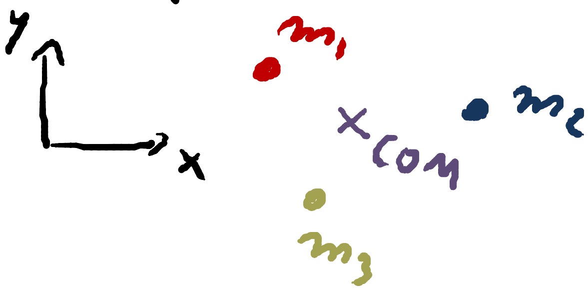


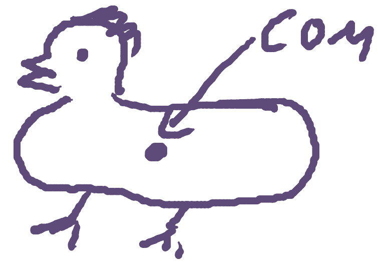


Center of mass

Center of Mass COM

Consider a collection of (point) particles or an object with distributed mass:





$\Rightarrow$ Where is the COM?

$$x_{com} = \frac{m_1 x_1 + m_2 x_2 + \dots}{m_1 + m_2 + \dots} = \frac{\sum_i m_i x_i}{m_{total}}$$

$$y_{com} = \frac{m_1 y_1 + m_2 y_2 + \dots}{m_1 + m_2 + \dots} = \frac{\sum_i m_i y_i}{m_{total}}$$

in vector notation:

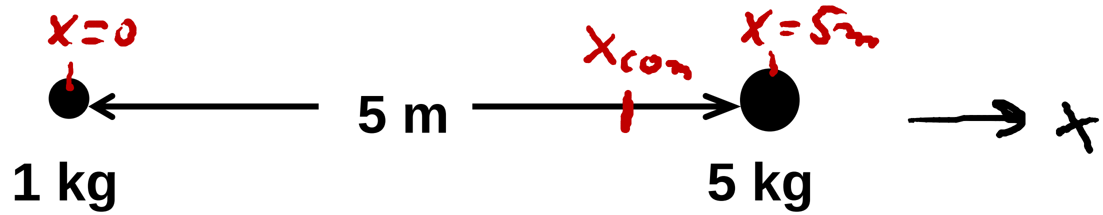
$$\vec{r}_{com} = \frac{\sum_i m_i \cdot \vec{r}_i}{m_{total}}$$

Solid object: break into small mass elements dm:

$$x_{com} = \frac{1}{m_{total}} \int x \, dm$$

$$y_{com} = \dots$$

Where is the center of mass?



A. between the masses, 2.5 m from each
B. between the masses, 1 m from the small mass
C. between the masses, 1 m from the large mass
D. between the masses, 0.8 m from the large mass
E. at the large mass

$$x_{com} = \frac{\sum_{i=1..2} m_i x_i}{m_{total}} = \frac{1\,kg \cdot 0\,m + 5\,kg \cdot 5\,m}{(1\,kg + 5\,kg)} = \frac{25}{6}\,m = \underline{4\tfrac{1}{6}\,m}$$

- Why is the COM useful?

$$\frac{d\vec{r}_i}{dt} = \vec{v}_i$$

$$\vec{r}_{com} = \frac{1}{m_{total}} \sum_i m_i \vec{r}_i \Rightarrow \vec{v}_{com} = \frac{d\vec{r}_{com}}{dt} = \frac{1}{m_{total}} \sum_i m_i \cdot \vec{v}_i$$

$$\frac{d\vec{v}_i}{dt} = \vec{a}_i$$

$$\Rightarrow \vec{a}_{com} = \frac{d\vec{v}_{com}}{dt} = \frac{1}{m_{total}} \sum_i m_i \vec{a}_i \leftarrow \text{acceleration of } i^{th} \text{ particle}$$

use NII for i^{th} particle:

$$\sum \vec{F}_{\substack{\text{on } i^{th} \\ \text{particle}}} = \underbrace{\vec{F}_{net,i}}_{\text{net force on } i^{th} \text{ particle}} = m_i \vec{a}_i$$

$$\Rightarrow \boxed{\sum_i \vec{F}_{net,i} = m_{total} \cdot \vec{a}_{com} = \vec{F}_{net,1} + \vec{F}_{net,2} + \ldots}$$

$$\sum_i \vec{F}_{net,i} = \vec{F}_{net,1} + \vec{F}_{net,2} + \ldots = \sum \vec{F}_{on\ particle\ 1} + \sum \vec{F}_{on\ particle\ 2} + \ldots$$

- forces on i^{th} particle:
  - forces from outside of system of particles (external forces)
  - forces due to other particles in the system (internal forces)

    $\Rightarrow$ internal force will be NIII interaction partner with force on other particles

$$\vec{F}_{1\,on\,2} = -\vec{F}_{2\,on\,1}$$

$\Rightarrow$ These pairs cancel out in sum $\sum_i \vec{F}_{net,i}$

$\Rightarrow$ only forces left are external forces

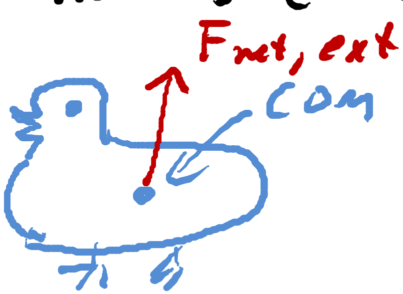
$$\Rightarrow \quad \sum_i \vec{F}_{net,i} = \sum \vec{F}_{ext\ on\ system}$$

$$\Rightarrow \boxed{\vec{F}_{net,ext} = \sum \vec{F}_{ext\ on\ system} = m_{total}\,\vec{a}_{COM}}$$

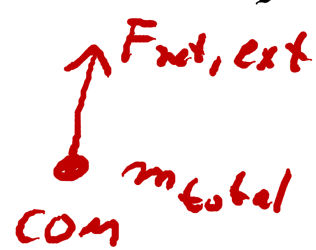
} NII for system of particles / composite object

$\Rightarrow$

- The COM point of an object or system of objects / particles moves translationally as though its mass is concentrated at $\vec{r}_{COM}$ and all <u>external</u> forces act there!
- Trajectory of COM determined by <u>net</u> external force only!
- Parts of system may individually undergo complicated motions (object might rotate, ...)



$\equiv$



} for motion of COM point

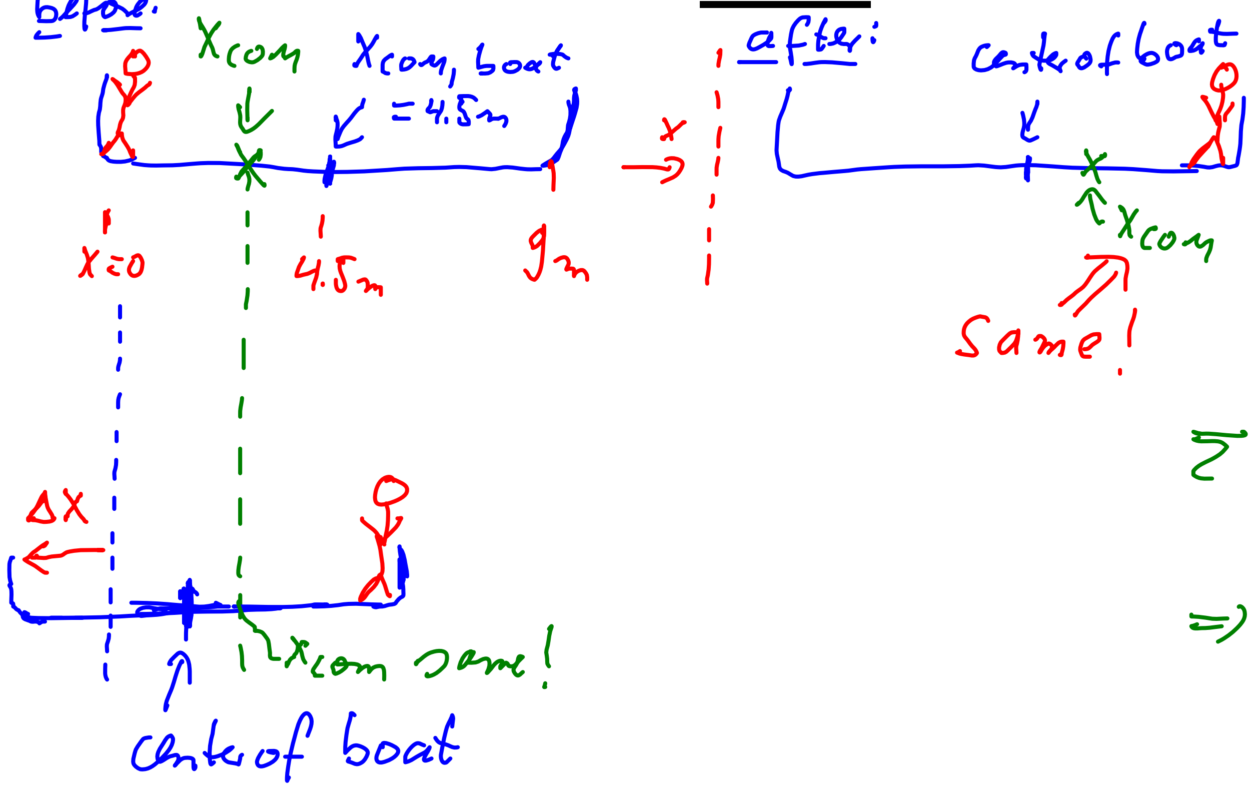
- Special case:

$$\boxed{\begin{array}{l} \text{if } \sum \vec{F}_{ext} = \vec{F}_{net,ext} = 0 \\ \Rightarrow \quad \vec{a}_{com} = 0 \quad \Rightarrow \vec{v}_{com} = \text{const} \end{array}} \left.\vphantom{\begin{array}{l} a \\ b \\ c \end{array}}\right\} \begin{array}{l} \text{NI for} \\ \text{system of} \\ \text{particles} \end{array}$$

$\Rightarrow$ if $\vec{v}_{com, initial} = 0 \Rightarrow \vec{r}_{com}$ is constant (stays at rest)

A 9 m long boat with a mass of 100 kg floats frictionlessly on the water. The boat is initially at rest. A sailor of mass 50 kg walks from the back to the front of the boat.

How far does the sailor move relative to the water?



A. 9 m

B. Less than 9 m

C. More than 9 m

$\sum \vec{F}_{ext} = 0 \Rightarrow \vec{V}_{com} = \text{const}$

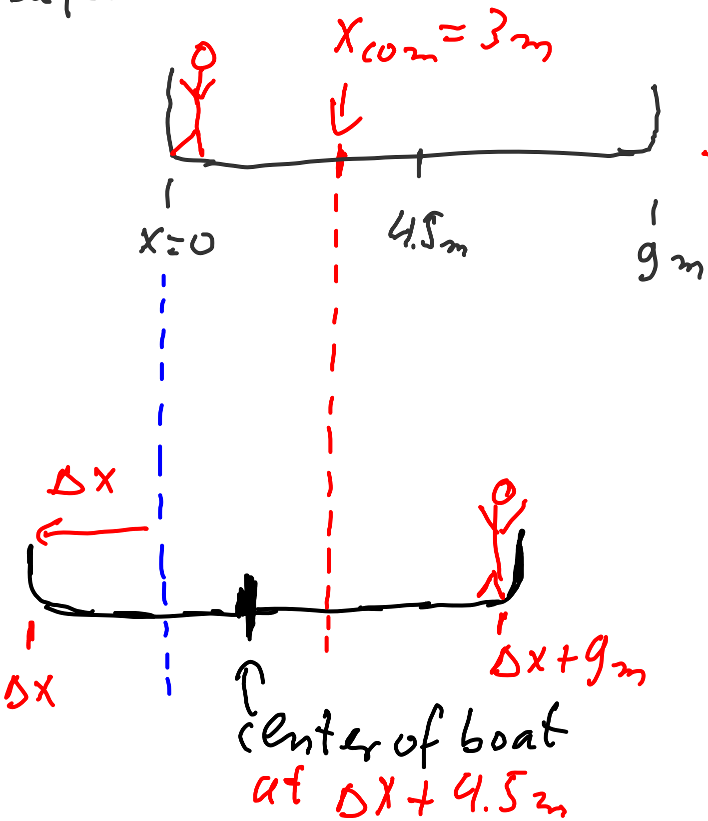
$= 0$ here

$\Rightarrow$ COM does not move!

A 9 m long boat with a mass of 100 kg floats frictionlessly on the water. The boat is initially at rest. A sailor of mass 50 kg walks from the back to the front of the boat.

How far does the **boat move relative to the water?**

before:



$|\Delta x| = ?$

A. 0 m

B. 3 m

C. 4.5 m

D. 6 m

$$x_{com,before} = \frac{50kg \cdot 0m + 100kg \cdot 4.5m}{50kg + 100kg} = \underline{\underline{3m}}$$

$$x_{com,after} = 3m$$

$$= \frac{50kg(\Delta x + 9m) + 100kg(\Delta x + 4.5m)}{50kg + 100kg}$$

solve for $\Delta x = -3m$ (moves to left)

Momentum:

$$\left(\begin{array}{c}\text{Linear Momentum}\\ \text{of a particle}\end{array}\right) = \boxed{\vec{p} \equiv m_{obj} \cdot \vec{v}_{obj}}$$

vector!

$$[p] = kg\,\frac{m}{s} = N \cdot s$$

NII: $\underline{\sum \vec{F}} = m\vec{a} = m\frac{d\vec{v}}{dt} = \underline{\frac{d\vec{p}}{dt}}$ (if $m = \text{const}$)

$$\Rightarrow \boxed{\sum \vec{F}_{\text{on object}} = \vec{F}_{net,obj} = \frac{d\vec{p}_{obj}}{dt} = \left(\begin{array}{c}\text{rate of}\\ \text{change of}\\ \text{momentum}\end{array}\right)}$$

$$\Rightarrow \boxed{\text{If } \vec{F}_{net,obj} = 0 \Rightarrow \vec{p}_{obj} = \text{const!}}$$

$$(\vec{a} = 0 \Rightarrow \vec{v} = \text{const} \Rightarrow \vec{p} = \text{const})$$