Collisions

Lecture 25

For any collision: $\vec{p}_{1,i} + \vec{p}_{2,i} = \vec{p}_{1,f} + \vec{p}_{2,f}$ } Momentum is conserved (if $\sum \vec{F}_{ext} = 0$)

Case A: Elastic collision: $\boxed{\sum K_i = \sum K_f}$ } kin. energy is also conserved

$\Rightarrow$ speed of separation = speed of approach

Case B: Inelastic collision: part of kinetic energy is lost

$\Rightarrow \boxed{\sum K_i > \sum K_f}$

special case: sticking collisions: $\boxed{V_{1,f} = V_{2,f}}$

have largest possible loss in kinetic energy

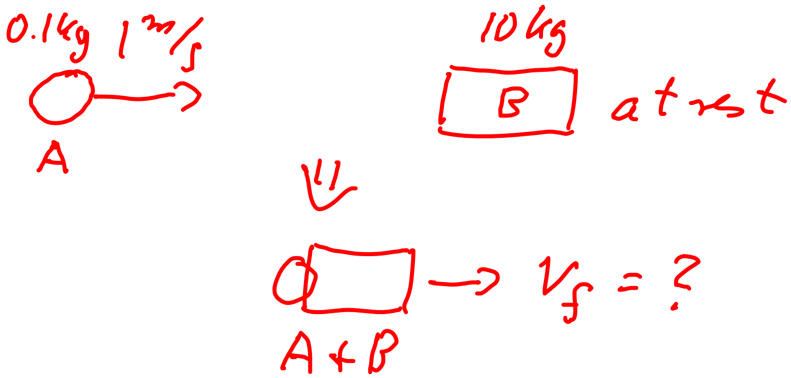
Case C: Superelastic collisions: $\boxed{\sum K_i < \sum K_f}$

e.g. compressed spring is released, objects explode during collision, ...

A **0.1 kg mass** with an **initial speed of 1 m/s** collides with a **10 kg mass** which is **initially at rest**, and **sticks** to it**.**

The **speed** of the **two masses** after the collision is *approximately***:**

- **A. 0 m/s**
- **B. 0.01 m/s**
- **C. 0.02 m/s**
- **D. 0.5 m/s**
- **E. 1 m/s**



$\Rightarrow$ Cons. of momentum:

$$m_A v_{i,A} + \underbrace{m_B v_{i,B}}_{=0 \text{ here}} = (m_A + m_B) v_f$$

$$\Rightarrow v_f = \frac{m_A}{m_A + m_B} v_{i,A} = \frac{0.1\,kg}{10\,kg + 0.1\,kg} \cdot 1\,\frac{m}{s} \approx \frac{1}{100}\,1\,\frac{m}{s}$$

Today:

- Sticking collisions
- Dinosaurs
- Rotational Motion
- Torque

Case B – Inelastic Collision:

Momentum is conserved, but kinetic energy is not!

$\sum K_i > \sum K_f \Rightarrow \Delta K_{loss} = K_{i,total} - K_{f,total} > 0$ } kinetic energy is lost

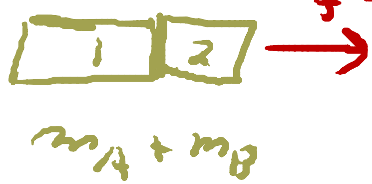
Special case: Sticking Collisions ("maximum inelastic collisions")

before:

cons. of momentum

$$m_1 v_{i,1} + m_2 v_{i,2} = (m_1 + m_2)\, v_f$$

after:



$v_f = v_{f,1} = v_{f,2}$

$$\Rightarrow \underline{\underline{v_f}} = \frac{m_1}{m_1 + m_2} v_{i,1} + \frac{m_2}{m_1 + m_2} v_{i,2}$$

$\Rightarrow$ Sticking collisions give the maximum possible loss of total kinetic energy ΔK_{loss} that is allowed by cons. of total momentum!

Proof:

conservation of momentum in <u>general</u> collision:

$$m_1 V_{i,1} + m_2 V_{i,2} = m_1 V_{f,1} + m_2 V_{f,2}$$

$$\Rightarrow V_{f,2} = \frac{m_1}{m_2}(V_{i,1} - V_{f,1}) + V_{i,2}$$

insert

$$\Delta K_{total} = K_{i,1} + K_{i,2} - K_{f,1} - K_{f,2} = \frac{1}{2} m_1 V_{i,1}^2 + \frac{1}{2} m_2 V_{i,2}^2 - \frac{1}{2} m_1 V_{f,1}^2 - \frac{1}{2} m_2 V_{f,2}^2$$

$$\Rightarrow \Delta K_{total} = \frac{1}{2} m_1 V_{i,1}^2 + \frac{1}{2} m_2 V_{i,2}^2 - \frac{1}{2} m_1 \underline{\underline{V_{f,1}}}^2 - \frac{1}{2} m_2 \left[\frac{m_1}{m_2}(V_{i,1} - \underline{\underline{V_{f,1}}}) + V_{i,2}\right]^2$$

$\Rightarrow$ find maximum of $\Delta K_{total}(V_{1,f})$

$$\frac{d(\Delta K_{total})}{d V_{f,1}} \overset{!}{=} 0 = -m_1 V_{f,1} - \frac{1}{2} m_2 2 \left[\frac{m_1}{m_2}(V_{i,1} - V_{f,1}) + V_{i,2}\right]\left(-\frac{m_1}{m_2}\right)$$

$$= -m_1 V_{f,1} + \frac{m_1^2}{m_2}(V_{i,1} - V_{f,1}) + m_1 V_{i,2} \quad \Big| \cdot \frac{m_2}{m_1}$$

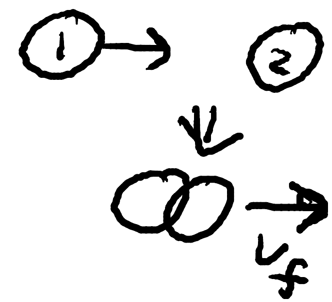
$$\Rightarrow -m_2 V_{f,1} + m_1 V_{i,1} - m_1 V_{f,1} + m_2 V_{i,2} = 0$$

$$\Rightarrow m_1 V_{i,1} + m_2 V_{i,2} = (m_1 + m_2) V_{f,1} \quad \Big\} \text{ this is true for sticking collision!}$$

$$\Rightarrow V_{f,1} = V_{f,2} \; !$$

Example: sticking collision with stationary target

$(v_{i,2} = 0 \Rightarrow K_{i,2} = 0)$



$$v_f = \frac{m_1}{m_1 + m_2} v_{i,1}$$

$$\Rightarrow \frac{\Delta K_{loss}}{\sum K_i} = \frac{\frac{1}{2} m_1 v_{i,1}^2 - \frac{1}{2}(m_1 + m_2) v_f^2}{\frac{1}{2} m_1 v_{i,1}^2 + 0} = \frac{m_2}{m_1 + m_2} \Big\} \; 0 \ldots 1$$

$\Rightarrow$ if $m_1 \ll m_2$

$$\frac{\Delta K_{loss}}{\sum K_i} = \frac{m_2}{m_1 + m_2} \approx 1 \Big\} \text{ all kinetic energy lost}$$

$\Rightarrow$ if $m_1 = m_2$:

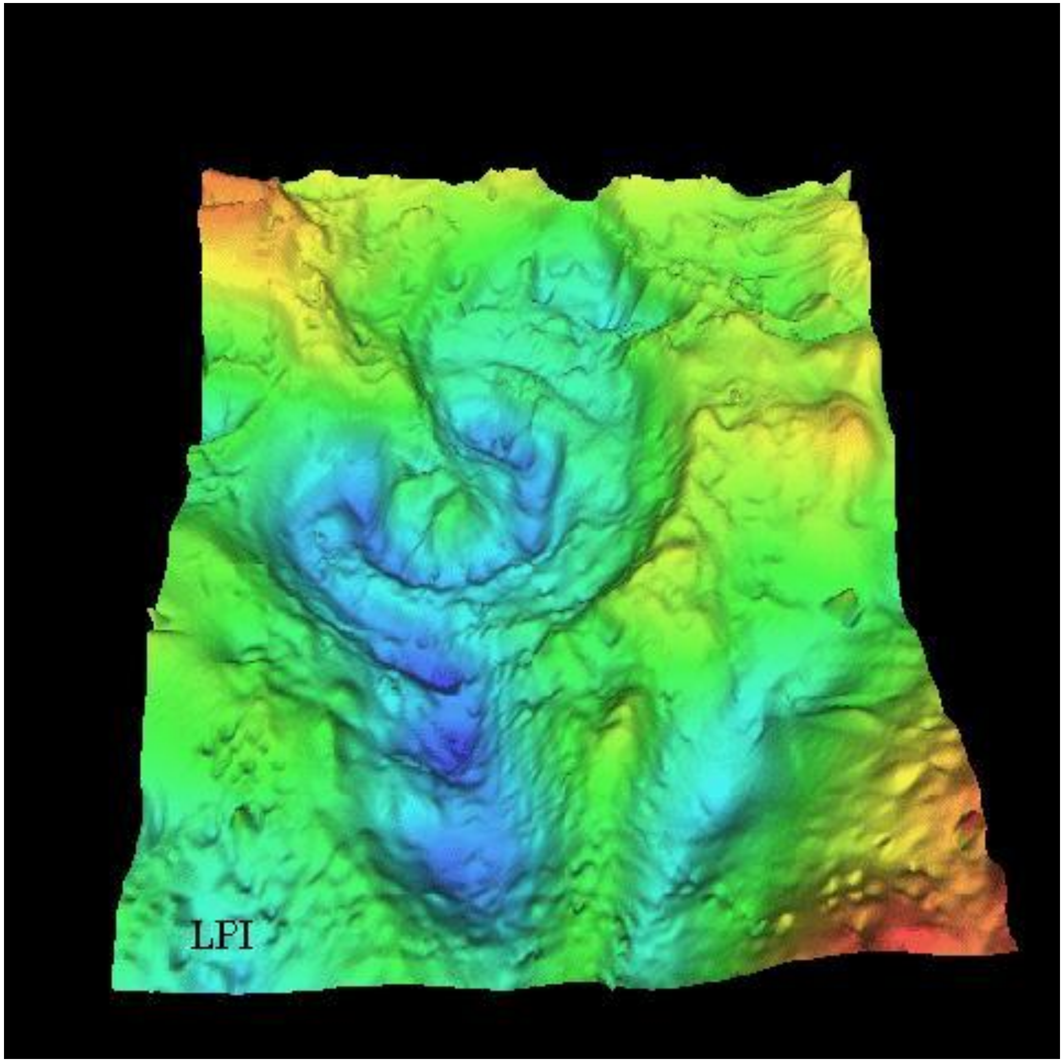
$$\frac{\Delta K_{loss}}{\sum K_i} = \frac{1}{2}$$

$\Rightarrow$ if $m_1 \gg m_2$

$$\frac{\Delta K_{loss}}{\sum K_i} = \frac{m_2}{m_1 + m_2} \rightarrow \text{tiny}$$

How the Dinosaurs Died

Gravitational Map of Buried Crater:



~65 million years old

~180 km in diameter

- caused by **impact of asteroid or comet**
  ~10 km in diameter

Chicxulub asteroid/comet numbers:

$R_a = 5$ km

$m_a = (4/3)\,\pi\, R_a^{\,3}\, \rho_a \quad \approx \quad 3 \times 10^{15}$ kg

$v_{a,I} = 20$ km/s

$\Rightarrow$ $\mathbf{K_{a,i} = (1/2)\, m_a\, v_{a,i}^{\,2} \approx 6 \times 10^{23}}$ **J**

Assume **sticking collision** with Earth:

$$m_a v_{a,i} + m_{earth} (0) = (m_a + m_{earth}) v_f$$

Since $m_{earth} \approx 6 \times 10^{24}$ kg >> $m_{a,}$

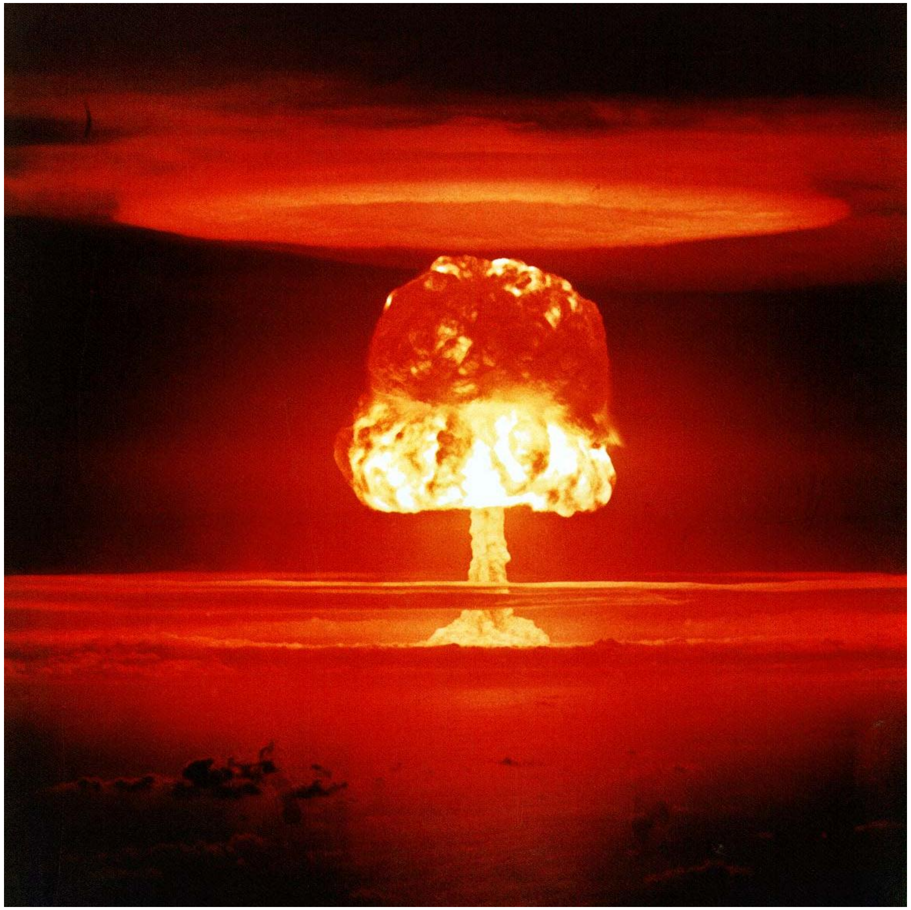
$$V_{f,earth} \approx (m_a / m_{earth}) v_{a,i} \approx \mathbf{1 \times 10^{-5}\ m/s\ !}$$

$$K_f \approx (1/2)\, m_{earth}\, v_f^{\,2} \approx 6 \times 10^{14}\ \text{J} << K_{a,i}$$

$\therefore$ **ΔK = $K_{a,i} - K_f \approx \underline{6 \times 10^{23}\ \text{J}}$**

= energy released upon impact

Mohawk, Eniwetok
Atoll
July 1956
350 kilotons



Romeo, Bikini Atoll
March 1954
11 Megatons

1 Megaton = $4 \cdot 10^{15}$ J

$\therefore\ \Delta K = K_{a,i} - K_f \approx 6 \times 10^{23}$ J

= energy released upon impact

= 1.5×10^8 Megatons

ΔK ~ 10^{10} energy released at Hiroshima

~ 10^4 energy of world's nuclear arsenals

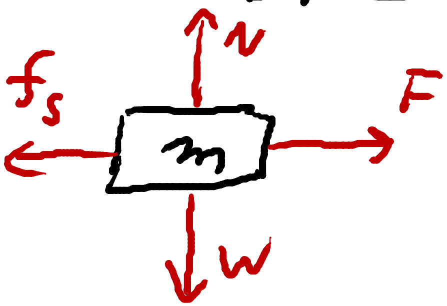
“The asteroid or comet "punched right through the Earth's crust, releasing molten magma from the mantle beneath. **Extraordinary earthquakes** tore at every seismic fault in the world as the crust buckled. **Waves kilometers high** crashed across the American continents. The blast threw **billions of tons of dust** and molten rock out into space. As the stuff fell back to earth the heat of its re-entry made the **sky glow like a furnace**, hot enough to **light forest fires all around the world**. When these burnt out, **all was blackness**; the **dust** hung in the sky like a wall of sooty brick. Eventually the sun returned, its warmth intensified. The comet had hit a seabed covered in limestone, and vaporized it by the cubic kilometer. That let **trillions of tons of carbon dioxide into the atmosphere**, enough to increase the temperature by perhaps 10° C.”

“The plankton in the sea died. So did most other marine creatures, except those safe in the depths. The terrestrial plants, able to stay dormant as seeds, did better. But **no land animal that weighed more than 30 kg (70 lb) survived**. The ***last dinosaurs were gone, along with 60% of all the species on the planet.*** A new geological era had arrived: the Cretaceous had given way to the Tertiary."

The Economist

September 11, 1993

Up to now:



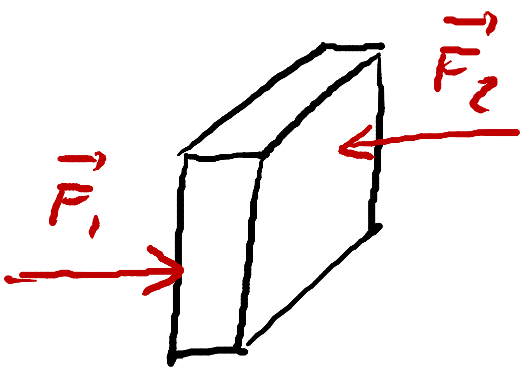
NII: $\sum \vec{F}_{ext} = m \, \vec{a}_{com}$

$\Rightarrow$ If $\sum \vec{F}_{ext} = 0 \Rightarrow \vec{a}_{com} = 0$

$\Rightarrow$ if $\vec{v}_{i,com} = 0 \Rightarrow$ COM point remains at rest

$\Rightarrow$ guarantees translational equilibrium

But: Objects might rotate!



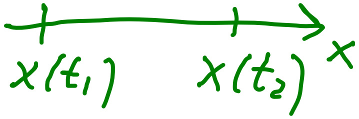
$\rightarrow$ rotates!

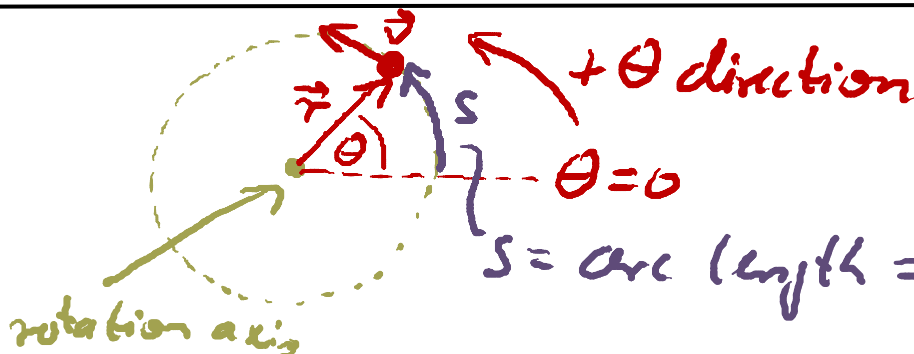
$\Rightarrow$ rotation depends on where the forces are acting (i.e "point of action" of $\vec{F}$)

$\sum \vec{F}_{ext} = 0 \Rightarrow$ no translational motion of COM point

Describing Rotational Motion

1-D lin. motion	Rotational Motion





s = arc length $= \theta \cdot r$

- position $x(t)$
  $[x] = m$

- **Angular position:** $\theta(t) = \dfrac{s(t)}{r}$

  $[\theta]$ = "rad" not deg!

- displacement:
  $\Delta x = x(t_2) - x(t_1)$

- **Angular displacement:**
  $\Delta\theta = \theta(t_2) - \theta(t_1) = \theta_2 - \theta_1$

- velocity:
  $v = \dfrac{dx}{dt}$
  $[v] = \dfrac{m}{s}$

- **Angular velocity:**
  $\omega = \dfrac{d\theta}{dt}$ = (rate of change of θ wrt. time) $[\omega] = \dfrac{\text{"rad"}}{s}$

if $|v|$ = const only

$$\omega = \frac{d\theta}{dt} = \frac{1}{r}\frac{ds}{dt} = \underline{\underline{\frac{v}{r}}} = \frac{1}{r}\frac{2\pi r}{T} = \frac{2\pi}{T}$$

T ← rotational period

What is the angular velocity ω of the minute hand of a clock?

angular velocity $= \omega = \frac{v}{r} = \frac{2\pi}{T}$

$= \frac{2\pi}{60 \cdot 60\,s}$

$= \frac{2\pi}{3660\,s}$

ω (in rad/s) = ?

A. 1 / 60

B. 2π / 60

C. 1 / 3600

D. 2π / 3600

1-D lin. motion	Rotational Motion

1-D lin. motion

• acceleration:

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

$[a] = m/s^2$

• $a = \text{const}$

$V = V_0 + at$

$\Delta x = V_0 t + \frac{1}{2} a t^2$

What causes translational acceleration?

$\Rightarrow$ Forces!

Rotational Motion

• Angular acceleration:

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} \qquad [\alpha] = \frac{\text{"rad"}}{s^2}$$

$\Rightarrow$ can do same graphs, equations, ..., as in 1-D linear motion:

example: if $\alpha = \text{const}$ (const ang. acceleration)

$$\omega = \omega_0 + \alpha t$$

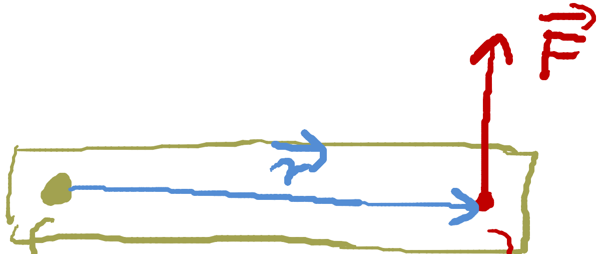
$$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

...

What causes angular acceleration?

$\Rightarrow$ $\boxed{\text{Torque}}$

Torque τ:



rotational axis A

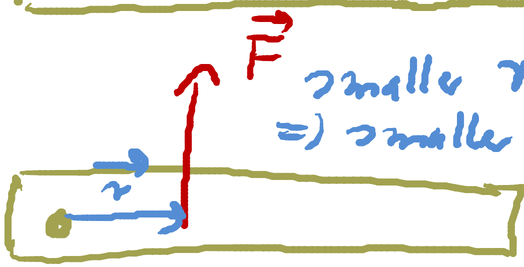
point of action of $\vec{F}$

τ of $\vec{F}$ about axis A $= F \, r_\perp$

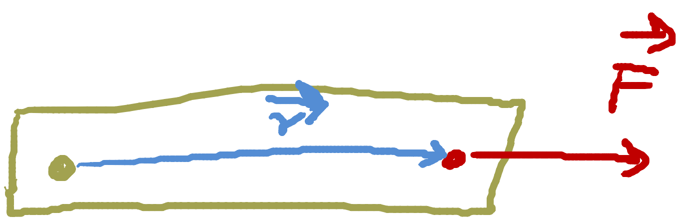
large force magnitude $\Rightarrow$ large τ, α

large perpendicular distance from A to point of action of $\vec{F}$
$\Rightarrow$ large "momentum arm"
$\Rightarrow$ large torque τ
$\Rightarrow$ large ang. accel. α

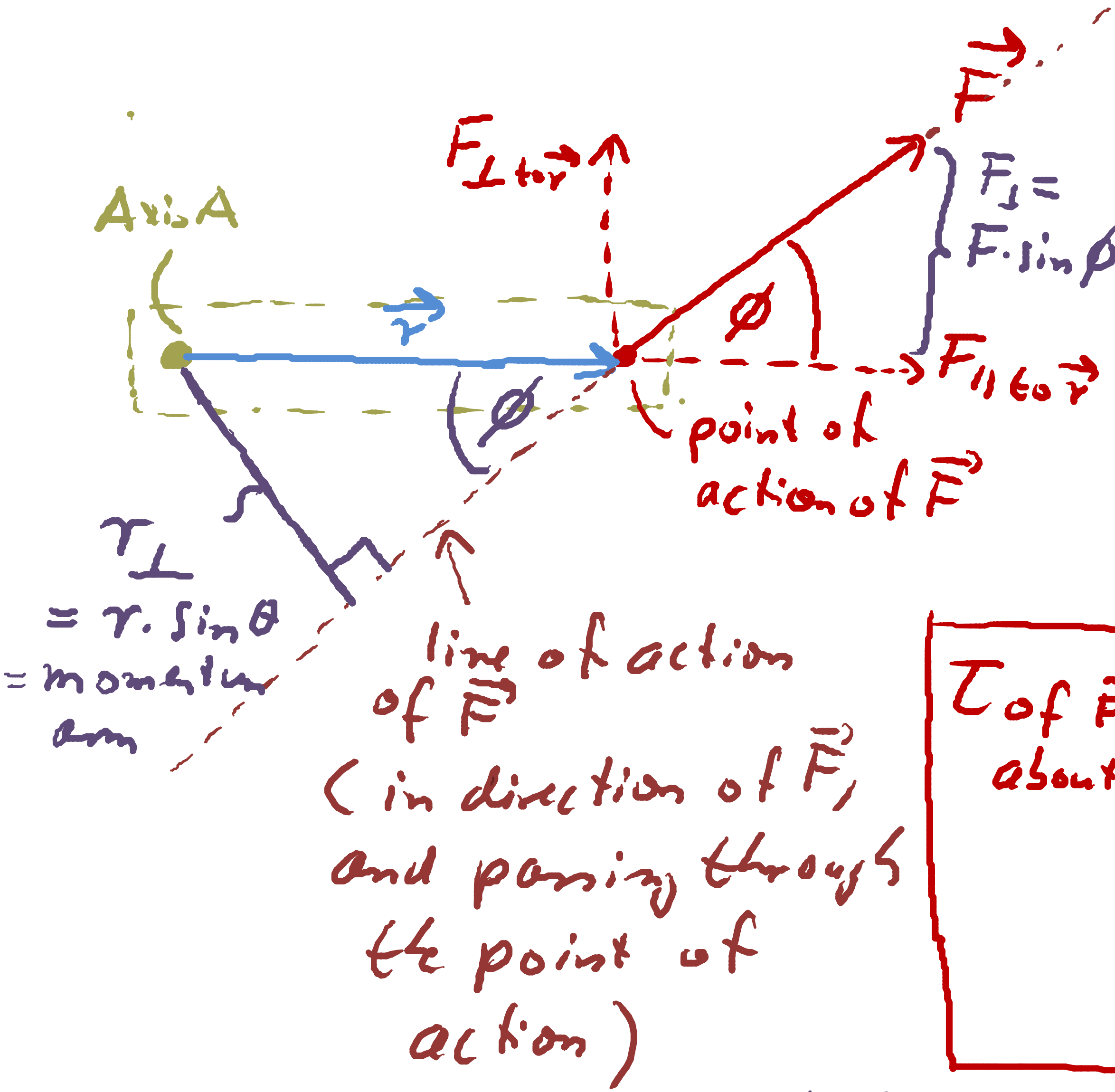
smaller $\vec{F}$
$\Rightarrow$ smaller τ
$\Rightarrow$ smaller α



smaller $r_\perp$
$\Rightarrow$ smaller τ
$\Rightarrow$ smaller α



τ of $\vec{F}$ about A $= 0$ here ($r_\perp = 0$)
$\Rightarrow$ direction of $\vec{F}$ is important!



ϕ: angle between $\vec{F}$ and $\vec{r}$

- for $F_{\parallel}$ to $\vec{r}$: produces no rotation and no torque about axis A
- for $F_{\perp}$ to $\vec{r}$

$$\boxed{\begin{aligned} \tau \text{ of } \vec{F} \text{ about axis A} &= F_{\perp} \cdot r \\ &= F\, r\, \underline{\sin}\, \phi \\ &= F\, \underbrace{r_{\perp}} \end{aligned}}$$

$\perp$ distance from axis A to the line of action of force $\vec{F}$ } "momentum arm" of $\vec{F}$ about axis A