Collisions

For any collision: \( \vec{p}_{1,i} + \vec{p}_{2,i} = \vec{p}_{1,f} + \vec{p}_{2,f} \) \{Momentum is conserved if \( \Sigma F_{\text{ext}} = 0 \)\}

Case A: Elastic collision: \( \Sigma k_i = \Sigma k_f \) \{kin. energy is also conserved\}

\( \Rightarrow \) speed of separation = speed of approach

Case B: Inelastic collision: part of kinetic energy is lost

\( \Rightarrow \Sigma k_i > \Sigma k_f \)

Special case: sticking collisions: \( v_{1,f} = v_{2,f} \)

have largest possible loss in kinetic energy

Case C: Superelastic collisions: \( \Sigma k_i < \Sigma k_f \)

E.g. compressed spring is released, objects explode during collision, ...
A 0.1 kg mass with an initial speed of 1 m/s collides with a 10 kg mass which is initially at rest, and sticks to it.

The speed of the two masses after the collision is approximately:

\[ V_f = \frac{m_A}{m_A + m_B} V_{i, A} = \frac{0.1 \text{ kg} \cdot 1 \text{ m/s}}{0.1 \text{ kg} + 10 \text{ kg}} \approx \frac{1 \text{ m}}{10.1} \approx 0.01 \text{ m/s} \]

- A. 0 m/s
- B. 0.01 m/s
- C. 0.02 m/s
- D. 0.5 m/s
- E. 1 m/s
Today:

- Sticking collisions
- Dinosaurs
- Rotational Motion
- Torque
Case B - Inelastic Collision:

Momentum is conserved, but kinetic energy is not!

\[ \Sigma K_i > \Sigma K_f \Rightarrow \Delta K_{loss} = K_{i,\text{total}} - K_{f,\text{total}} > 0 \] kinetic energy is lost

Special case: Sticking Collision ("maximum inelastic collisions")

before:

\[ \begin{array}{c}
1 \\
\rightarrow \\
2 \\
V_{i,1} \\
V_{i,2} \\
\end{array} \]

\[ \text{Cons. of momemum,} \]

\[ m_1 V_{i,1} + m_2 V_{i,2} = (m_1 + m_2) V_f \]

\[ V_f = V_{i,1} = V_{i,2} \]

\[ \Rightarrow V_f = \frac{m_1 V_{i,1} + m_2 V_{i,2}}{m_1 + m_2} = \]

after:

\[ \begin{array}{c}
1 \\
\rightarrow \\
2 \\
\frac{m_1}{m_1 + m_2} \\
\end{array} \]

\[ \Rightarrow \text{Sticking collisions give the maximum possible loss of total kinetic energy} \]

\[ \text{D.Kin., that is allowed by cons. of total momentum!} \]
Proof:

conservation of momentum in general collision:

\[ m_1 V_{i,1} + m_2 V_{i,2} = m_1 V_{f,1} + m_2 V_{f,2} \]

\[ \Rightarrow V_{f,2} = \frac{m_1}{m_2} (V_{i,1} - V_{f,1}) + V_{i,2} \]

\[ \Delta K_{\text{total}} = K_{i,1} + K_{i,2} - L_{f,1} - K_{f,2} = \frac{1}{2} m_1 V_{i,1}^2 + \frac{1}{2} m_2 V_{i,2}^2 - \frac{1}{2} m_1 V_{f,1}^2 - \frac{1}{2} m_2 V_{f,2}^2 \]

\[ \Rightarrow \Delta K_{\text{total}} = \frac{1}{2} m_1 V_{i,1}^2 + \frac{1}{2} m_2 V_{i,2}^2 - \frac{1}{2} m_1 V_{f,1}^2 - \frac{1}{2} m_2 V_{f,2}^2 \]

\[ \Rightarrow \text{find maximum of } \Delta K_{\text{total}} \text{ (V}_{1,f} \text{)} \]

\[ \frac{d(\Delta K_{\text{total}})}{d V_{f,1}} = 0 = -m_1 V_{f,1} + \frac{m_1}{m_2} (V_{i,1} - V_{f,1}) + m_1 V_{i,2} \left( -\frac{m_1}{m_2} \right) \]

\[ = -m_1 V_{f,1} + \frac{m_1^2}{m_2} (V_{i,1} - V_{f,1}) + m_1 V_{i,2} \left( -\frac{m_1}{m_2} \right) \]

\[ \Rightarrow -m_2 V_{f,1} + m_1 V_{i,1} - m_1 V_{f,1} + m_2 V_{i,2} = 0 \]

\[ \Rightarrow m_1 V_{i,1} + m_2 V_{i,2} = (m_1 + m_2) V_{f,1} \text{ } (\text{this is true for sticking collision!}) \]

\[ \Rightarrow V_{f,1} = V_{f,2} ? \]
Example: sticking collision with stationary target 
\( V_{i,2} = 0 \Rightarrow H_{i,2} = 0 \)  
\[
V_f = \frac{m_1}{m_1 + m_2} V_{i,1}
\]

\[
\frac{\Delta K_{100}}{\sum K_i} = \frac{\frac{1}{2} m_1 V_{i,1}^2 - \frac{1}{2} (m_1 + m_2) V_f^2}{\frac{1}{2} m_1 V_{i,1}^2 + 0} = \frac{m_2}{m_1 + m_2} \}
\]

\( \Rightarrow \) if \( m_1 \ll m_2 \)

\[
\frac{\Delta K_{100}}{\sum K_i} = \frac{m_2}{m_1 + m_2} \times 1 \] all kinetic energy lost

\( \Rightarrow \) if \( m_1 = m_2 \):

\[
\frac{\Delta K_{100}}{\sum K_i} = \frac{1}{2}
\]

\( \Rightarrow \) if \( m_1 \gg m_2 \)

\[
\frac{\Delta K_{100}}{\sum K_i} = \frac{m_2}{m_1 + m_2} \rightarrow \text{Lig}
\]
How the Dinosaurs Died
Gravitational Map of Buried Crater:

- ~65 million years old
- ~180 km in diameter
-造成的 by **impact of asteroid or comet**
- ~10 km in diameter
Chicxulub asteroid/comet numbers:

\[ R_a = 5 \text{ km} \]
\[ m_a = \frac{4}{3} \pi R_a^3 \rho_a \approx 3 \times 10^{15} \text{ kg} \]
\[ v_{a,l} = 20 \text{ km/s} \]

\[ \Rightarrow K_{a,i} = \frac{1}{2} m_a v_{a,i}^2 \approx 6 \times 10^{23} \text{ J} \]
Assume **sticking collision with Earth**:

\[ m_a v_{a,i} + m_{\text{earth}} (0) = (m_a + m_{\text{earth}}) v_f \]

Since \( m_{\text{earth}} \approx 6 \times 10^{24} \text{ kg} >> m_a, \)

\[ V_{f,\text{earth}} \approx \left( \frac{m_a}{m_{\text{earth}}} \right) v_{a,i} \approx 1 \times 10^{-5} \text{ m/s} ! \]

\[ K_f \approx \left( \frac{1}{2} \right) m_{\text{earth}} v_f^2 \approx 6 \times 10^{14} \text{ J} << K_{a,i} \]

\[ \therefore \Delta K = K_{a,i} - K_f \approx 6 \times 10^{23} \text{ J} \]

= energy released upon impact
Mohawk, Eniwetok Atoll
July 1956
350 kilotons

Romeo, Bikini Atoll
March 1954
11 Megatons

1 Megaton = 4 \cdot 10^{15} \text{ J}
\[ \Delta K = K_{a,i} - K_f \approx 6 \times 10^{23} \text{ J} \]

= energy released upon impact

= \(1.5 \times 10^8\) Megatons

\[ \Delta K \sim 10^{10} \text{ energy released at Hiroshima} \]

\[ \sim 10^4 \text{ energy of world's nuclear arsenals} \]
“The asteroid or comet "punched right through the Earth's crust, releasing molten magma from the mantle beneath. Extraordinary earthquakes tore at every seismic fault in the world as the crust buckled. Waves kilometers high crashed across the American continents. The blast threw billions of tons of dust and molten rock out into space. As the stuff fell back to earth the heat of its re-entry made the sky glow like a furnace, hot enough to light forest fires all around the world. When these burnt out, all was blackness; the dust hung in the sky like a wall of sooty brick. Eventually the sun returned, its warmth intensified. The comet had hit a seabed covered in limestone, and vaporized it by the cubic kilometer. That let trillions of tons of carbon dioxide into the atmosphere, enough to increase the temperature by perhaps 10° C.”
“The plankton in the sea died. So did most other marine creatures, except those safe in the depths. The terrestrial plants, able to stay dormant as seeds, did better. But no land animal that weighed more than 30 kg (70 lb) survived. The last dinosaurs were gone, along with 60% of all the species on the planet. A new geological era had arrived: the Cretaceous had given way to the Tertiary.”

The Economist
September 11, 1993
Up to now:

\[ \sum \mathbf{F}_{\text{ext}} = m \mathbf{a}_{\text{com}} \]

\[ \Rightarrow \text{If } \sum \mathbf{F}_{\text{ext}} = 0 \Rightarrow \mathbf{a}_{\text{com}} = 0 \]

\[ \Rightarrow \text{If } V_{c,\text{com}} = 0 \Rightarrow \text{COM point remains at rest} \]

\[ \Rightarrow \text{Guarantees translational equilibrium} \]

**But:** Objects might rotate!

\[ \sum \mathbf{F}_{\text{ext}} = 0 \Rightarrow \text{no translational motion of COM point} \]

\[ \Rightarrow \text{rotation depends on where the forces are acting (i.e., \textit{point of action} of } \mathbf{F}) \]
Describing Rotational Motion

Rotational Motion

\[ \theta(t) = \frac{s(t)}{r} \]

<table>
<thead>
<tr>
<th>Position</th>
<th>( x(t_1) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Displacement</td>
<td>( x(t_2) - x(t_1) )</td>
</tr>
<tr>
<td>Velocity</td>
<td>( \frac{dx}{dt} )</td>
</tr>
</tbody>
</table>

\[ [V] = \frac{m}{s} \]

\[ \omega = \frac{d\theta}{dt} \triangleq \text{(rate of change of } \theta \text{ wrt. time)} \]

\[ \omega = \frac{d\theta}{dt} = \frac{1}{r} \frac{ds}{dt} = \frac{V}{r} = \frac{1}{r} \frac{2 \pi r}{T} = \frac{2 \pi}{T} \text{ (rotational period)} \]

**Angular Position**

\[ \theta(t) = \frac{s(t)}{r} \]

\[ [\theta] = \text{"rad" not deg!} \]

**Angular Displacement**

\[ \Delta \theta = \theta(t_2) - \theta(t_1) = \theta_2 - \theta_1 \]

**Angular Velocity**

\[ \omega = \frac{d\theta}{dt} \]

\[ \omega = \frac{d\theta}{dt} = \frac{1}{r} \frac{ds}{dt} = \frac{V}{r} = \frac{1}{r} \frac{2 \pi r}{T} = \frac{2 \pi}{T} \text{ (rotational period)} \]
What is the angular velocity $\omega$ of the minute hand of a clock?

Angular velocity $= \omega = \frac{v}{r} = \frac{2\pi}{T}$

$= \frac{2\pi}{60 \cdot 60}$

$= \frac{2\pi}{3600}$

$\omega$ (in rad/s) = ?

A. $\frac{1}{60}$
B. $\frac{2\pi}{60}$
C. $\frac{1}{3600}$
D. $\frac{2\pi}{3600}$
1-D linear motion

- Acceleration:
  \[ a = \frac{dv}{dt} = \frac{d^2x}{dt^2} \]
  \[ [a] = \text{m/s}^2 \]

- \( a = \text{const} \)
- \( v = v_0 + at \)
- \( x = v_0 t + \frac{1}{2} at^2 \)

What causes translational acceleration?
\[ \Rightarrow \text{Forces!} \]

Rotational motion

- Angular acceleration:
  \[ \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} \]
  \[ [\alpha] = \text{rad/s}^2 \]

\[ \Rightarrow \] can do same graphs, equations, ..., as in 1-D linear motion:

- Example: if \( \alpha = \text{const} \) (const angular acceleration)
  \[ \omega = \omega_0 + \alpha t \]
  \[ \theta = \omega_0 t + \frac{1}{2} \alpha t^2 \]

... What causes angular acceleration?

\[ \Rightarrow \text{Torque} \]
Torque $\tau$:

- Point of action of $F$
  - Smaller $\tau$ = Smaller $\alpha$

- Smaller perpendicular distance from $A$ to point of action of $F$
  - Larger "moment arm"
  - Larger $\tau$

- Larger $\alpha$

- $T_{\text{of } F \text{ about } A} = 0$ here ($\tau = 0$)
  - Direction of $F$ is important!
\[ T_{\text{of } F} = \frac{F_L \cdot r}{\sin \phi} \]

- for \( F_{\parallel} \) to \( \vec{r} \):
  - produce no rotation and no torque about axis \( A \)
- for \( F_{\perp} \) to \( \vec{r} \):
  - distance from axis \( A \) to the line of action of force \( F \)

Where:
- \( F_{\parallel} \) and \( F_{\perp} \) are components of force \( F \)
- \( F \) is the magnitude of force \( F \)
- \( r \) is the distance between the point of action of \( F \) and the axis \( A \)
- \( \phi \) is the angle between \( F \) and \( \vec{r} \)
- \( F_{\parallel} \) is parallel to \( \vec{r} \)
- \( F_{\perp} \) is perpendicular to \( \vec{r} \)
- \( T_{\text{of } F} \) is the torque of force \( F \) about axis \( A \)