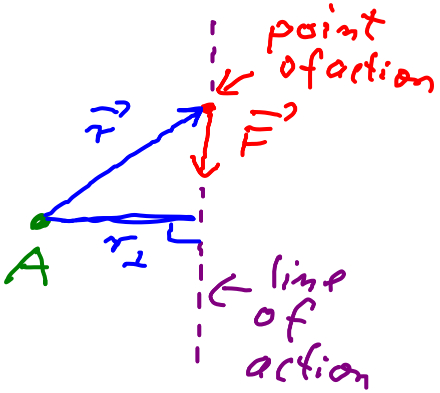
Lecture 28

# Recap: Solving for Equilibrium

- Torque $= \tau = F r \sin\phi = F_\perp r = F r_\perp$



- Mechanical advantage : $MA = \dfrac{\text{Output force}}{\text{Input force}}$
- Equilibrium:

$$\sum \vec{F}_{ext} = 0 \qquad \sum \tau_{\text{about }\underline{\text{any}}\text{ axis}} = 0$$

  - forces act at specific points ⇒ show on FBD!
  - weight $= mg$ acts through center of mass of object
  - $\tau$ has sign! ⇒ Define positive direction!
  - can calculate $\sum \tau$ about *any* axis
    ⇒ place axis at point where unknown forces act

# Today:

- Elastic response of materials
- The Titanic
- Proportions of Animals
- Temperature and thermal equilibrium

# Elastic Response of Materials:

- **Spring**

$L + \Delta L$

↗

$\Delta L = 0$ for $F = 0$

Hooke's "Law"

$$|F| = K \, \Delta L$$

$$\Rightarrow \Delta L = \frac{1}{K} |F|$$

↗

K depends on material, size, shape of spring

- **General Solid Material**

Under tension / compression

$L + \Delta L$

↗

$\Delta L = 0$ for $F = 0$

$$\Delta L \propto \frac{L}{A} F$$

← like springs in series

← like springs in parallel

$$\Rightarrow \underbrace{\frac{F}{A}}_{\text{"stress"}} \propto \underbrace{\frac{\Delta L}{L}}_{\text{"strain"}}$$

$\left[\frac{F}{A}\right] = \frac{N}{m^2}$ (dimensionless)

## ⇒ Elasticity:

$$\boxed{\frac{F}{A} = E\,\frac{\Delta L}{L}}$$

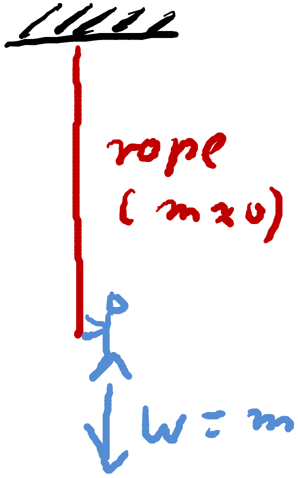
stress = E strain

E = Young's Modulus

- bulk material property
- varies alot between materials
- rubber tube: small E, steel: large E
- $[E] = \frac{[F]}{[A]} = \frac{N}{m^2}$
- often similar for tension and compression

A Nylon rope used by mountaineers has a length of **50 m** and cross-sectional area of **0.004 m$^2$**.  When an **80 kg climber** hangs from the rope it **extends by 1 m**.

What is the **stress** in the rope?



$$\text{stress} = \frac{F}{A} = \frac{mg}{A} = \frac{800N}{0.004\,m^2} = 200{,}000\ N/m^2$$

$W = mg = 800N$

- ~~A.~~ 80 N
- ~~B.~~ 800 N
- C. 2 N/m$^2$
- D. 20 N/m$^2$
- [✓ E. 200,000 N/m$^2$]

A Nylon rope used by mountaineers has a length of **50 m** and cross-sectional area of **0.004 m$^2$**. When an **80 kg climber** hangs from the rope it **extends by 1 m**.

What is the **Young's modulus E** of the Nylon?

$$\frac{F}{A} = E \frac{\Delta L}{L}$$

$$\Rightarrow E = \frac{\text{stress}}{\text{strain}} = \frac{F/A}{\Delta L/L}$$

$$= \frac{200{,}000 \text{ N/m}^2}{1\text{m}/50\text{m}}$$

$$= 10{,}000{,}000 \text{ N/m}^2$$

$$= 10^7 \text{ N/m}^2$$

**E ≈ ?**

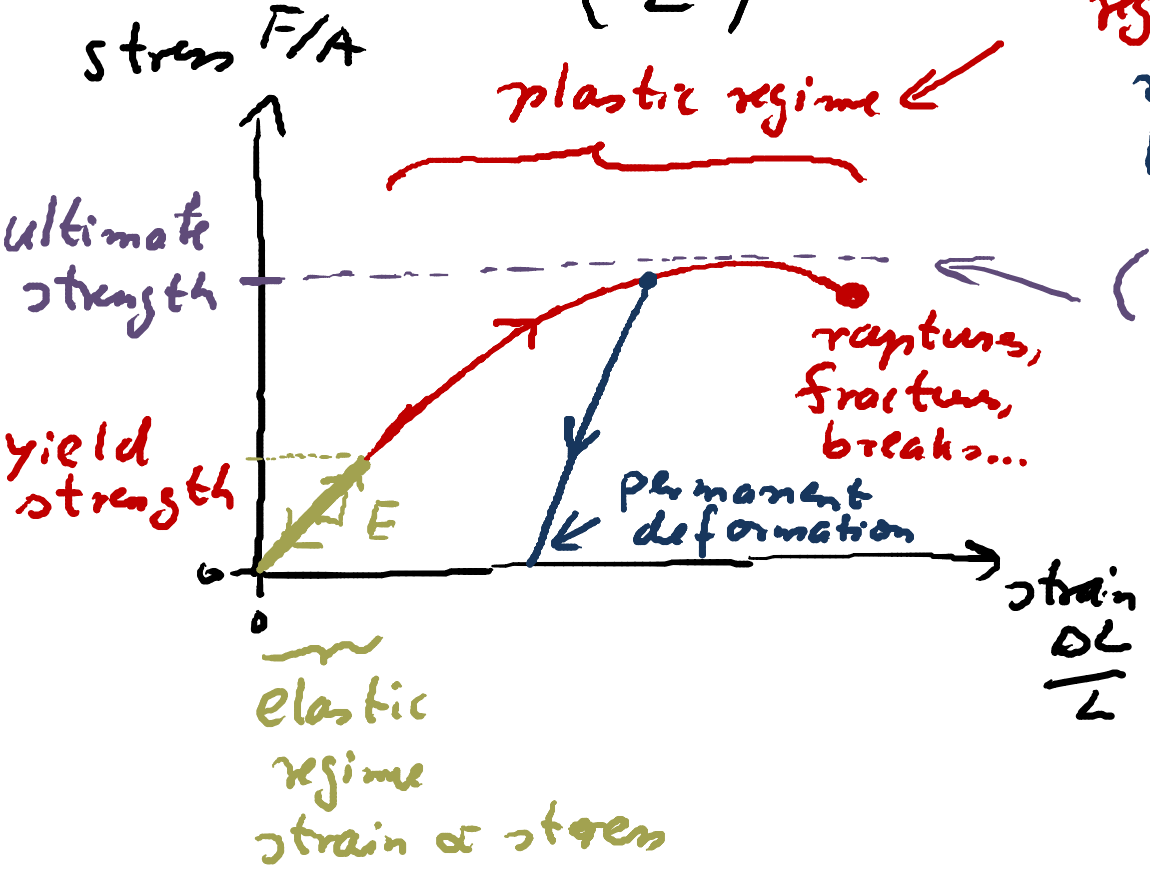
**A. 200 N/m$^2$**

**B. 200,000 N/m$^2$**

**C. 1,000,000 N/m$^2$**

**[✓ D.] 10,000,000 N/m$^2$**

- For small strains ($\frac{\Delta L}{L}$) solid materials behave elastically: return to original length when stress is removed.
- For large ($\frac{\Delta L}{L}$)



(permanent deformation regime): when stress is removed, object is permanently elongated

$\left(\frac{F}{A}\right)_{max}$ = ultimate strength

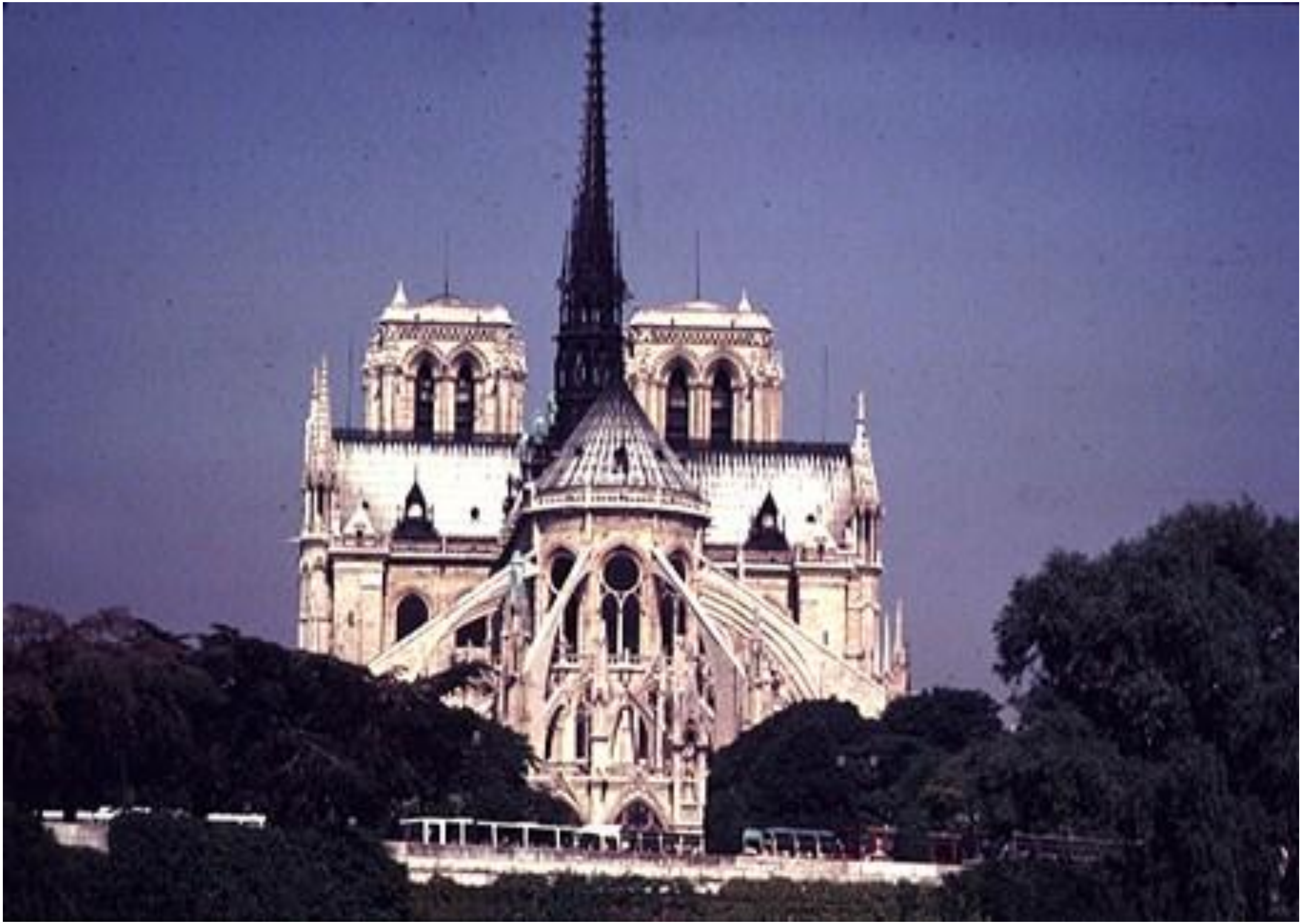
= maximum stress that can be applied (often different for tension and compression)

# The Pyramids at Giza

## Constructed: 2650 - 2490 BC

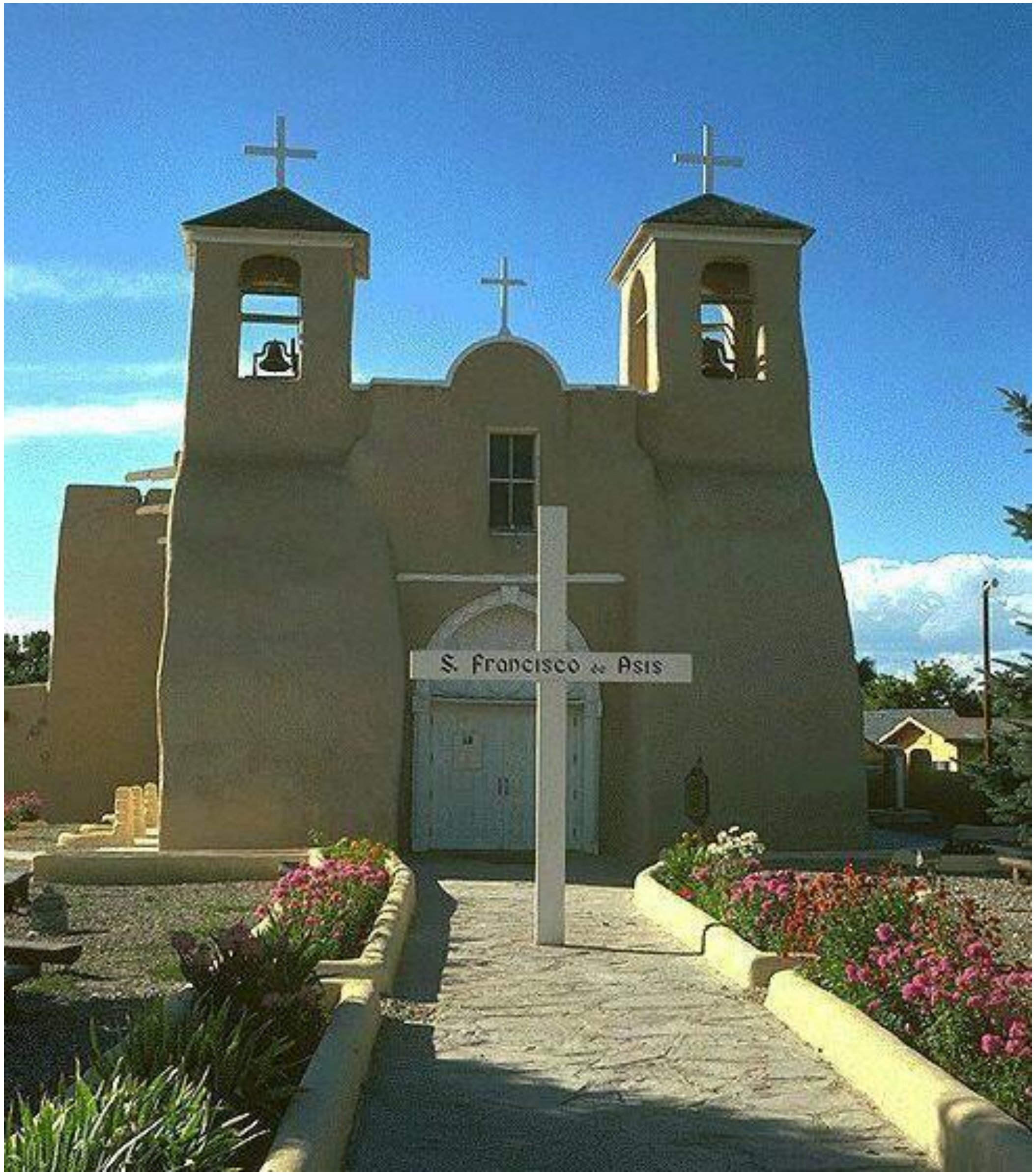
# Cathedrale de Notre-Dame-de-Paris

## Constructed: 1163 – 1345 AD



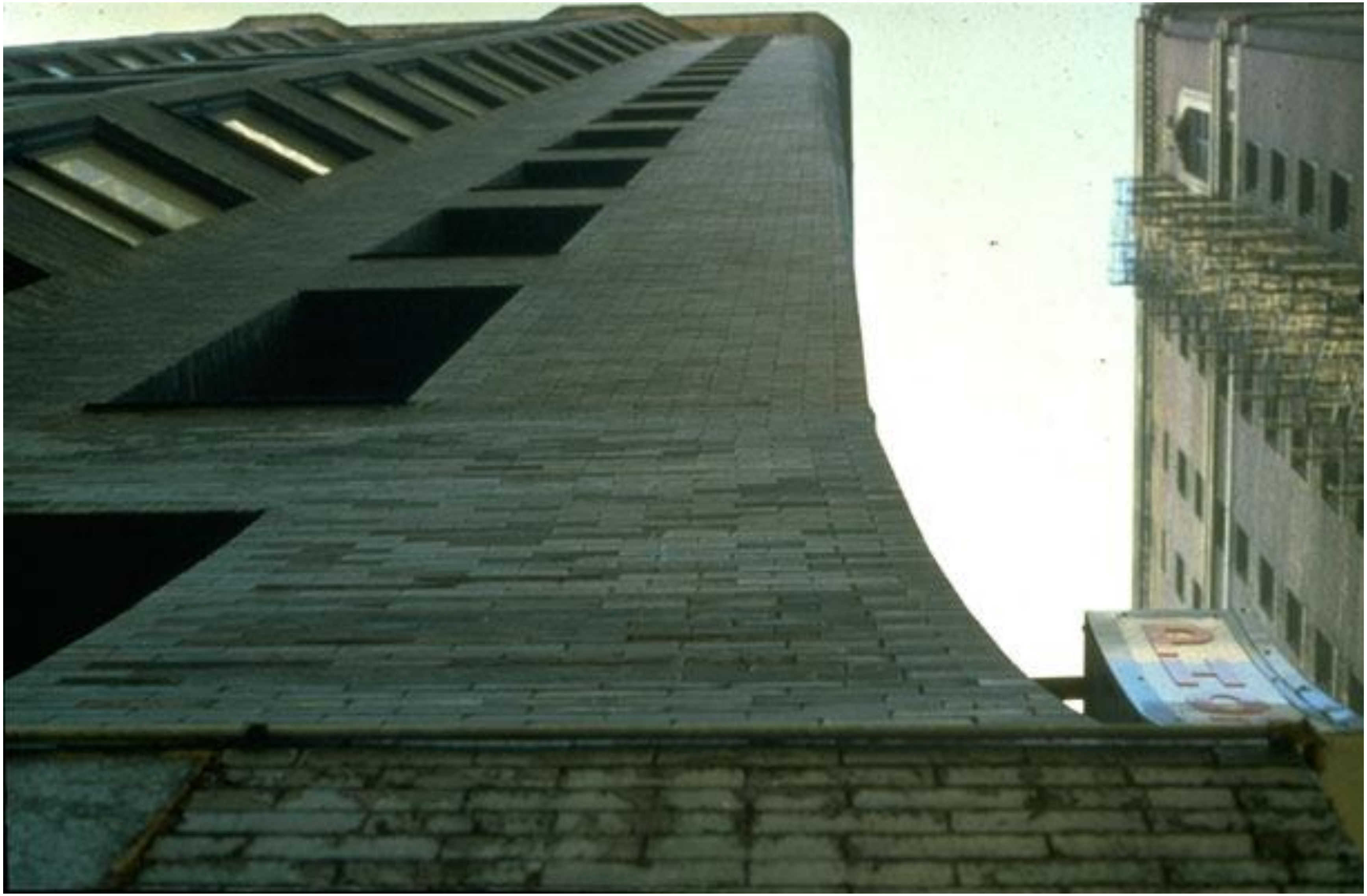
# San Francisco de Asis, Taos, New Mexico

## Constructed: 1772-1816



# Monadnock Building, Chicago

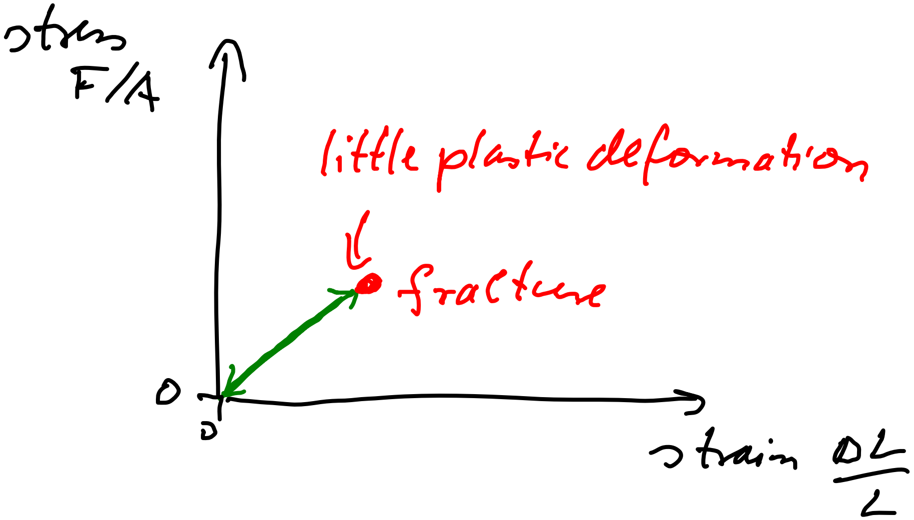
## Constructed: 1893



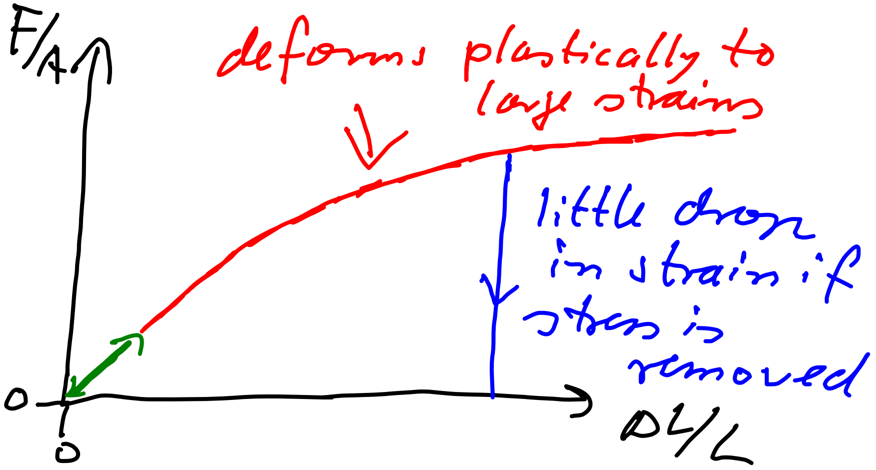
The 627 ft tall **“Turning Torso”**, an apartment complex in Malmö, Sweden, designed by Spanish architect Santiago Calatrava.

- Detailed behavior depends on material

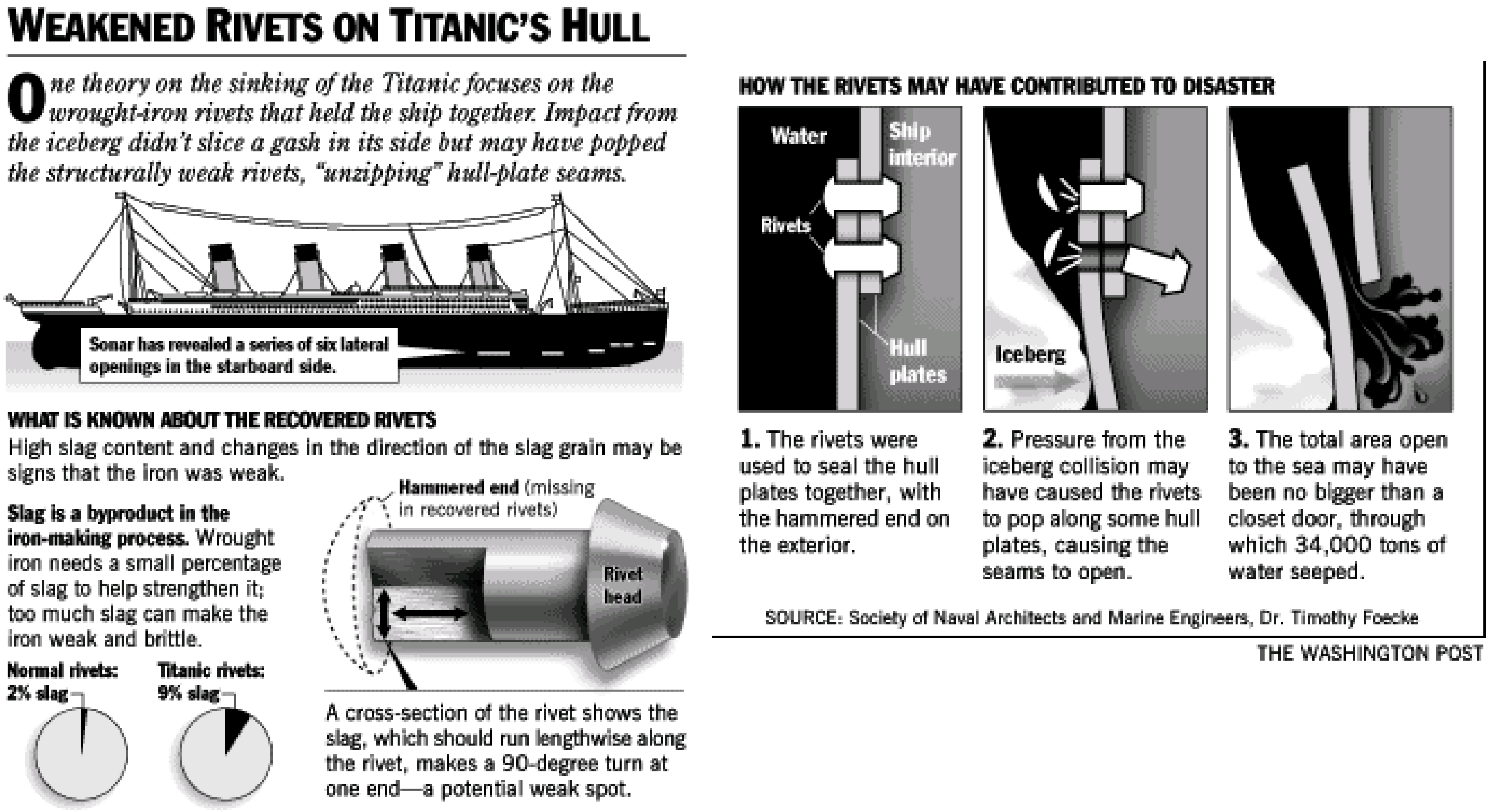
Glass - brittle



Gold - ductile



# TITANIC



## WEAKENED RIVETS ON TITANIC'S HULL

*One theory on the sinking of the Titanic focuses on the wrought-iron rivets that held the ship together. Impact from the iceberg didn't slice a gash in its side but may have popped the structurally weak rivets, "unzipping" hull-plate seams.*

**Sonar has revealed a series of six lateral openings in the starboard side.**

### WHAT IS KNOWN ABOUT THE RECOVERED RIVETS

High slag content and changes in the direction of the slag grain may be signs that the iron was weak.

**Slag is a byproduct in the iron-making process.** Wrought iron needs a small percentage of slag to help strengthen it; too much slag can make the iron weak and brittle.

A cross-section of the rivet shows the slag, which should run lengthwise along the rivet, makes a 90-degree turn at one end—a potential weak spot.

### HOW THE RIVETS MAY HAVE CONTRIBUTED TO DISASTER

**1.** The rivets were used to seal the hull plates together, with the hammered end on the exterior.

**2.** Pressure from the iceberg collision may have caused the rivets to pop along some hull plates, causing the seams to open.

**3.** The total area open to the sea may have been no bigger than a closet door, through which 34,000 tons of water seeped.

SOURCE: Society of Naval Architects and Marine Engineers, Dr. Timothy Foecke

THE WASHINGTON POST

The minimum **cross-sectional area of the femur** of a human adult is **$6 \times 10^{-4}$ m$^2$**. The **ultimate strength** of bone under compression is **$17 \times 10^7$ N m$^{-2}$**.

What is the **maximum compressional force $F_{max}$** that can be applied (e.g., during a fall) before the bone breaks?

ultimate strength $= \left(\frac{F}{A}\right)_{max}$

$=$ max. stress

$\Rightarrow F_{max} = (U.S.) \cdot A$

$= 17 \cdot 10^7 \frac{N}{m^2} \cdot 6 \cdot 10^{-4} m^2$

$= 100,000$ N

$\hat{=}$ weight of 10 tons

**$F_{max} \approx$ ?**

**A. 1000 N**

**[✓ B.] 100,000 N**

**C. $3 \times 10^{11}$ N**

**D. None of the above**

The minimum **cross-sectional area of the femur** of a human adult is **$6 \times 10^{-4}$ m$^2$**. The **ultimate strength** of bone under compression is **$17 \times 10^7$ N m$^{-2}$**, and its Young's modulus **E** is **$9 \times 10^9$ N m$^{-2}$.**

$\Delta L / L = ?$

What is the **strain** at which **fracture occurs**?
(Assume that the stress-strain relation is linear up to when fracture occurs.)

$$\frac{F}{A} = E\,\frac{\Delta L}{L}$$

$$\Rightarrow \left(\frac{\Delta L}{L}\right)_{max} = \frac{(F/A)_{max}}{E} = \frac{\text{ultimate strength}}{E}$$

$$= \frac{17 \cdot 10^7\ N/m^2}{9 \cdot 10^9\ N/m^2} = 0.02 = 2\%$$

**Strain ≈ ?**

**A. 0.01**

**[✓ B.] 0.02**

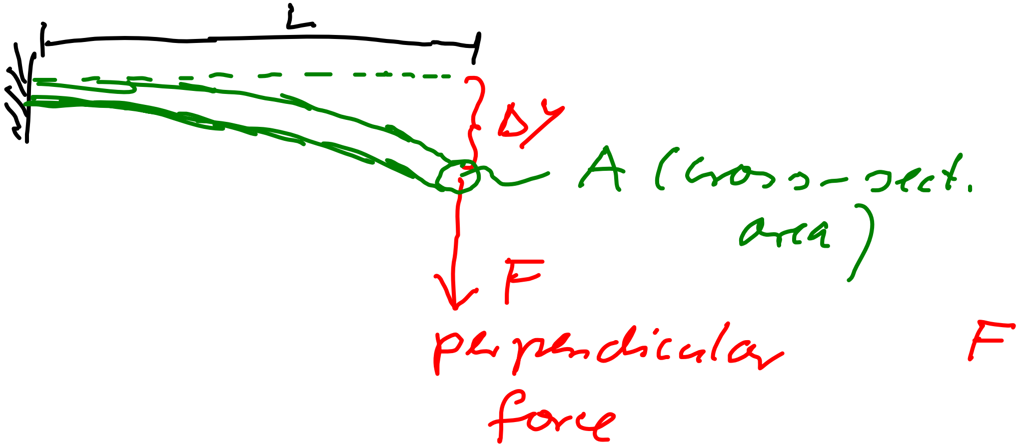
**C. 0.04**

**D. 50**

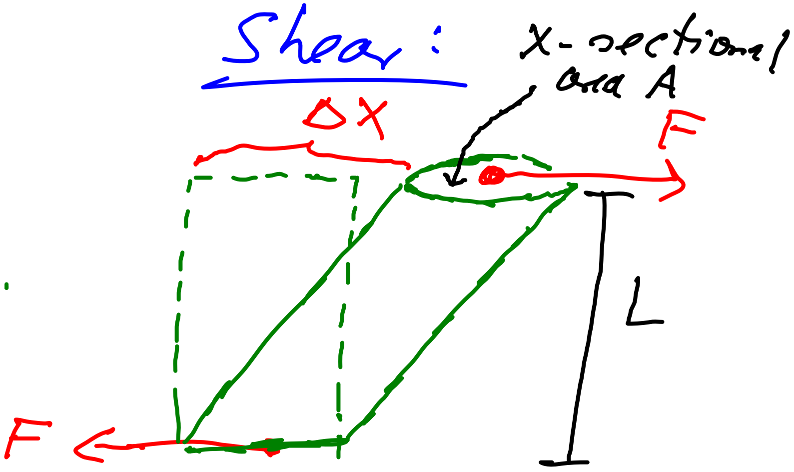
**E. 100**

# • Other forms of deformation :

## Bending



## Shear :



$$\underbrace{\frac{F}{A}}_{\text{bending stress}} = \underset{\substack{\uparrow \\ \underline{\text{bending}} \\ \underline{\text{modulus}}}}{B} \frac{\Delta y}{L}$$

$$\underbrace{\frac{F}{A}}_{\text{shear stress}} = \underset{\substack{\uparrow \\ \underline{\text{shear}} \\ \underline{\text{modulus}}}}{G} \frac{\Delta X}{L}$$

# Proportions of Animals

## Geometric Similarity:

Individuals ***of the same species*** usually have the ***same shape***.  Larger individuals are just magnified versions of smaller individuals.

# E.g., Regal Horned Lizards:

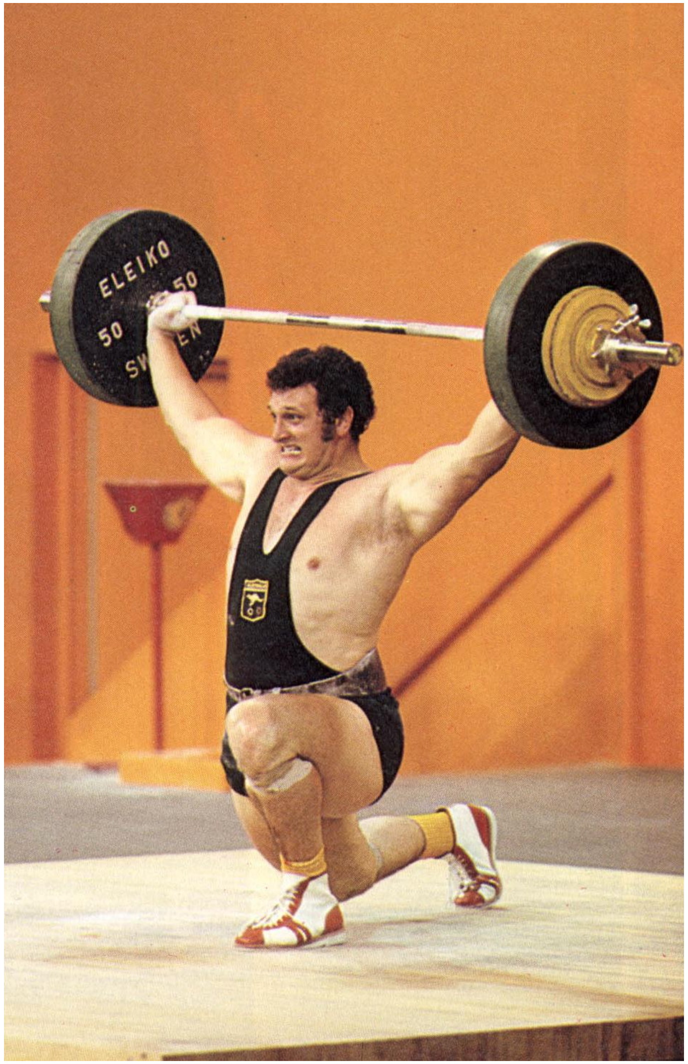
m=0.086 kg

m=0.012 kg

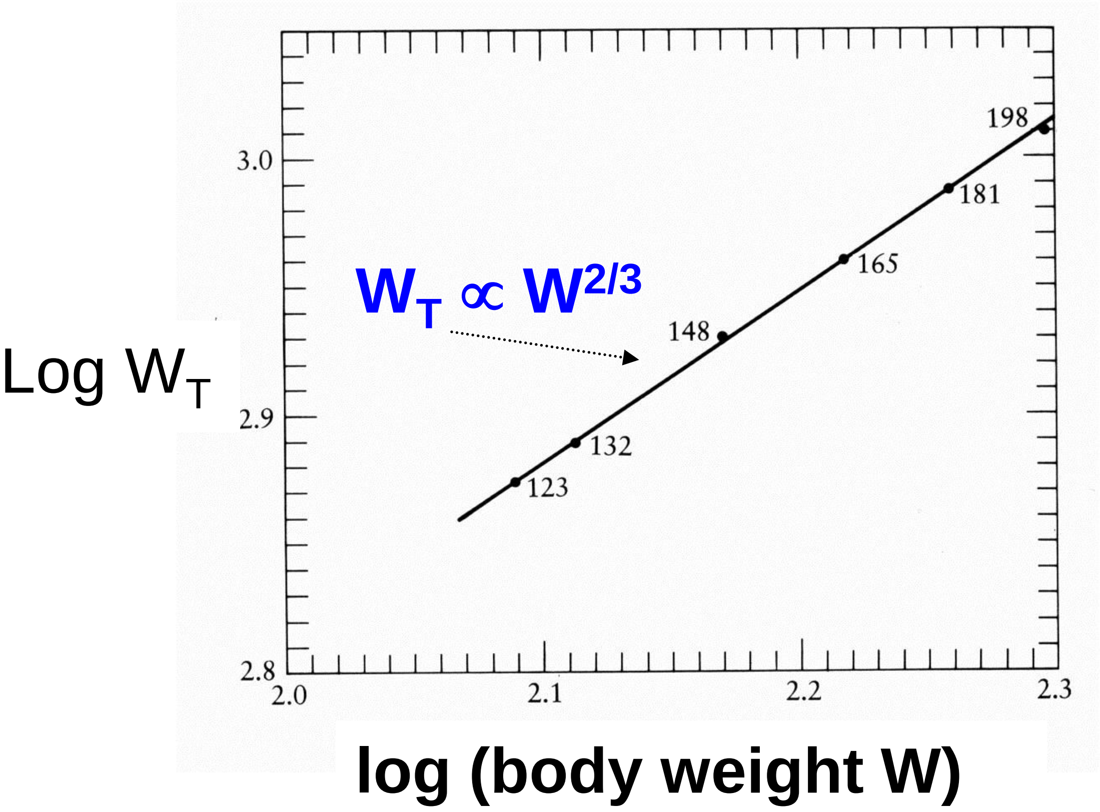
# Geometric Similarity:

**$\Rightarrow$ characterized by a single length scale $\ell$**

- length, diameter of limbs $\propto \ell$
- bone, muscle x-sectional area A $\propto \ell^2$
- mass m $\propto \ell^3$
- muscle strength $\propto$ A $\propto \ell^2$



$W_T$ = world record weight lifted



## Geometric Similarity:

$W_T \propto$ strength $\propto \ell^2$

$W \propto$ mass $\propto \ell^3$

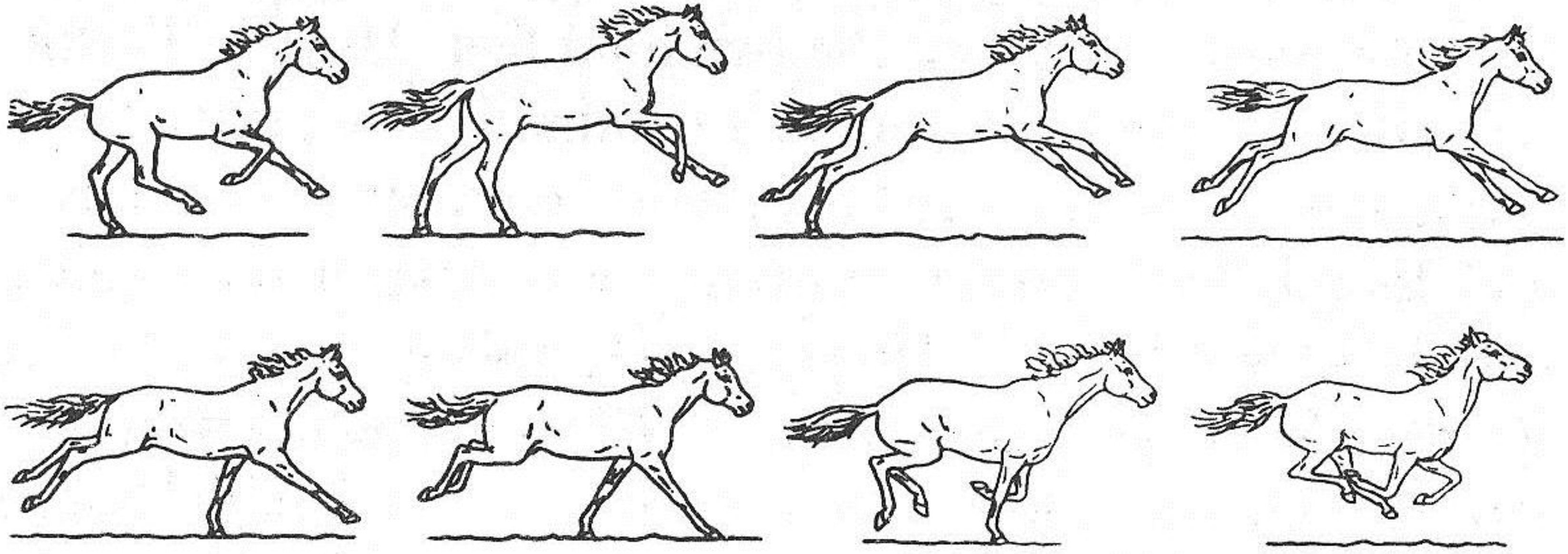
$\Rightarrow \quad W_T \propto W^{2/3}$

# Elastic Similarity:

The shapes of individuals ***of different species*** are characterized by two length scales:

- **long bone and muscle *length*** $\ell$
- **bone and muscle *diameter d***

Animals of increasing size must be designed so that the elastic bending of their limbs during, e.g., running, does not cause the limbs to break.



For no increase in bone breakage (same $\Delta y/L$), need

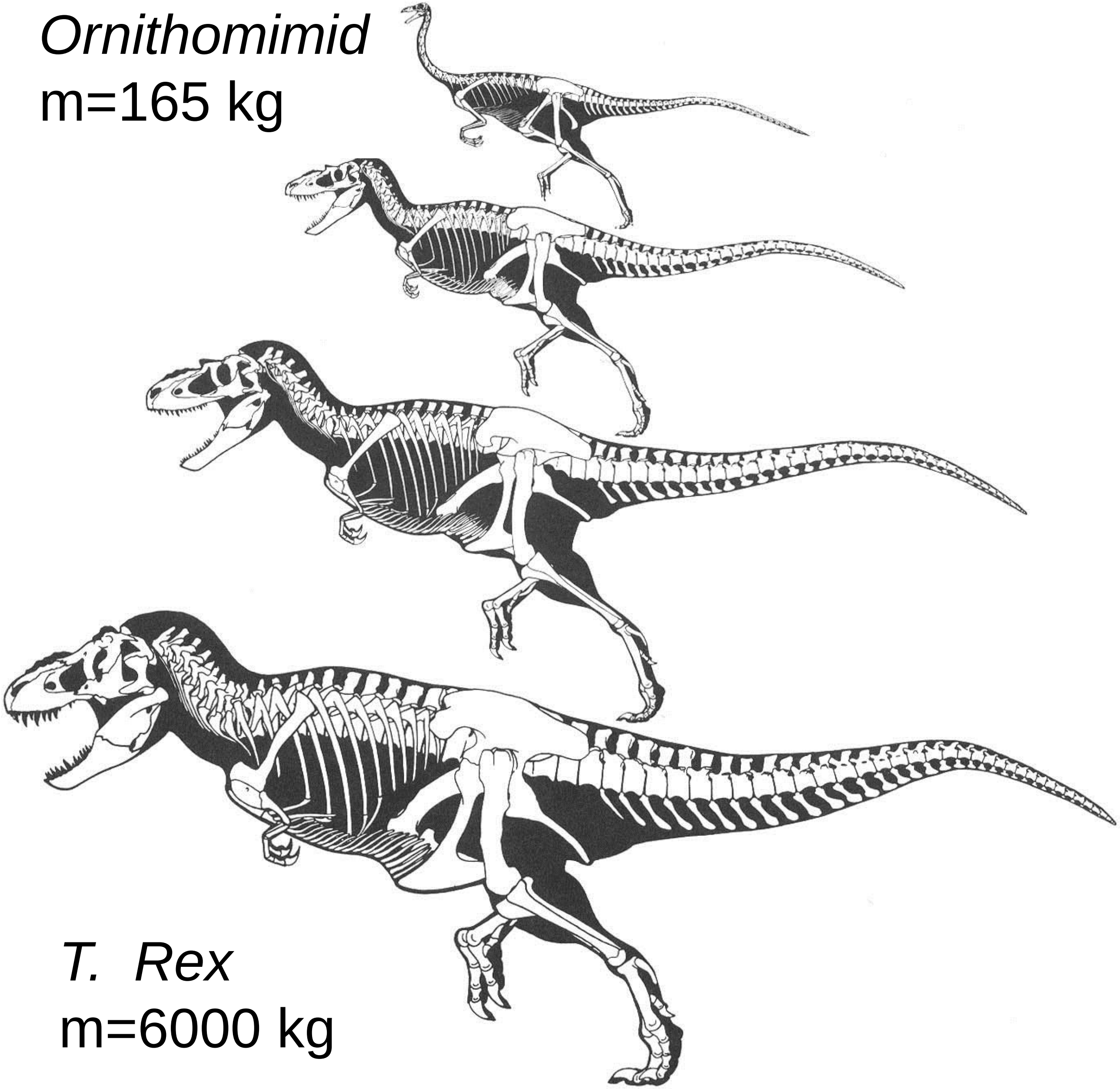
$\ell^3 \propto \mathbf{d}^2$ $\longleftarrow$ $\left\{ \frac{F}{A} = B\frac{\Delta y}{L} = \text{same} \propto \frac{\ell^3}{d^2} \right.$

$F \propto W \propto \ell^3 \qquad A \propto d^2$

**⇒ limb diameter $d \propto$ limb length $\ell^{3/2}$**

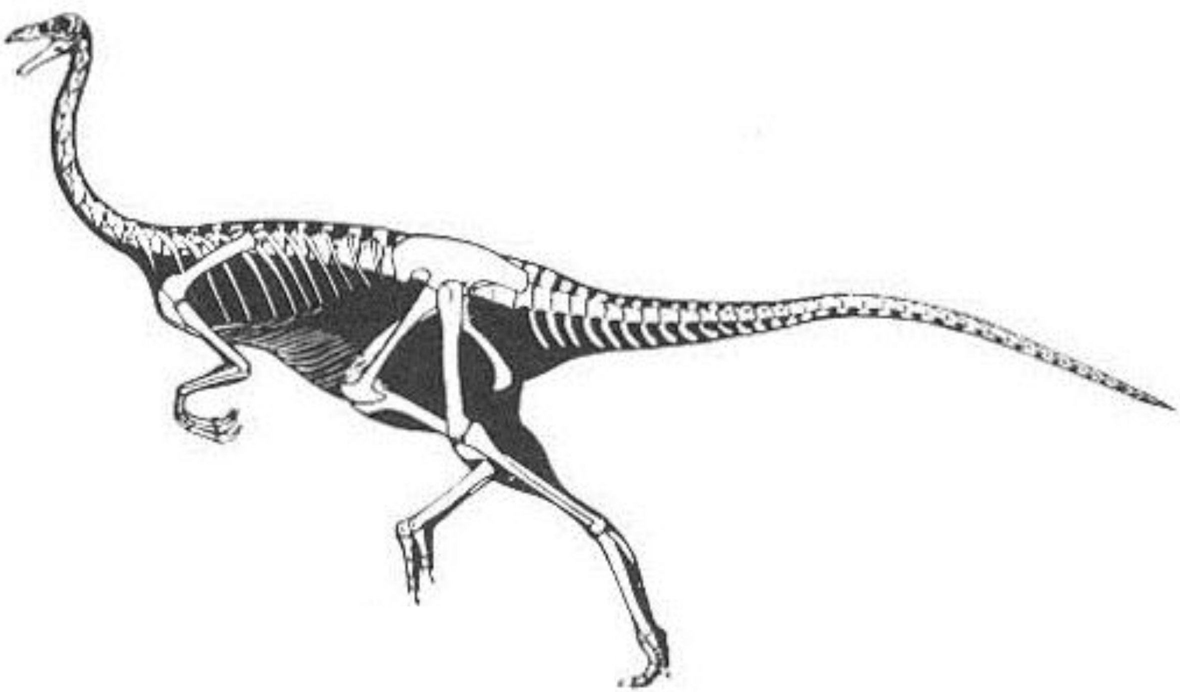
**∴ Larger species tend to have proportionately thicker limbs.**

# Theropod Dinosaurs:

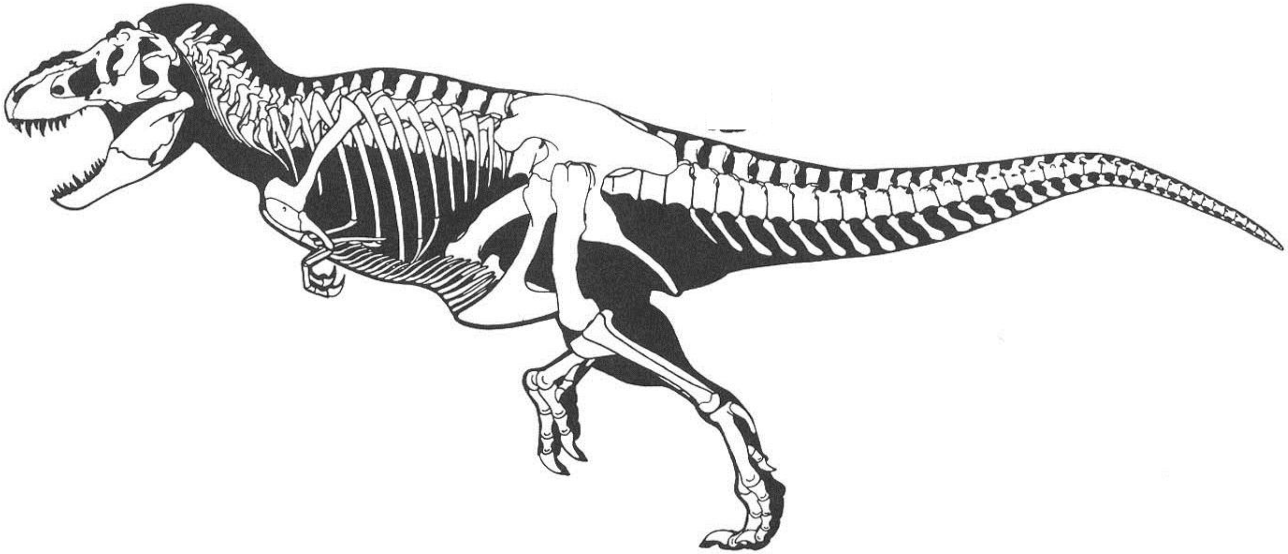


*Ornithomimid*
m=165 kg

*T. Rex*
m=6000 kg



*Ornithomimid*
m=165 kg
**magnified 2.7 ×**



*T. Rex*
m=6000 kg

# Thermodynamics and Heat:

- **Temperature:** measure of the internal energy from motion of atoms of an object

$$T({}^\circ C) = T({}^\circ \underset{\uparrow \text{ "Kelvin"}}{K}) - 273.15^\circ$$

$$T({}^\circ F) = \left(\frac{9}{5}\right) T({}^\circ C) + 32^\circ \implies 32^\circ F = 0^\circ C$$

Lower limit: $T = 0$ Kelvin $= -273.15\,{}^\circ C$

- **Thermal equilibrium**

  - temperatures of objects same
  - no net heat flow between objects