Lecture 29

Recap

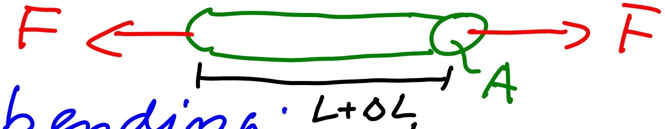
- Elasticity: stress & strain

Young's modulus

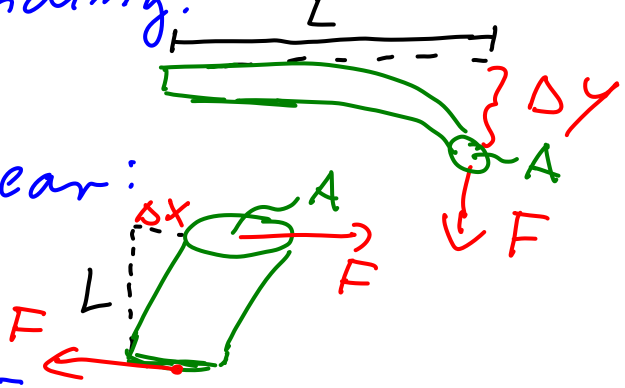
$$\left.\begin{aligned} \frac{F}{A} &= E\,\frac{\Delta L}{L} \\ \frac{F}{A} &= B\,\frac{\Delta Y}{L} \\ \frac{F}{A} &= g\,\frac{\Delta X}{L} \end{aligned}\right\}$$

for small strains

  - tension / compression:

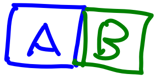


  - bending:
  - shear:



  - $(F/A)_{max}$ = ultimate strength = max. stress that can be applied

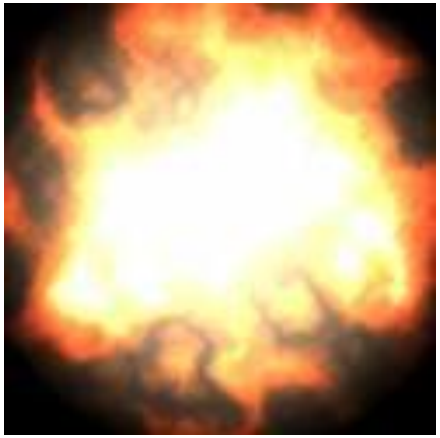
- Thermal Equilibrium:



  - no net heat flow between objects
  - same temperature $T_A = T_B$

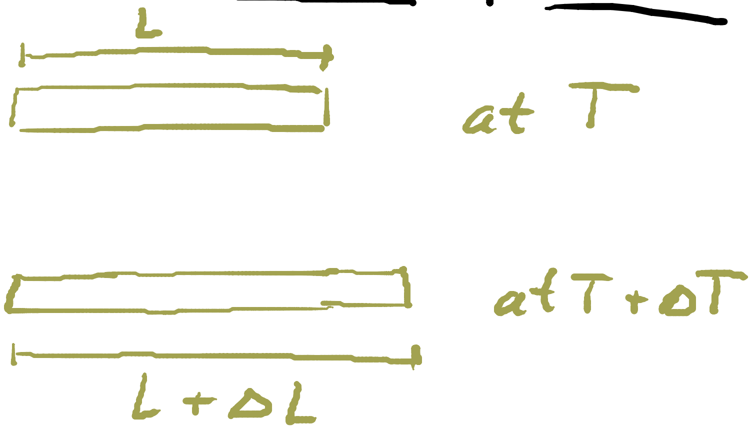
Today:

- Thermal expansion
- Heat energy
- Latent heat
- Thermal conduction



• Thermal Expansion:

→ Linear expansion:



temperature change

$$\boxed{\frac{\Delta L}{L} = \alpha \, \Delta T}$$

fractional change in length (strain)

coefficient of linear expansion

$[\alpha] = [\frac{1}{T}] = \frac{1}{K}$

- α depends on material type
- can be >0 or <0

⇒ applies to every linear dimension of object!

On a cold winter day with **T=-10 °C**, the Eiffel Tower is **300 m** tall.

How tall is it on a hot summer day when **T=40 °C**?

The Eiffel Tower is made from steel with a thermal expansion coefficient

α **=11 × 10^{-6}/K**.

$$\frac{\Delta L}{L} = \alpha \Delta T$$

$$\Delta T = 40°C - (-10°C) = 50K$$

$$\Rightarrow \Delta L = \alpha \Delta T L$$

$$= 11 \cdot 10^{-6} /K \cdot 50K \cdot 300m$$

$$= 0.16m$$

$$\Rightarrow L(40°C) = L(-10°C) + \Delta L$$

$$= 300m + 0.16m$$

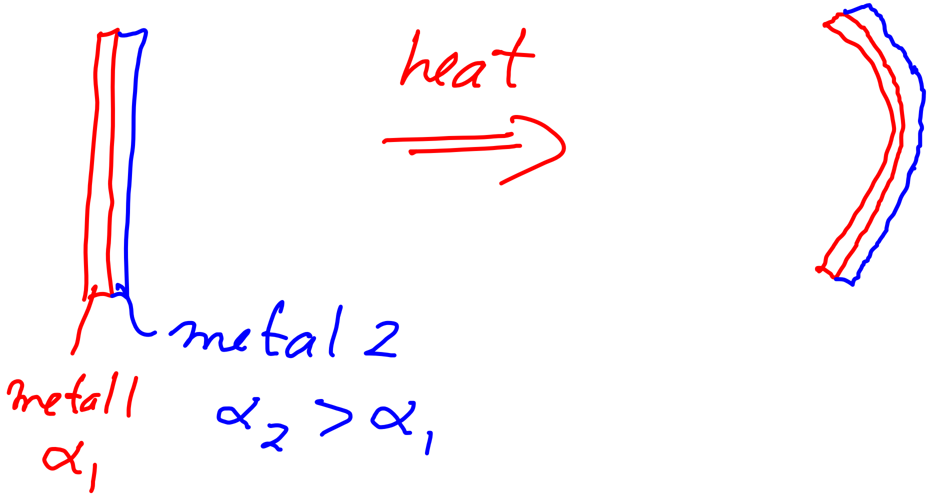
A. **300.001 m**

[✓ **B.**] **300.16 m**

C. **316 m**

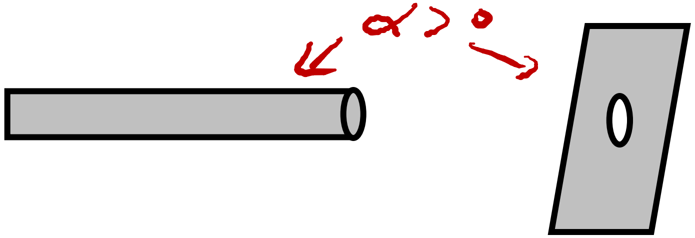
D. **420 m**

Demo: Bimetal strip

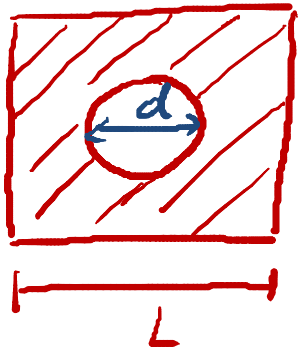


$\Rightarrow$ used in thermostats...

A **cylindrical rod** is just **slightly too large** to fit through a circular **hole**.



If you want the rod to fit through the hole, should you **heat the plate or cool it**?



heat ⇒ <u>all</u> linear dimensions grow (if $\alpha > 0$)

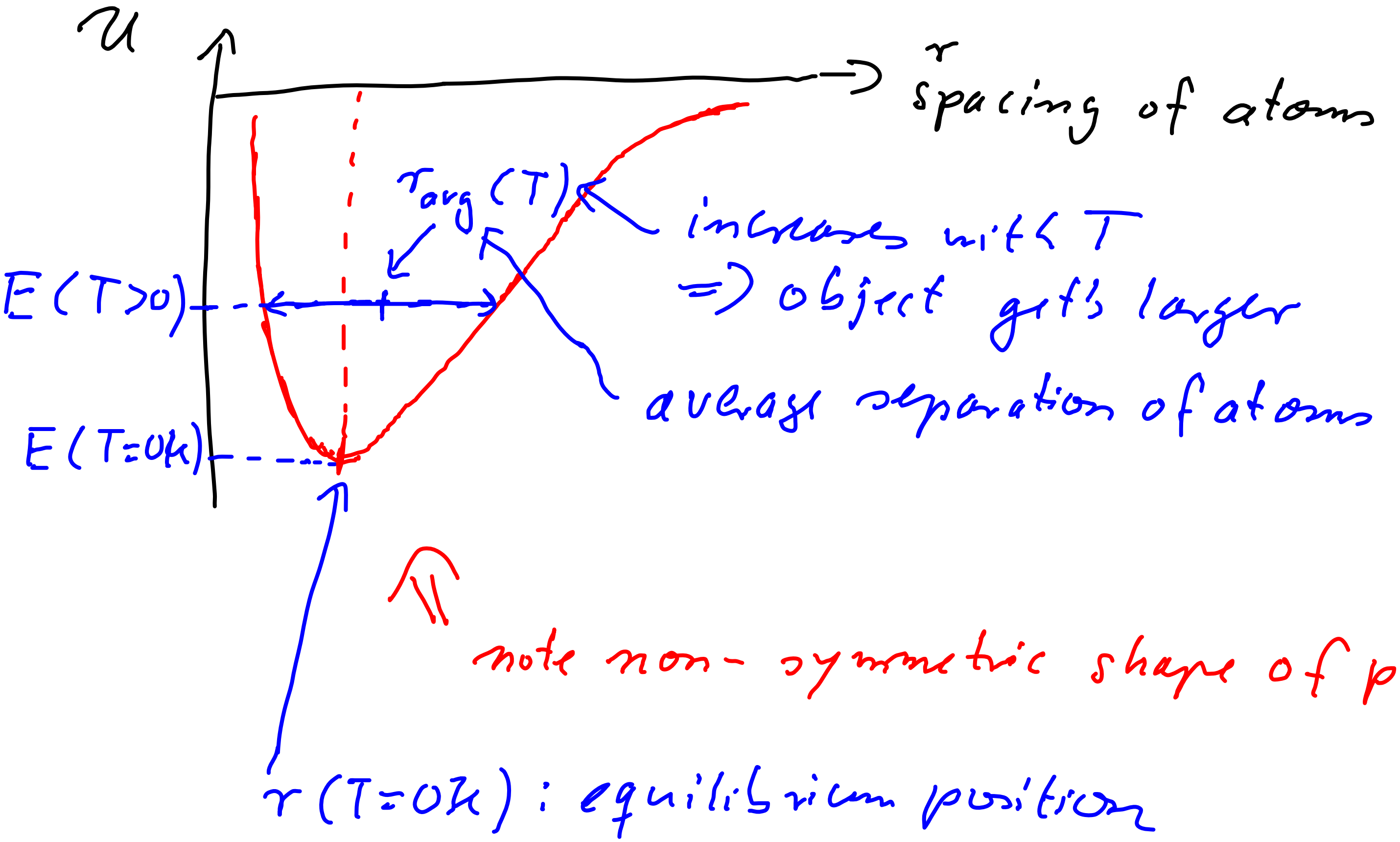
⇒ diameter d of hole gets bigger!

[✓ A.] **Heat**

B. **Cool**

Why do objects expand?

→ potential energy between a pair of atoms:



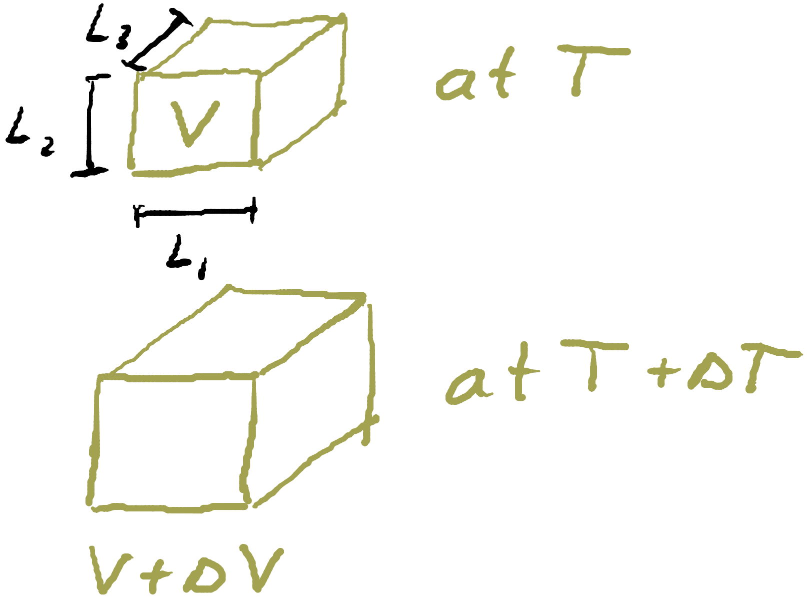
increases with T ⇒ object gets larger

average separation of atoms

note non-symmetric shape of potential!

r(T=0K): equilibrium position

→ Volume Expansion:



$$\boxed{\frac{\Delta V}{V} = \beta \Delta T}$$

↑ coefficient of volume expansion

$$\boxed{\beta = 3\alpha}$$

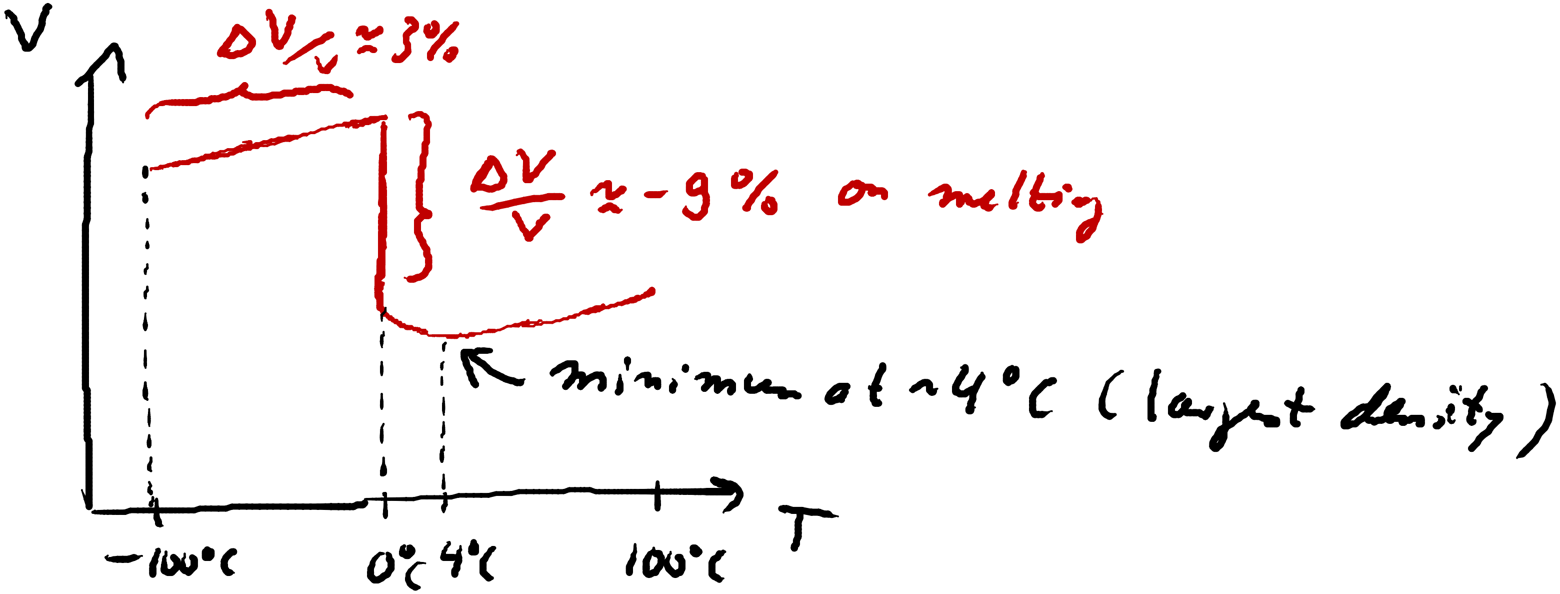
Note: In general, $\alpha = \alpha(T)$, $\beta = \beta(T)$

⇒ above equations only valid for small ΔT

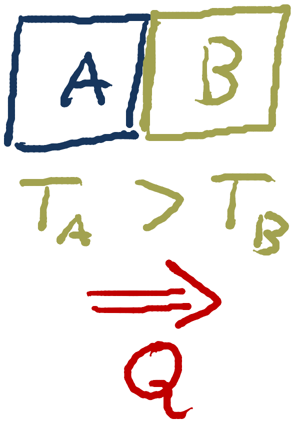
⇒ $\alpha, \beta \approx$ const

→ Also big changes in volume of object by phase transitions:

Example: water $\xrightarrow[0°C]{}$ ice



→ Heat Energy Q:



when $|\Delta T| > 0$ between two objects heat energy Q will be transferred between them until $\Delta T \to 0$

$[Q] = J$

⇒ Heat energy required to change the temperature of an object from T_i to T_f ?

$$Q \propto (T_f - T_i) = \Delta T$$

$Q > 0$: heat energy is transferred <u>to</u> object

$Q < 0$: heat energy is transferred <u>from</u> object

$$\Rightarrow \quad Q = C\,(T_f - T_i) = C\,\Delta T$$

$\uparrow$ change!

C = heat capacity of the object

$[C] = \mathrm{J}/\mathrm{K}$

$$C \propto m \quad \Rightarrow \quad C = c\, m \leftarrow \text{mass}$$

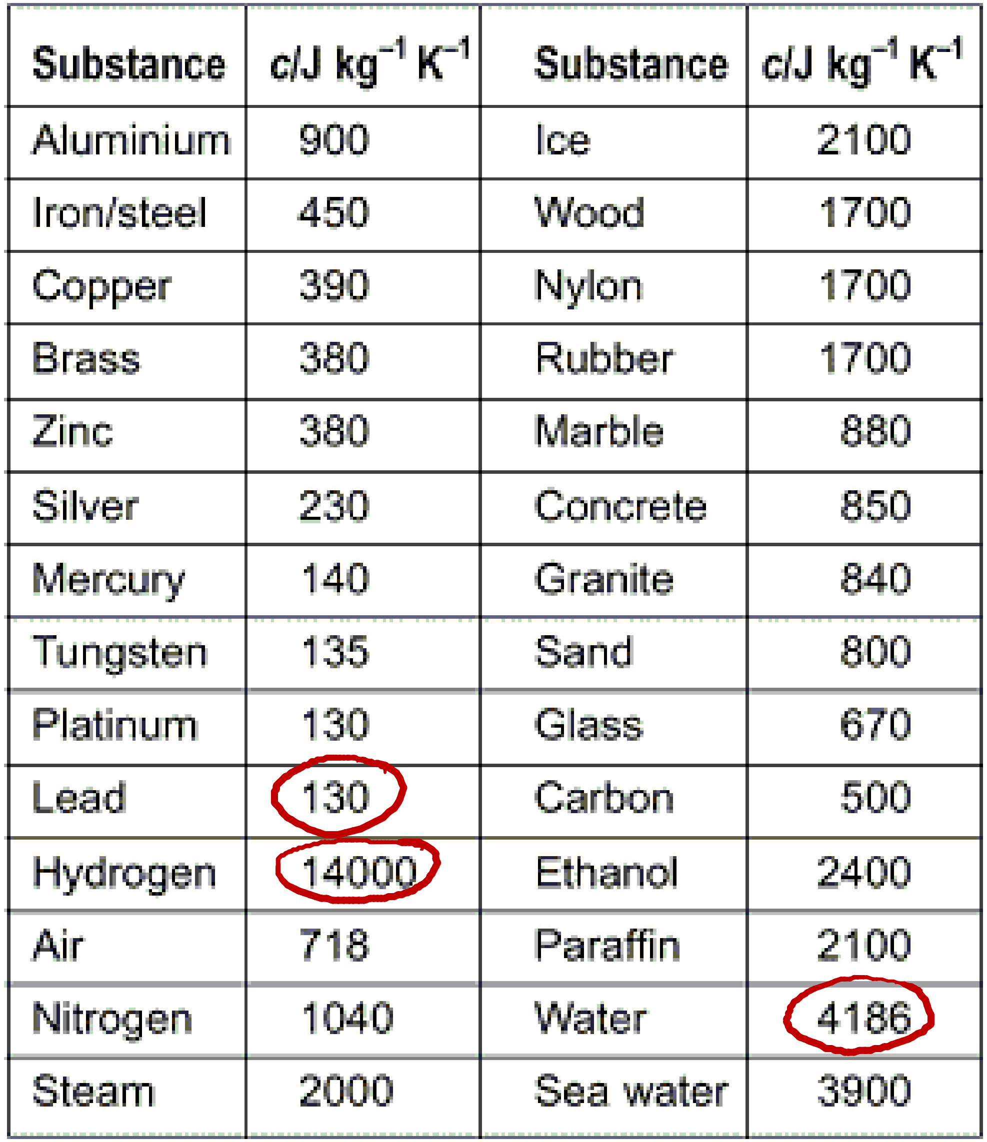
$$\Rightarrow \quad \boxed{Q = c\, m\,(T_f - T_i) = m\, c\, \Delta T}$$

c = specific heat
intrinsic property
of material

mass of object

$[c] = \mathrm{J}/(\mathrm{K}\cdot\mathrm{kg})$

Specific Heat c:



Substance	c/J kg^{-1} K^{-1}	Substance	c/J kg^{-1} K^{-1}
Aluminium	900	Ice	2100
Iron/steel	450	Wood	1700
Copper	390	Nylon	1700
Brass	380	Rubber	1700
Zinc	380	Marble	880
Silver	230	Concrete	850
Mercury	140	Granite	840
Tungsten	135	Sand	800
Platinum	130	Glass	670
Lead	130	Carbon	500
Hydrogen	14000	Ethanol	2400
Air	718	Paraffin	2100
Nitrogen	1040	Water	4186
Steam	2000	Sea water	3900

→ Heat of Phase Transition:

Q that must be added at constant T to convert a substance from one phase to another.

Example: Ice $\xrightarrow{\quad}$ water

$0°C \quad Q_{s\to L} \quad 0°C$

- Solid ⟷ Liquid

at $\underset{\text{fusion}}{\overset{\nearrow}{T_F}}$, $\underset{\text{melting}}{\overset{\uparrow}{T_M}}$

$$Q_{s\to L} = \underset{\text{Latent heat of fusion}}{L_F} \cdot \overset{\text{mass}}{m}$$

$\Big\}$ note: $Q_{L\to s} = -Q_{s\to L}$

$[L_F] = J/kg$

- Liquid ⟷ gas/vapor

at $\underset{\text{vapor}}{T_V}$, $\underset{\text{boiling}}{T_b}$

$$Q_{L\to g} = \underset{\text{Latent heat of vaporization}}{L_V} \cdot m$$

$\Big\}$ $Q_{g\to L} = -Q_{L\to g}$

for water:

$c = 4190\ \text{J/kgK}$ in liquid phase

$$\left.\begin{array}{l} L_F = 333\ \text{kJ/kg} \\ \\ L_V = 2256\ \text{kJ/kg} \end{array}\right\} \text{large!}$$

$L_V >> L_F$: since more bonds broken

$\Uparrow$

H-bonding in water

Suppose that it takes an amount of heat Q to bring a pot of water initially at 20 °C to a boil.

How much additional heat must you add to boil all water away?

c = specific heat of water

L_v = latent heat of vaporization

$$Q\,(20°C \to 100°C) = c\, m\, \underbrace{\Delta T}$$

$$\Delta T = 100°C - 20°C = 80K$$

$$\Rightarrow m = \frac{Q}{c\, 80K}$$

$$\downarrow$$

$$Q_{\text{to boil}} = L_v\, m = \underbrace{L_v \frac{1}{c\, 80K}}_{\simeq 6.7 \text{ for water}} Q$$

all water into vapor

Additional heat =

A. $Q \times L_v$

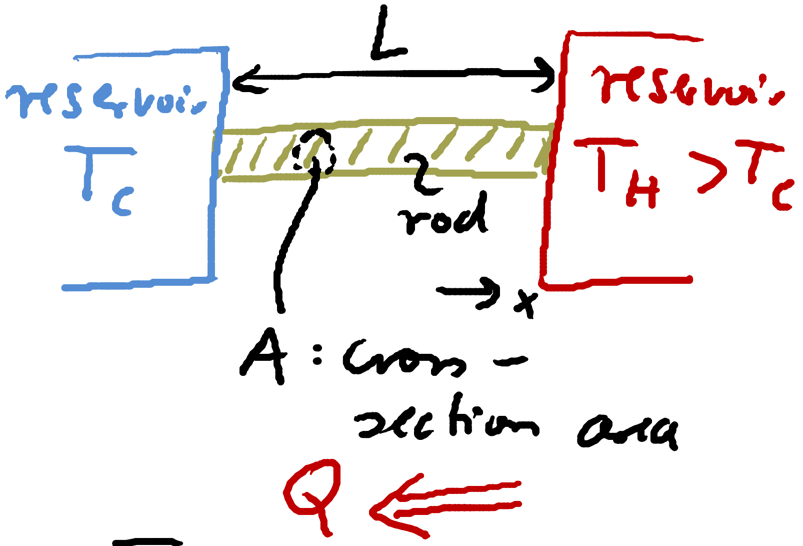
B. $Q \times L_v / c$

[✓ **C.**] $Q \times L_v / (c \times 80°)$

D. $Q \times L_v / (c \times 100°)$

→ Heat Transfer Mechanisms:

① Conduction

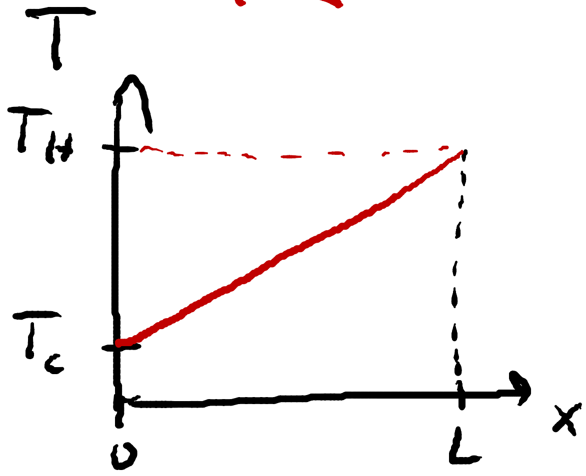


- direct physical contact
- no transfer / motion of mass in conduction
- assume steady state (wait long enough)

⇒ all T's and T-profiles are constant

⇒ no heat energy goes into heating up objects

⇒ heat energy Q that enters at T_H leaves at T_C



$\Rightarrow$ in steady state:

$$\text{Power} = P = \frac{\Delta Q}{\underbrace{\Delta t}} = \left(\begin{array}{l}\text{rate of heat transfer}\\ \text{from } T_H \text{ to } T_c \text{ accross}\\ \underline{\text{any}} \text{ vertical plane in rod}\end{array}\right)$$

amount of heat energy transported per time interval Δt

$$P \propto \frac{A}{L}\left(T_H - T_c\right)$$

$$\Rightarrow \boxed{P = \kappa \frac{A}{L}\left(T_H - T_c\right)} \quad \text{in } \underline{\text{conduction}}$$

$[P] = W = \frac{J}{s}$

κ = thermal conductivity; material property

$$[\kappa] = \frac{W}{m \cdot K}$$

Material	Thermal conductivity *K* W/(m·K) at 25C
Air	0.025
Wood	0.04 – 0.4
Alcohol or oil	0.15
Soil	0.15
Rubber	0.16
Epoxy (unfilled)	0.19
Water (liquid)	0.6
Glass	1.1
Ice	2
Stainless steel	15
Lead	35.3
Copper	401
Silver	429
Diamond	900 – 2320