Recap:

Static Liquid:

- Buoyant Force:

\[ F_{\text{buoy}} = \rho V \text{fluid displaced} \]

• Weight of fluid displaced by object
• Net force on object from fluid pressure on its surface

Note: \( F_{\text{buoy}} \) is a consequence of pressure variation with depth \( h \) in a fluid!
Recap:

- **Ideal Fluid**: no fluid friction; $S = \text{const}$; laminar flow

- **Ideal Flow**:
  
  - **Volume flow rate**: $R = \frac{\text{Volume passing}}{\Delta t}$
    
    $x\text{-sectional area}$
    
    $= A \cdot \nu$

    $\uparrow$
    
    $\text{speed of flow}$

  - **Continuity**:
    
    $R_1 = R_2 \Rightarrow A_1 \nu_1 = A_2 \nu_2$
Today:

- Fluids in motion
- Bernoulli’s Equation
- Measuring air speed
**Bernoulli's Equation**

Consider a tube with flow:

1. \( \vec{F}_1 = \vec{P}_1, A_1 \)
2. \( \vec{F}_2 = \vec{P}_2, A_2 \)
3. \( \vec{P}_1, A_1 \) (pressure, cross-sectional area)
4. \( \vec{U}_1 \) (speed of flow)

**Note:** For ideal fluid, i.e. no friction here!

\[ P_1 = \rho \cdot \vec{V}_1 \]

\[ A_1 \times \text{ext. area} \]

\[ \vec{U}_1 \] (speed of flow)

\[ \rightarrow \text{Consider a tube with flow:} \]

\( \vec{F}_1 = \vec{P}_1, A_1 \)

\( \vec{F}_2 = \vec{P}_2, A_2 \)

\( \vec{P}_1, A_1 \) (pressure, cross-sectional area)

\( \vec{U}_1 \) (speed of flow)

\[ \text{Note: for ideal fluid, i.e. no friction here!} \]

\[ \begin{align*}
\text{Continuity: } & R_1 = R_2 \\
\Rightarrow & A_1, U_1 = A_2, U_2 \\
\frac{\partial V_1}{\partial t} &= \frac{\partial V_2}{\partial t}
\end{align*} \]

\[ \begin{align*}
\text{Use Work-Kinetic Energy Theorem:} \\
\Delta W = W_{\text{fluid by force}} \\
&= W_{\text{gravity}} + W_{\text{fluid by applied force}} \\
&= -\Delta U_g
\end{align*} \]
FIG. 14-20 Fluid flows at a steady rate through a length $L$ of a tube, from the input end at the left to the output end at the right. From time $t$ in (a) to time $t + \Delta t$ in (b), the amount of fluid shown in purple enters the input end and the equal amount shown in green emerges from the output end.

The fluid flow is given by

$$W_{on\ fluid} = \frac{\partial U_g}{\partial t} + \frac{\partial \phi}{\partial x} = \frac{\partial E}{\partial t} \quad \text{during time } \Delta t$$

- Pressure difference between (1) and (2) drive flow:

$$W_{on\ fluid} = F_1 \Delta x_1 - F_2 \Delta x_2$$

by op

$$= \rho \bar{A}_1 \Delta x_1 - \rho \bar{A}_2 \Delta x_2$$

$$\frac{\Delta V_1}{\Delta V_2} = \frac{\Delta V_1}{\Delta V_2} = \Delta V$$

$$\Rightarrow W_{on\ fluid} = (\rho \bar{A}_1 - \rho \bar{A}_2) \Delta V \quad \text{(A)}$$

- $\Delta U_g = U_2 - U_1 = m_2 g y_2 - m_1 g y_1$

$m_2 = m_1 = m$ (with $\Delta V_1 = \Delta V_2$)

$$\Rightarrow \Delta U_g = mg(y_2 - y_1) = \rho \bar{A} V g(y_2 - y_1) = \text{const} \quad \text{(B)}$$

- $\Delta X = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$

$$= \frac{1}{2} \rho \bar{A} V (v_2^2 - v_1^2) \quad \text{(C)}$$
\( \text{insert (A), (B), (C) into:} \)
\[
W_{\text{on fluid}} = \frac{\partial U_g}{\partial p} + \frac{\partial K}{\partial t} \quad \text{during flow in some } \sigma \text{ at }
\]
\[
\Rightarrow (P_1 - P_2) \Delta V = \frac{\partial V}{\partial (\gamma_2 - \gamma_1)} + \frac{1}{2} \frac{\partial V}{\partial (v_2^2 - v_1^2)}
\]
\[
(P_1 - P_2) = \rho g (\gamma_2 - \gamma_1) + \frac{1}{2} \rho [(v_2^2 - v_1^2)]
\]

\[\begin{align*}
\frac{\text{Work}}{\text{Volume}} &= \frac{\Delta U_g}{\text{Volume}} + \frac{\Delta K}{\text{Volume}} \\
\rho \frac{g}{m^3} &= \frac{\gamma_2 - \gamma_1}{m^3}
\end{align*}\]

**Bernoulli's Equation**

Relate change in pressure in flowing fluid to change in height and change in flow speed.
\[ R = \frac{d}{dt} \frac{\Delta \text{Volume}}{\text{Area}} = \nu_1 A_1 = \nu_2 A_2 = \text{const} \]

\[ P_1 + \rho g y_1 + \frac{1}{2} \rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2} \rho v_2^2 + \frac{1}{\rho} \text{ const} \]
Air flows through a tube with a constriction. Where is the pressure highest?

A. at A
B. at B
C. same at both A and C
D. not enough information

\[ A = \frac{1}{2} S U_A^2 = \frac{1}{2} S U_B^2 = \frac{1}{2} S U_C^2 \]

\[ P_A + \frac{1}{2} S U_A^2 = P_B + \frac{1}{2} S U_B^2 = P_C + \frac{1}{2} S U_C^2 \]

\[ \Delta P = P_A - P_B > 0 \text{ for constant height: when flow speed } \]

\[ \text{of } A \text{ to } B > U_A \]
Bernoulli:

For flow at constant height, if \( v \uparrow \), \( p \downarrow \)

Some Applications
Measuring Air Speed

At Hole A, air speed $v_A = 0$.
At Hole B, air speed $v_B > 0$.

By Bernoulli’s equation, $p_A + \frac{1}{2} \rho v_A^2 = p_B + \frac{1}{2} \rho v_B^2$

$\Rightarrow v_B = \left[\frac{2(p_A - p_B)}{\rho}\right]^{1/2}$
Perfume atomizers

Flowing air creates $\Delta p$ that pushes fluid out of container.
Airfoils in aircrafts?

Air must travel a **larger distance** over the top of an airfoil than over the bottom.

\[ \Rightarrow \text{air velocity } v_{\text{top}} > v_{\text{bot}}, \quad p_{\text{top}} < p_{\text{bot}} \]

\[ \Rightarrow \text{Lift Force } F_L \sim \Delta p A_{\text{wing}} \]

**But:**

- Bernoulli's equation is for laminar flow only!!
- The flow of air is highly turbulent here!
The Bernoulli principle acting on an umbrella

⇒ Same principle used in sailing ⇒ can go faster than wind!
Wind damage to buildings

\[ v_{\text{inside}} \sim 0, \quad v_{\text{outside}} \gg 0 \quad \Rightarrow \quad \Delta p = p_{\text{in}} - p_{\text{out}} \gg 0 \]

\[ \Rightarrow \text{building "explodes"!} \]

E.g.:

\[ v_{\text{outside}} = 360 \text{ km/h} \quad (\sim 220 \text{ mi/h}) \]
\[ \Delta p = (1/2) \rho_{\text{air}} v_{\text{out}}^2 \sim 6000 \text{ Pa} \quad (\sim 0.06 \rho_{\text{atm}}) \]

\[ \Rightarrow \text{Upward force on 80 m}^2 \text{ house roof:} \]
\[ \Delta p \ A \sim 5 \times 10^5 \text{ N} \quad \sim 50 \text{ tons!} \]
Hancock Building (Boston, 1973):
Ventilation of prairie dog burrows

\[ V_1 > V_2 \]

\[ \Rightarrow p_1 < p_2 \]
What is the pressure at the top of the siphon?

A. \( p_0 \)
B. \( p_0 + \rho \, gh \)
C. \( p_0 - \rho \, gh \) (static case!)
D. less than \( p_0 - \rho \, gh \)
E. greater than \( p_0 + \rho \, gh \)

\( \rho g h + \frac{1}{2} \rho \, v^2 = \text{const} \)

\( \Rightarrow \) \( p_1 > p_2 > p_3 \)

\( p_4 = p_0 \) outlet must be at \( p_0 \)
→ at point \( Q \):
\[ P_1 = P_0 , \quad Y_1 = 0 , \quad U_1 = 0 \quad (\text{At tank} \Rightarrow \text{At tube}) \]

→ at point \( 2 \):
\[ P = P_2 , \quad Y_2 = 0 , \quad U_2 > 0 \]
\[ P_2 < P_1 \]

\[ \text{Pressure difference from} \quad 1 \to 2 \quad \text{produces change in speed of flow} \quad (\Delta Y = 0) \]

→ at point \( 3 \):
\[ P = P_3 , \quad Y_3 = h , \quad U_3 = U_2 > 0 \]
\[ P_3 < P_2 \]

\[ \text{Work done by} \quad DP_2 \to 3 \quad \text{to raise fluid by} \quad h \quad \text{while} \quad \Delta X = 0 \]

→ at point \( 4 \):
\[ P_4 = P_0 , \quad Y = Y_4 = -d , \quad U_4 = U_3 = U_2 \]

Outlet must be at atm. pressure! \((p_0)\)
\[ \Rightarrow \text{use Bernoulli's equation for } (1) \text{ and } (2) \]
\[ P_1 + 8g h_1 + \frac{1}{2} g v_1^2 = P_3 + 8g h_3 + \frac{1}{2} g v_3^2 \]
\[ \Rightarrow P_0 + 0 + 0 = P_3 + 8g h + \frac{1}{2} g v_3^2 \]
\[ \Rightarrow P_3 = P_0 - 8g h - \frac{1}{2} g v_3^2 \Rightarrow \text{answer D!} \]

To find \( v_3 = v_2 = v_4 = v_{tube} \): use points \( 1 \) and \( 4 \)
\[ \Rightarrow P_0 + 0 + 0 = P_0 - 8g d + \frac{1}{2} g v^2 \]
\[ \Rightarrow v^2 = 2gd \Rightarrow v = \sqrt{2gd} \]
\[ \Rightarrow \text{as expected from 1-D motion for free fall from height } d! \]