Recap: Ideal Fluids in Motion

- **Ideal Fluid**: no fluid friction; $S = \text{const}$; laminar flow
- **Ideal Flow**:

\[ F_2 = P_2 A_2 \]

\[ \gamma_2 \]

\[ \gamma_1 = \frac{F_1}{P_1 A_1} \]

\[ A_2 \]

\[ U_2 \]

\[ P_2 \]

\[ \Rightarrow P_1 + Sg \gamma_1 + \frac{1}{2} S U_1^2 = P_2 + Sg \gamma_2 + \frac{1}{2} S U_2^2 \]

\[ \text{Work by } F_i \]

\[ \text{volume} \]

\[ \text{Ug} / \partial V \]

\[ \text{K} / \partial V \]

\[ \Rightarrow \text{for flow at const. height: where } U \text{ is big, } P \text{ is small!} \]
Today:

- Fluid friction
- Viscous flow though tube
- Viscous and turbulent drag
- Terminal speed
When blood flows though a blood vessel, it experiences **viscous drag, i.e. friction forces.**

Two **blood vessels** of **equal length** are connected in parallel so that they have **same pressure drop** $\Delta p$. The radius of the first vessel is $r_1$. Due to formation of atheriosclerotic plaques, the radius $r_2$ of the second vessel has been reduced by 50%.

What is the ratio $R_2/R_1$ of the **volume flow rates** in the two vessels?

$$R_2 / R_1 = ?$$

### Choices

- A. 1
- B. 1/2
- C. 1/4
- D. 1/8
- E. 1/16

**Solution:**

The volume flow rate $Q$ is given by

$$Q = \frac{\Delta p}{L} \left( \frac{2r}{8\eta} \right)$$

Since $r_2 = \frac{1}{2} r_1$, we have

$$\frac{R_2}{R_1} = \left( \frac{r_2}{r_1} \right)^4 = \left( \frac{1}{2} \right)^4 = \frac{1}{16}$$

Therefore, the correct answer is **E. 1/16**.
Fluid Friction:

Recall: Solid on Solid friction:
- $f_s, f_k$ oppose relative motion of the surface in contact!

Fluid-Solid Friction:

- $U_f = U_f(y)$ depends on height above solid surface
- $U_f(y=0) = 0$ at $y = 0$ (i.e., at fluid-solid interface) relative to surface (no relative motion at surface)
- Example:
  - Rain drops on car windshield
  - Air near earth's surface
- **Fluid-Fluid Friction:**

- Friction forces (drag) between layers at different speeds
- Fluid friction opposes relative motion of adjacent fluid layers

$\Rightarrow$ Viscosity $\eta = \text{"eta"}$
specific geometry

$F_{drag \ from \ fluid \ friction}$

area $A$

 Fluid velocity $U_{top}$

$U_{top \ plate} = \text{constant}$

$U_{bottom \ plate} = 0$

$\dot{U}_{fluid}(y)$

$\sum F_x = 0 \Rightarrow U_{com} = \text{const}$

$F_{top}: \text{pull on top plate}$

fluid between plates
no relative motion to bottom plate

\[ \text{slope} = \frac{dV}{dy} = \frac{U_{\text{top}}}{L} \]

\[ F_{\text{fluid drag}} \propto A \frac{U_{\text{top}}}{L} \]
- large \( A \) \( \Rightarrow \) more force
- small \( L \) \( \Rightarrow \) more force
- \( \frac{U_{\text{top}}}{L} = \frac{dV}{dy} \)

\[
F_{\text{fluid drag}} = \eta A \frac{dV}{dy}
\]

\[ \eta = \text{viscosity} \quad \eta = \frac{k_{\text{g}}}{2m} = 1.8 \times 10^{-5} \text{Pa.s} \]

\[ \text{at} \ t = 10^{-2} \text{Pa.s} \]
Viscous, laminar flow through tube (const. height)

Assume: \( y = \text{const.}, A = \text{const} \)

Fluid speed in the tube:

\[
U(r) = \frac{1}{4} \frac{1}{\gamma} \frac{\Delta P}{L} (R_{\text{tube}}^2 - r^2)
\]

Radial distance from tube axis

Work by pressure difference required to provide energy lost by friction and to maintain constant flow!

\[
\eta \text{ (viscosity)} \quad \text{Note: Bernoulli's equation only for ideal fluids, i.e. no friction, } \eta = 0
\]
Volume flow rate for tube:

\[ R = A \cdot \text{V}_{\text{avg}} = \pi \tau_{\text{tube}}^2 \cdot \text{V}_{\text{avg}} \]

average of flow speed over cross-sectional area of tube

\[ \frac{\pi \tau_{\text{tube}}^2}{32 \eta} \cdot \frac{1}{8} \frac{1}{\eta} \frac{\Delta P}{L_{\text{tube}}} \tau_{\text{tube}}^4 \]

from area

from \( \eta \)

Poiseuille's equation

\[ R_{\text{tube}} = \frac{\pi \tau_{\text{tube}}^2}{32 \eta} \frac{\Delta P}{L_{\text{tube}}} \tau_{\text{tube}}^4 \]

- for laminar, \( \nu \) is con flow only?

- \( \nu \) obeyed in human circulatory system
**Fluid drag on moving objects**

Two regimes (extreme cases):

<table>
<thead>
<tr>
<th>Turbulent drag regime</th>
<th>Viscous drag (laminar flow) regime</th>
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<tbody>
<tr>
<td>- high velocities, large objects, small ( \eta )</td>
<td>- low velocities, small objects, large ( \eta )</td>
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</tbody>
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\[
F_{\text{drag}} = \frac{1}{2} C S_{\text{fluid}} A_{\text{obj}} U^2
\]

\[
F_{\text{drag}} = \frac{1}{4} S_{\text{fluid}} \frac{\pi}{4} r_{\text{sp}}^2 U^2
\]

\[
A_{\text{sp}} = \pi r_{\text{sp}}^2
\]

\[
F_{\text{drag}} \propto \eta U
\]

\[
F_{\text{drag}} = 6 \pi \eta r_{\text{sp}} U
\]

\[
\text{Stokes' Law}
\]

\[
\Rightarrow \text{for sphere: } C = \frac{1}{2}
\]

\[
\Rightarrow \text{for sphere: } (r_{sp} = \text{radius of sphere})
\]
Fluid flow pattern around a moving cylinder

**Turbulent flow**: High velocities / larger diameters / "thin" fluids:
Laminar Flow: Low velocities / small diameters / "thick" fluids

Fluid flow pattern around a moving cylinder
What determines whether we are in the viscous regime or turbulent regime?

1) compare force in regimes:

\[
\frac{F_{\text{drag, turb}}}{F_{\text{drag, visc}}} \propto \text{on sphere} = \frac{1}{24} \left( \frac{8 \pi r_{sp} \nu}{\eta} \right) \]

\[
\text{if } D > 1 = \text{turbulent}
\]

\[
\text{if } D < 1 = \text{viscous}
\]

Reynolds' number is (dimensional)
Reynold's Number:

\[ \text{Re} = \frac{\rho \text{fluid} \cdot r \cdot V}{\eta} \]

\( \eta \): "typical" dimension of the object

- for sphere: \( r = r_{\text{sphere}} \) moving through fluid:
  \( \text{Re} \gg 24 \rightarrow \) turbulent \( \rightarrow \) use F
drag, turb
  \( \text{Re} \ll 24 \rightarrow \) viscous/laminar \( \rightarrow \) use F
drag, visc.

- for flow through pipe: \( r = \text{radius of pipe} \)
  \( \text{Re} > 3000 \rightarrow \) turbulent
  \( \text{Re} < 2000 \rightarrow \) viscous
determined experimentally
Terminal speed \( v_t \) of a sphere:
- at \( v = v_t \) when \( a \rightarrow 0 \)
  \[ \Rightarrow \mathbf{F}^\prime = 0 \quad \text{at} \quad v = v_t \]
  \[ \Rightarrow |F_{\text{drag}}| = |W| - |F_{\text{ buoy}}| \]
  \[ \Rightarrow \text{In viscous regime:} \]
  \[ F_{\text{drag, visc}} (v = v_t) = 6 \pi \eta r_{sp} v_t \]
\[ W = mg = V_{sp} \rho_{sp} g = \frac{4}{3} \pi r_{sp}^3 \rho_{sp} g \]
\[ F_{\text{ buoy}} = W_{\text{ fluid dim}} = \frac{4}{3} \pi r_{sp}^3 \rho_{\text{ fluid}} g \]
\[ v_{t, \text{ visc}} = \frac{2}{9} \left( \frac{8}{9} \frac{\rho_{sp} - \rho_{\text{ fluid}}}{\rho} \right) \frac{g r_{sp}^2}{\eta} \propto r_{sp}^2 \]