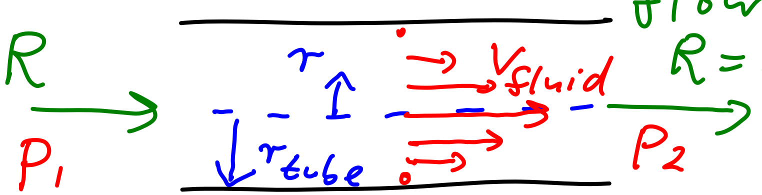
Recap:

• Fluid Friction

- v_{fluid} wrt solid surface $= 0$ at solid-fluid interface
- Fluid friction/drag opposes relative motion

$$F_{drag} = \eta A \frac{dv}{dy} \qquad \eta = \text{viscosity}$$

- Viscous/laminar flow through pipes ($A = \text{const}$, height $=$ const)



$\Delta p = p_1 - p_2 > 0$

$$R = \frac{\pi}{8\eta} \frac{\Delta p}{L} r_{tube}^4 \propto r_{tube}^4$$

Need $p_1 > p_2$ to provide power that is dissipated by fluid friction! [without friction: use Bernoulli's equation $\Rightarrow$ gives $\Delta p = 0$ for $A = \text{const}$ and $y = \text{const}$]

Recap

- Fluid Drag on a Sphere moving through a Fluid:

– Turbulent regime (big r, v; small η) : $F_{Drag} = \frac{1}{4}\rho_f \pi r_{sp}^2 v^2 \Big\}$ if $Re = \frac{\rho_f r v}{\eta} \gg 24$

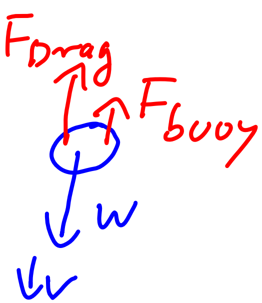
– Viscous regime (small r, v; large η) : $F_{Drag} = 6\pi\eta r_{sp} v \Big\}$ if $Re \ll 24$

$\Rightarrow$ Reynold's Number:

$$Re = \frac{\rho_{fluid} \cdot r v}{\eta}$$

r: "typical" dimension of the object

- Terminal Speed of a Sphere:



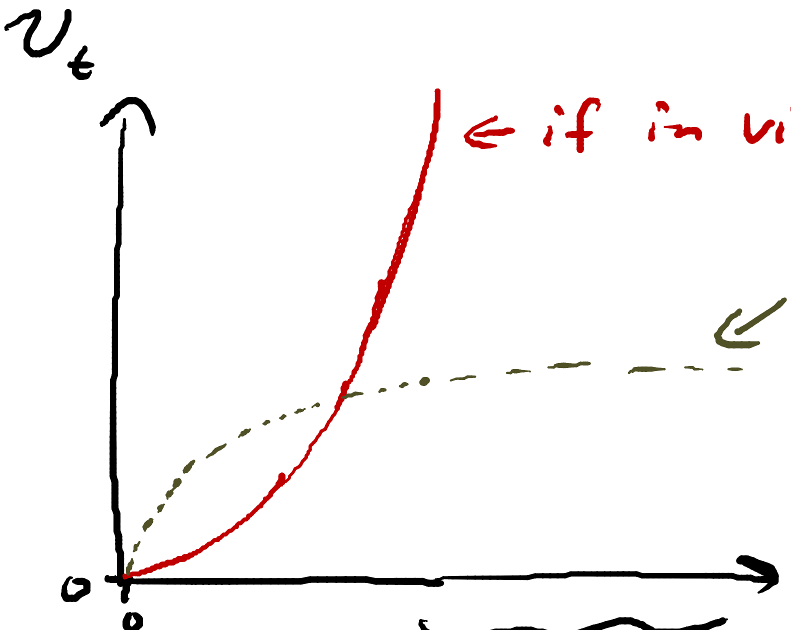
$$V_{t,turb} = \sqrt{\frac{16}{3}\frac{(\rho_{sp}-\rho_f)}{\rho_f} g r} \quad \text{or} \quad V_{t,visc} = \frac{2}{9}\frac{\rho_{sp}-\rho_f}{\eta} g r^2$$

$\Rightarrow$ to calculate v_t: use formula that gives smallest V_t (biggest F_{drag})

→ In the turbulent regime:

$$F_{drag,turb}(v=v_t) = \frac{1}{4}\,\rho_{fluid}\,\pi\,r_{sp}^2\,v_t^2$$

$$\Rightarrow \quad v_{t,turb} = \sqrt{\frac{16}{3}\,\frac{(\rho_{sp}-\rho_{fluid})}{\rho_{fluid}}\,g\,r_{sp}} \;\propto\; \sqrt{r_{sp}}$$



← if in viscous regime: $v_{t,visc} \propto r_{sp}^2$

if in turbulent regime $v_{t,turb} \propto \sqrt{r_{sp}}$

r_{sp} $(\rho_{sp}, \rho_{fluid} = \text{const})$

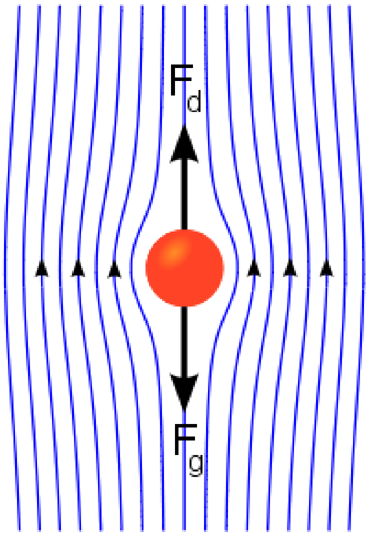
Use formula for $V_{t,visc}$

Use formula for $V_{t,turb}$

Use formula that gives **smallest** v_t

Today:

- **Surface tension**
- **Bubbles**
- **Liquid-solid-gas interfaces**



What happens?

A. Air flows from the *large* to the *small* balloon

B. Air flows from the *small* to the *large* balloon

C. Insufficient information

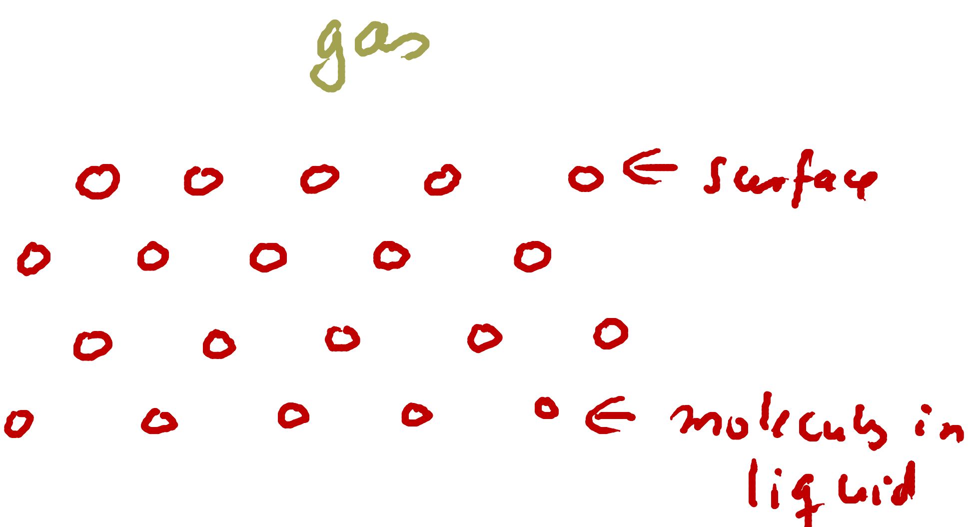
Surface Tension:

→ in absence of gravity and contact with solid surface

→ liquids will form spheres!

→ sphere: geometry with <u>smallest surface area</u> for a given volume!

→ <u>why?</u>



- Molecules in liquid attract each other

⇒ minimize their energy E by getting close to each other

→ In the bulk of the liquid

N_{bulk} = number of nearest neighbors of a molecule in the bulk

Energy reduction in bulk : $\dfrac{|E_{bulk}|}{\text{molecule}} \propto N_{bulk} \cdot \Delta E_{\text{per neighbor}}$

→ At the surface:

N_{surf} = number of nearest neighbors of a molecule at the surface

Energy reduction at surface : $\dfrac{|E_{surf}|}{\text{molecule}} \propto N_{surf} \cdot \Delta E_{\text{per neighbor}}$

$$N_{surf} < N_{bulk} \Rightarrow \frac{|E_{surf}|}{\text{molecule}} < \frac{|E_{bulk}|}{\text{molecule}}$$

$\Rightarrow$ larger energy reduction in bulk

$\Rightarrow$ minimize surface area $\Rightarrow$ form spheres

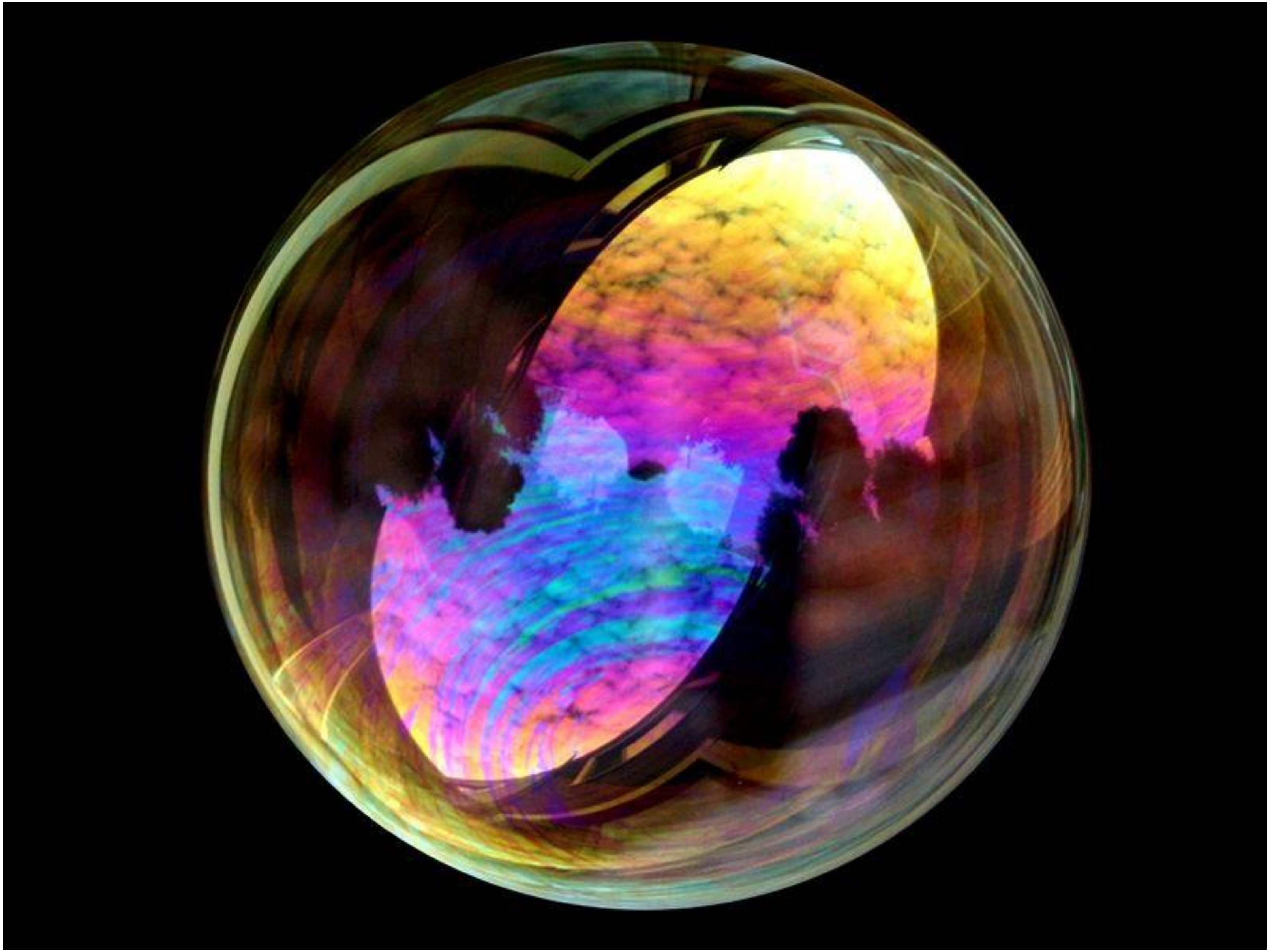
$\Rightarrow$ Energy cost to create (or increase) surface area:

$$\boxed{\gamma = \frac{\text{energy cost to add surface area}}{\text{surface area created}} = \text{surface tension}}$$

$$[\gamma] = \underbrace{\frac{J}{m^2}}_{\frac{\text{energy}}{\text{area}}} = \frac{Nm}{m^2} = \underbrace{\frac{N}{m}}_{\frac{\text{force}}{\text{length}}}$$

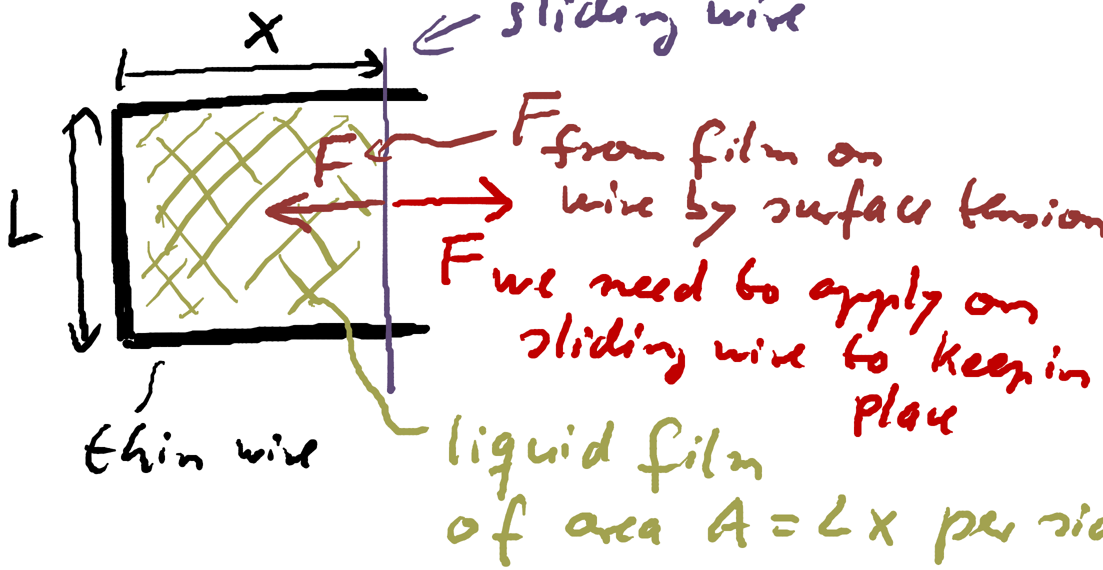
$\Rightarrow$ Surface tension tries to make spheres, which have the smallest surface area / volume!

Bubbles…

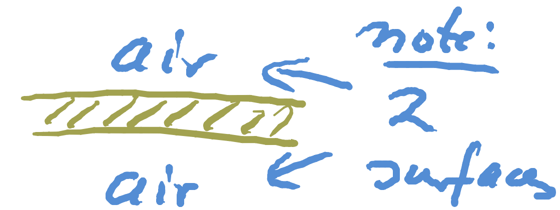


$m \sim 0.01$ g, $W \sim 10^{-4}$ N

Example: Flat liquid film in a wire frame:



side view:



2 surfaces: top + bottom

$$\Rightarrow \text{Energy of film surfaces} = E_{surf} = \gamma A \cdot 2$$

$$= \gamma L x \cdot 2$$

if $x \longrightarrow x + \Delta x$

$$\Rightarrow \Delta E_{surf} = 2\gamma \overbrace{L\Delta x}^{\Delta A} \quad \Big\} \text{ increase in energy}$$

$$= \text{work we do by force we apply}$$

$$= F\Delta x$$

$\Rightarrow 2\gamma L \, \Delta x = F \, \Delta x$

$\Rightarrow |F_{\text{we apply}}| = 2\gamma L = |F_{\text{by surf. tension}}|$

$\Rightarrow \boxed{\frac{F}{L} = 2\gamma} = \frac{\text{force}}{\text{length}}$ that the fluid film exerts on its edge ($\perp$ to edge)

2 surfaces

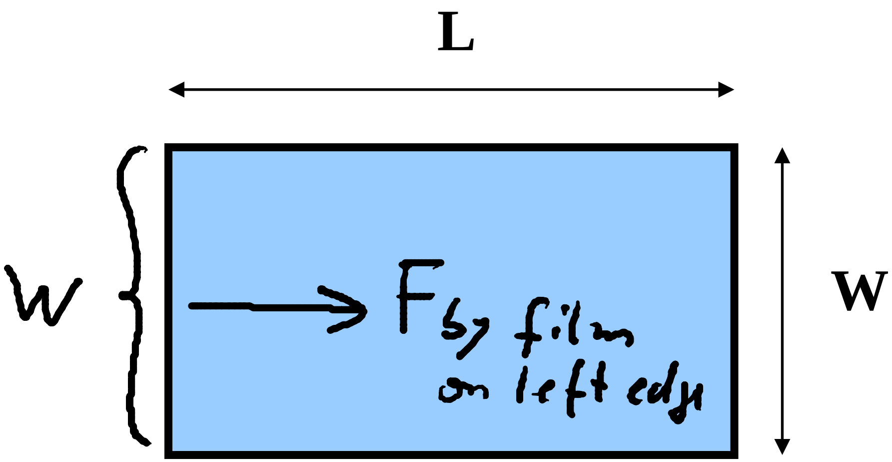
$[\gamma] = \frac{N}{m}$

<u>water:</u> $\gamma_{H_2O} = 0.07 \, \frac{J}{m^2}$

$\rightsquigarrow$ for $L = 0.1\,m \Rightarrow F_\gamma = 0.014\,N$ ... tiny!

<u>But:</u> in the microworld, F_γ is large compared to weight (e.g. insects...)

A soap film with **surface tension** γ stretches across a rectangular **plastic frame** as shown.



What is the force F exerted by the film on the **left edge** of the frame?

$$\frac{F}{\text{length}} = 2\gamma$$

$$\Rightarrow F = 2\gamma\,\text{length} = 2\gamma W$$

A. γ ***L***

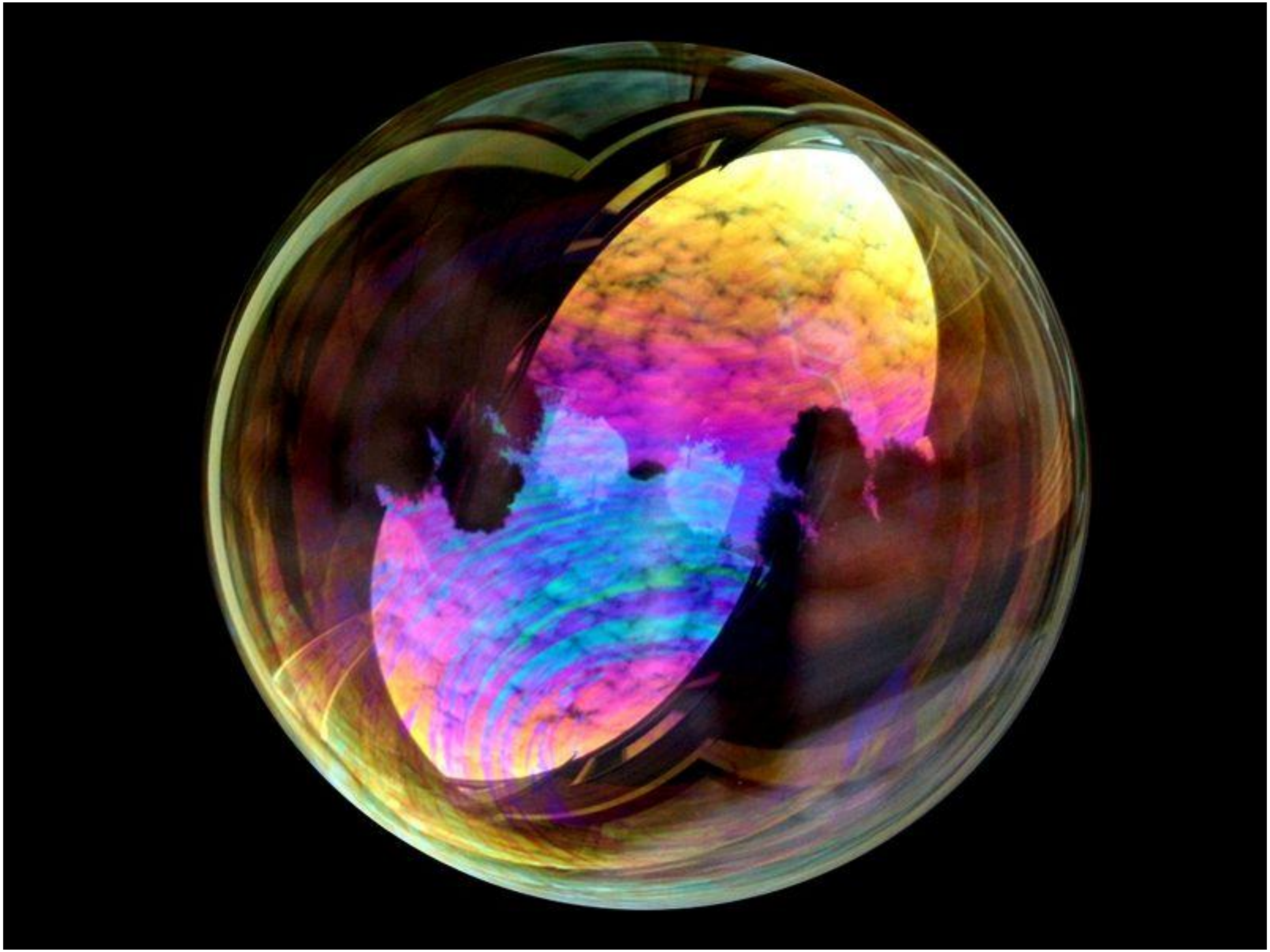
B. γ ***W***

C. ***2*** γ ***L***

D. ***2*** γ ***W***

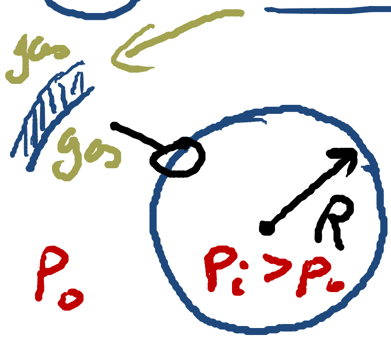
E. Insufficient information

Bubbles…

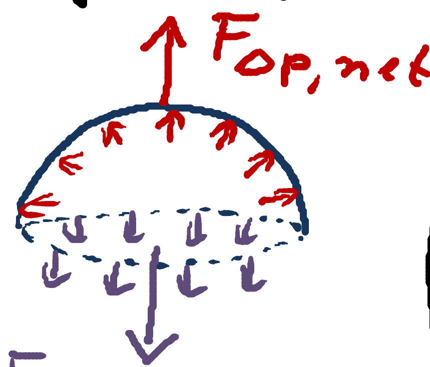


Bubbles:

(A) 'Two-sided' bubble (e.g. soap bubble)



FBD for top half



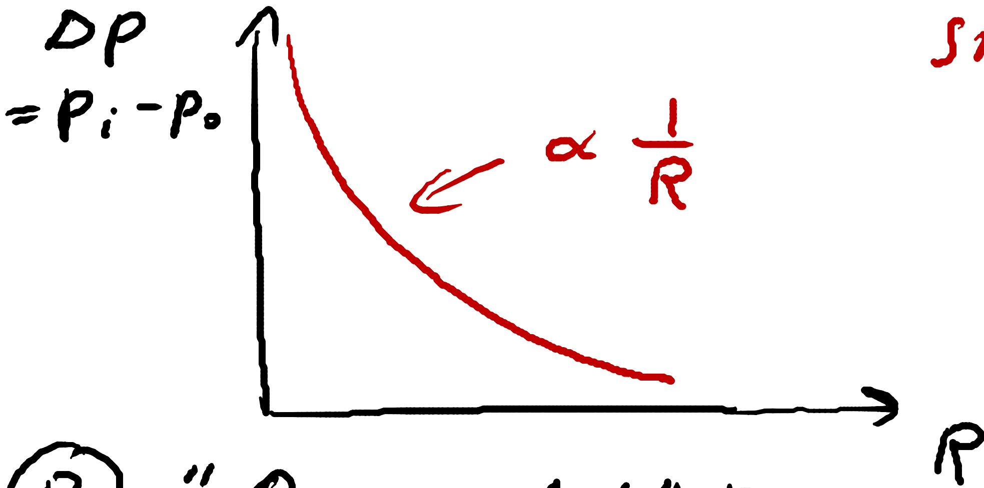
$F_{\gamma, net}$: net force from surface tension by bottom half of bubble

- surface tension ⇒ bubble wants to collapse to minimize surface area
- but: $P_i > P_0$ prevents this!

$\rightarrow F_{\Delta p, net} = \underbrace{(p_i - p_0)}_{\Delta p} \pi R^2$ (recall hemisphere question)

$\rightarrow F_{\gamma, net} = \underset{\text{2 surfaces}}{2}\, \gamma\, \underbrace{(2\pi R)}_{\text{circumference = length of edge of film}}$

$\Rightarrow$ need $\sum \vec{F} = 0 \Rightarrow |F_{\Delta p, net}| = |F_\gamma| \Rightarrow \boxed{\Delta P = P_i - P_0 = \dfrac{4\gamma}{R}}$



Small bubble $\Rightarrow$ needs large inside pressure

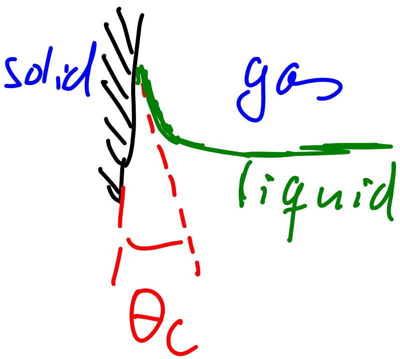
(B) "<u>One-sided</u>" <u>Bubble</u>: (e.g. bubble in soda, water droplets)

only <u>one</u> gas-liquid interface

$$\Rightarrow \quad F_{\gamma,\,net} = \gamma\,(2\pi R)$$

$$\Rightarrow \quad \boxed{\Delta P = P_i - P_o = \frac{2\gamma}{R}}$$

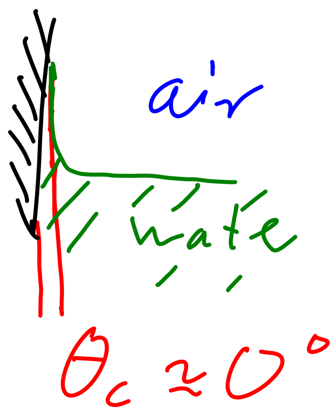
Liquid - Solid - Gas Interface:



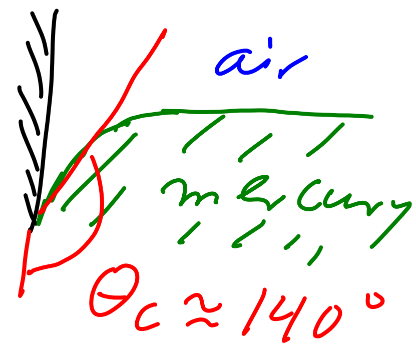
θ_c : contact angle

depends on liquid and solid interaction

"good wetting"
(water on clean glass)



"poor wetting"
(mercury on clean glass)

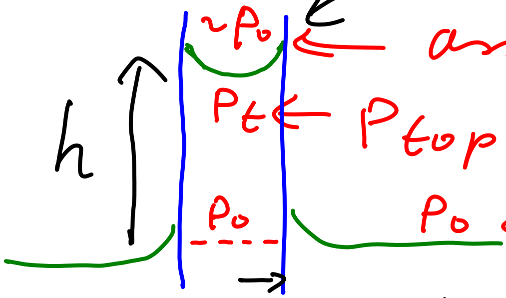


Difference? Interaction between molecules in liquid with surface

attractive	repulsive
⇒ wet solid surface is energetically favorable	⇒ don't wet

→ Capillary Rise (for $\theta_c < 90°$)

small diameter tube (capillary)



assume ~ const pressure in gas

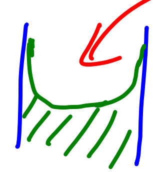
$P_{top} = P_0 - \rho g h$ (static fluid!)

P_0 ← same height in container → same pressure

R = radius of tube

liquid in container

→ from bubbles: if $\theta_c = 0°$

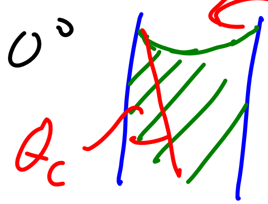


1/2 of one-sided bubble

$\Rightarrow \Delta p = P_0 - P_t = \frac{2\gamma}{R}$

$P_0 - P_{top}$ "inflates" bubble; surface tension opposes "inflation"

if $\theta_c > 0°$



less than 1/2 of a one-sided bubble

$\Rightarrow \Delta p = P_0 - P_t$

$= \frac{2\gamma}{R} \cdot \underline{\underline{\cos\theta_c}}$

$\Rightarrow \Delta P = P_0 - P_{top} = \rho_\ell g h = \frac{2\gamma}{R} \cos\theta_c$

$$\Rightarrow \boxed{h = \frac{2\gamma}{\rho_\ell g R} \cos\theta_c} \propto \frac{1}{R}$$

$\Rightarrow$ small R gives large h !