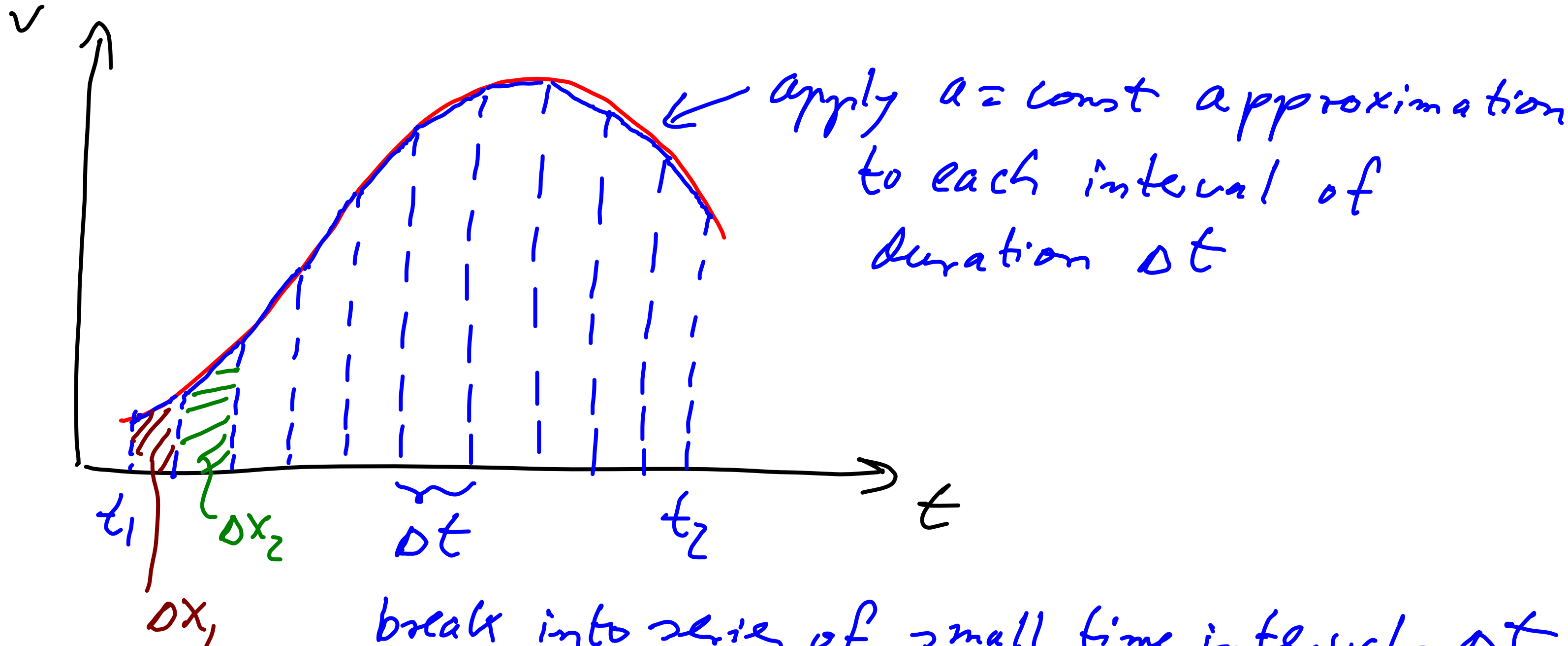
Recap: Motion with const. acceleration

- acceleration: $a(t) = a = \text{const}$
  (special case $a = 0$)
- change in velocity: $\Delta v = v(t) - v_0 = a t =$ area "under" a-t graph
- change in position: $\Delta x = x(t) - x_0 = v_0 t + \frac{1}{2} a t^2$
  = area "under" v-t graph
- $v^2 - v_0^2 = 2 a \Delta x$

- General advice:
  - start by drawing v-t graph
  - define "+" direction for the problem
  - <u>then</u> use formulas or solve graphically!

② General case:



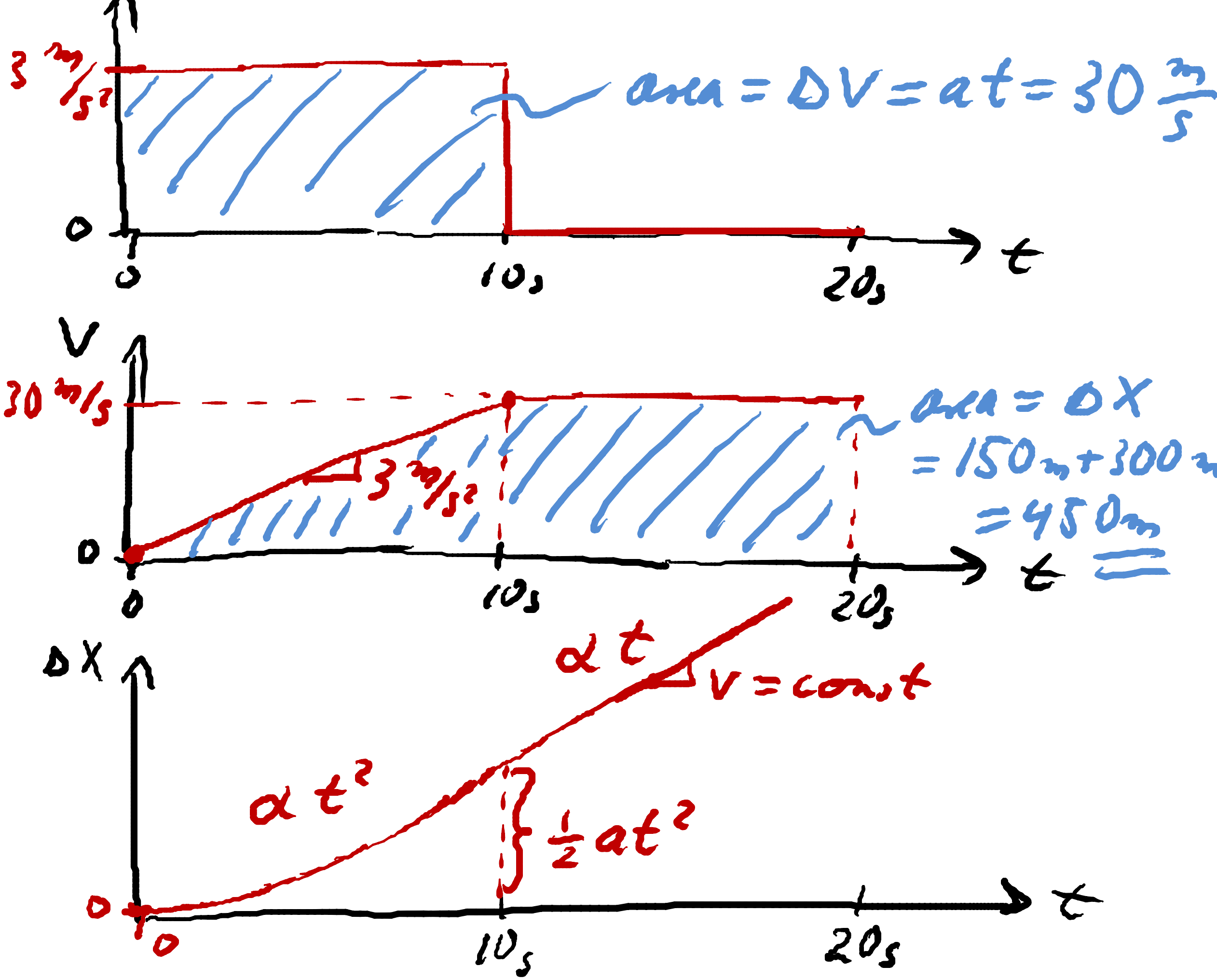
break into series of small time intervals Δt

⇒ apply a = const analysis to each interval Δt

⇒ then get sum

A car starts from rest at t=0 and accelerates at 3 m/s^2 for 10 s. It then drives at constant speed for another 10 s.

How far does the car travel?



distance car travels = ?

A. 150 m

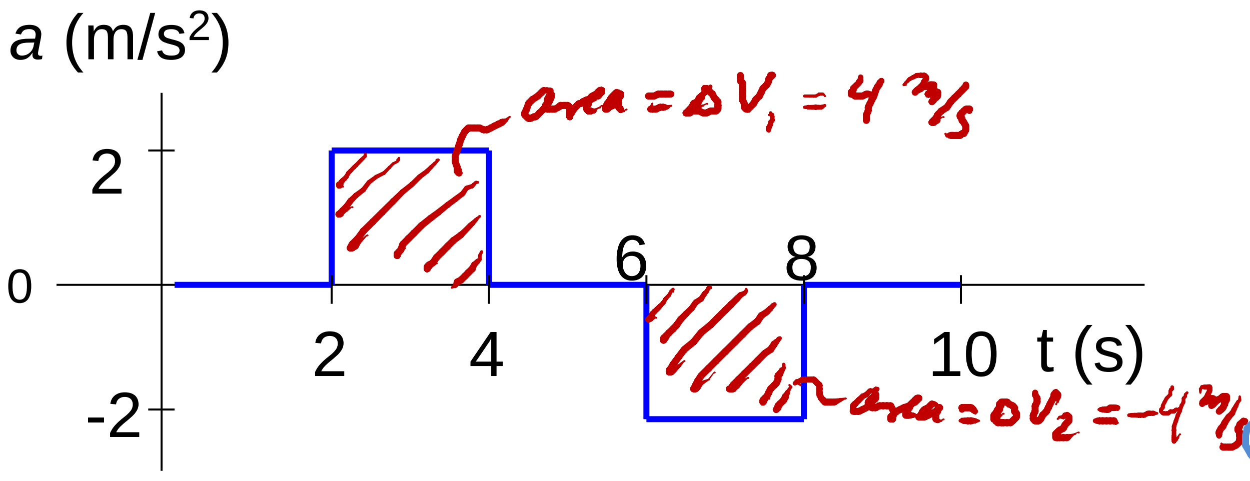
B. 300 m

C. 450 m

D. 600 m

An object starts with v=0 and x=0 at t=0.

If $a(t)$ is as shown below, what is $x(t$=10 s)?



$x(t$=10 s) =?

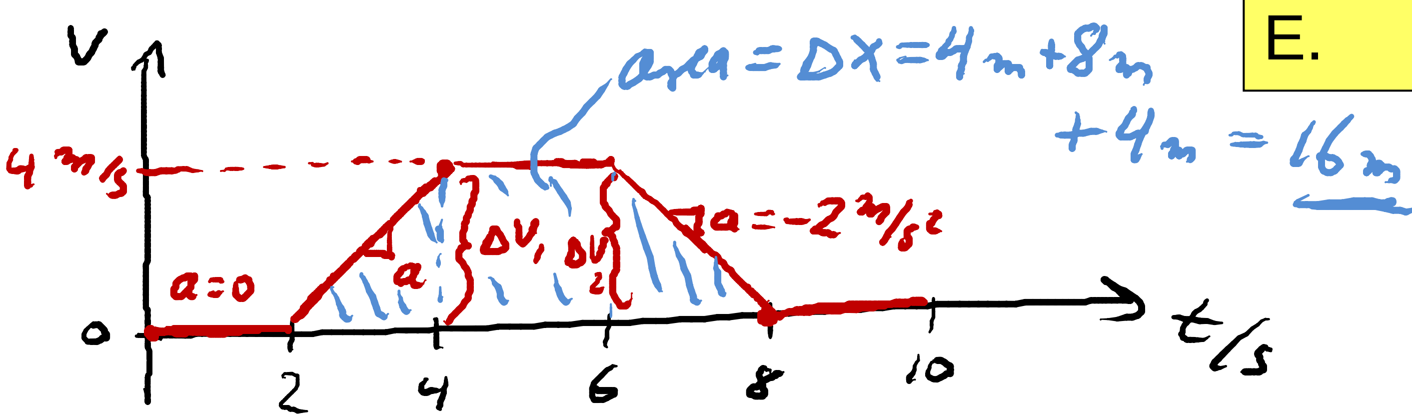
A. 0 m

B. 4 m

C. 8 m

D. 16 m

E. 24 m



Can a Boing 767 glide, i.e. fly without engine power?

A. Yes

B. Probably, but nobody has tried this yet

C. No way

Today:

- Checking Answers
  - Order of magnitude
  - Number of significant figures
  - Units
    - Why units matter: Air Canada Flight 143
    - Dimensional analysis

Solving physics problems: Tactics

I Identify the type of problem:

e.g. 1-D motion, 2-D motion, relative motion, force, ...

II Draw + collect information: (define directions, sign, add values ...)

e.g. v(t) graph, free body diagram, coordinate system

III Solve (use equations)

IV Check your answer

Checking Answers:

① Does the numeric value make sense?
Is the "order of magnitude" reasonable?
Is the "power of 10" reasonable?

Some Order of Magnitudes:

Speeds: (m/s)

10^8	light, electromagnetic waves, electrons in particle accelerators
10^7	electrons in TV picture tubes
10^6	comets, stars
10^5	planet orbital speeds around the Sun
10^4	satellite orbital speed around the Earth
10^3	supersonic aircraft, bullets
10^2	aircraft, high speed trains
10	cars, larger animals, birds
1	rivers, walking
10^{-1}	ground speed during magnitude 7 earthquake
10^{-2}	centipede
10^{-7}	molecular diffusion speeds in liquids

Some Order of Magnitudes:

Acceleration: ($g=10\ m/s^2$)

10^3g car crash

10^2g boxing blow to head

10g fighter aircraft, ground acceleration during magnitude 7 earthquake

1g hard braking in car, rockets, space shuttle

10^{-1}g cars, trains, planes during powered acceleration

10^{-2}g elevators

10^{-5}g vibration from passing truck

② check # of significant figures!
Have you used an appropriate #
of significant figures?

<u>Example:</u> $\Delta x = 1.0\ m$, $v_0 = 0.0\ m/s$ $a = 7.0\ m/s^2$

$\Rightarrow t = \sqrt{2\Delta x / a} = 0.53~~4522748~~\ s$

③ <u>check units!</u>

Example: $h = \frac{1}{2} a t^2$

$[h] = [a][t^2]$

↑
"units of"

$m \overset{\checkmark}{=} \frac{m}{s^2} s^2$

Why Units Matter

Air Canada Flight 143, July 23, 1983

Arctic Ocean
Ellesmere Island
Alert
Kalaallit Nunaat
(Grønland)
(Denmark)
Iceland
North Magnetic Pole
Arctic Circle
Alaska
(USA)
Beaufort Sea
Banks I
Resolute
Baffin Bay
Arctic Circle
Inuvik
Victoria
Island
Davis Strait
Baffin Island
Dawson
Yukon Territory
Great Bear Lake
Mackenzie
Gjoa Haven
Whitehorse
Northwest Territories
Nunavut
Iqaluit
Labrador Sea
Yellowknife
Baker Lake
River
Fort Nelson
Great Slave Lake
CANADA
Ungava Bay
Atlantic Ocean
Nain
Newfoundland
British Columbia
Fort McMurray
Kuujjuaq
Prince George
Saskatchewan
Churchill
Hudson Bay
Churchill Falls
Alberta
St John's
Edmonton
Manitoba
L Winnipeg
Sept-Îles
La Grande Rivière
James Bay
Vancouver
Fraser River
Calgary
Gimli
Quebec
Victoria
Kamloops
Saskatoon
Prince Edward Island
Charlottetown
Regina
Winnipeg
New Brunswick
Ontario
Québec
Nova Scotia
Thunder Bay
Halifax
Fredericton
United States of America
Montréal
Ottawa
Lake Superior
Toronto
Lake Ontario
LEGEND
National capital
Provincial or Territorial capital

Airplane fuel loads specified in kg. Fuel pumped in liters. Needed to convert liters of fuel to kg of fuel.

Errors:

Calculated mass of fuel already on plane

= **liters of fuel** × **lb/liter**

Assumed result was in kg (actually in lb)

∴ Overestimated fuel on plane by factor of 2.2

Calculated fuel to be loaded

= **(kg required — kg already on plane)** × **liters/lb**

Assumed result was in liters.

Result: total fuel on plane ≈1/2 that required.

From the Official Report of the Transportation Safety Board of Canada:

"Mr. Bourbeau (Certified Aircraft Technician, Category 1) testified that he started off himself to make some calculations but he was not too sure about what he was doing and he was not going so fast and he therefore gave up."

From the Official Report of the Transportation Safety Board of Canada:

"First Officer Ouellet testified that he started to do some calculations but never finished them because all the figures got "so crowded" that he ran out of paper. "

From the Official Report of the Transportation Safety Board of Canada:

“No one involved in making the calculations in Montreal (i.e., neither maintenance technicians nor flight crew) seemed to know how to convert liters to kilograms."

Metric mishap caused loss of NASA orbiter

September 30, 1999
Web posted at: 4:21 p.m. EDT (2021 GMT)

In this story:

Metric system used by NASA for many years

Error points to nation's conversion lag

RELATED STORIES, SITES

NASA's Climate Orbiter was lost September 23, 1999

By Robin Lloyd
CNN Interactive Senior Writer

(CNN) -- NASA lost a $125 million Mars orbiter because a Lockheed Martin engineering team used English units of measurement while the agency's team used the more conventional metric system for a key spacecraft operation, according to a review finding released Thursday.

Contractor provided thruster firing data in English units while NASA was using metric…

Dimensional Analysis:

Important formulas can often be determined by "guessing" the relevant independent variables and then comparing their units (i.e. dimensions).

"independent": can't be calculated from each other

Example:

How does the stopping distance Δx of a car depend on initial speed v_i and braking acceleration a_b?

independent variables that might be relevant	v_i	a_b	m (mass)
	$\frac{m}{s}$	$\frac{m}{s^2}$	kg

Assume: $\Delta x = \underbrace{\text{const}}_{\text{no units}} \, v_i^{\alpha} a_b^{\beta} m^{\gamma}$

$\Rightarrow$ units: $m = \left(\frac{m}{s}\right)^{\alpha} \left(\frac{m}{s^2}\right)^{\beta} (kg)^{\gamma}$

$\Rightarrow \gamma = 0, \underbrace{\beta = -\alpha/2}_{\text{get rid of "s"}}, \alpha = 2 \Rightarrow \beta = -1$

conclude:

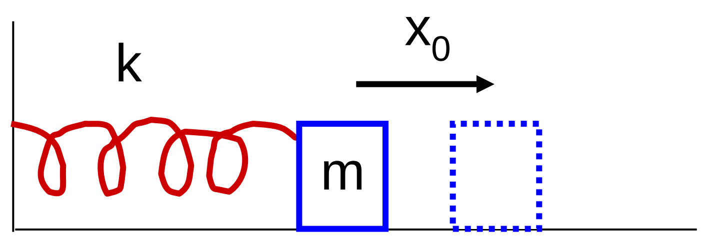
$$\Delta x \propto \frac{v_i^2}{a_b}$$ indep. of mass of car

↑ can't determine dimensionless constant to get "="

check:

$$m \stackrel{?}{=} \frac{(m/s)^2}{m/s^2} = m \checkmark$$

A mass m connected to a spring of spring constant k is initially displaced a distance x_0 from its equilibrium position.



Using **dimensional analysis**, determine a relation between the maximum velocity v_{max} of the mass during its oscillation and the quantities m, k, and x_0.   **Note:** $[k]$=kg/s^2

V_{max}	mass	k	x_0
m/s	kg	kg/s^2	m

$V_{max} \propto m^{\alpha} k^{\beta} x_0^{\gamma}$

$\Rightarrow \gamma = 1 \quad \beta = 1/2 \quad \Rightarrow \alpha = -\frac{1}{2}$

$\Rightarrow V_{max} \propto \sqrt{\frac{k}{m}} \cdot x_0$

A. $v_{max} \propto x_0\,(k/m)$

B. $v_{max} \propto x_0\,k\,m$

C. $v_{max} \propto x_0\,(k/m)^{1/2}$

D. $v_{max} \propto x_0\,(k/m)^{1/2}$

E. $v_{max} \propto x_0\,(m/k)^{1/2}$