Recap: Vectors

- have magnitude + direction: \( \vec{r}, \vec{v}, \vec{a}, \vec{F}, \ldots \)

Specifying vectors:

1. by magnitude \( A \) and angle \( \theta \)
   
   \[
   A_x = A \cos \theta \\
   A_y = A \sin \theta
   \]

   \[
   \vec{A} = A_x \hat{i} + A_y \hat{j}
   \]

   unit vector

2. by components \( A_x, A_y \)
   
   \[
   \vec{A} = A_x \hat{i} + A_y \hat{j}
   \]

Adding vectors:

1. “tail to tip”, then
   “tail to tip”

2. by components:
   
   \[
   \vec{C} = \vec{A} + \vec{B} = (A_x + B_x) \hat{i} + (A_y + B_y) \hat{j}
   \]

Never mix \( x \) and \( y \) components!
Three trajectories for a thrown ball are shown below.

Which trajectory has the longest time of flight?

A. a  
B. b  
C. c  
D. all the same

Key: great x- and y-components of 2-D motion in dependence?
=) same height =) same vertical motion
=) "time of flight" is determined by y-motion here
=) same ΔT!
Today:

- Motion in a plane (2-D motion)
  - Position $\mathbf{r}(t)$, velocity $\mathbf{v}(t)$, acceleration $\mathbf{a}(t)$
  - Projectile motion

A projectile often moves horizontally as it moves upward and/or downward.
\( \vec{r} = \text{position vector} = \vec{r}(t) = x(t) \hat{x} + y(t) \hat{y} \)

Separate out components of motion!

Displacement:

\[ \Delta \vec{r} = \vec{r}(t_2) - \vec{r}(t_1) \]

Velocity:

Direction of \( \vec{v} \) points along path always! (tangent to path)
*average velocity*: \( \bar{v}_{\text{avg}} = \frac{\text{displacement}}{\Delta t} = \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} \)

*instantaneous velocity*: \( \vec{v}(t) = \frac{d\vec{r}}{dt} = \frac{dx}{dt} \hat{i} + \frac{dy}{dt} \hat{j} \)

- slope of \( x - t \) graph
- slope of \( x - t \) graph

The direction of \( \vec{v}(t) \) is tangent to the path at \( \vec{v}(t) \).

*average acceleration*: \( \bar{a}_{\text{avg}} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} \)

- slope of \( V_x - t \) graph

*instantaneous acceleration*: \( \vec{a}(t) = \frac{d\vec{v}}{dt} = \frac{dV_x}{dt} \hat{i} + \frac{dV_y}{dt} \hat{j} \)

- slope of \( V_x - t \) graph
- slope of \( V_y - t \) graph

\[ = a_x \hat{i} + a_y \hat{j} \]

\[ = \frac{d^2x}{dt^2} \hat{i} + \frac{d^2y}{dt^2} \hat{j} \]
\[ \Rightarrow X - \text{and } Y - \text{components of motion can be treated independently!} \]

\[ X(t) \overset{\text{slope}}{\longleftarrow} V_x(t) \overset{\text{slope}}{\longrightarrow} A_x(t) \]
\[ \text{area} = \Delta X \]

\[ Y(t) \overset{\text{slope}}{\longleftarrow} V_y(t) \overset{\text{slope}}{\longrightarrow} A_y(t) \]
\[ \text{area} = \Delta Y \]

\[ \Rightarrow \text{2D motion} = \text{two 1-D motions!} \]
The velocity $\vec{v}$ of a particle at two different times $t_1$ and $t_2$ is shown below ($t_2 > t_1$).

Which vector best represents the **average acceleration** of the particle between these two times?

\[
\vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t}
\]

\[\Delta \vec{v} = \text{later} - \text{earlier} = \vec{v}_2 - \vec{v}_1 = \vec{v}_2 + (-\vec{v}_1)
\]
Example of 2-D motion: Projectile motion

1. Object is given initial \( \vec{v}_0 = V_0x \hat{c} + V_0y \hat{g} \) and moves in vertical plane.

2. 2D motion in the vertical plane defined by \( \vec{v}_0 \) and direction of gravity \( \vec{g} \).

3. Only acting force/acceleration is gravity (neglect air)

\[
\vec{a}(t) = 0 \hat{c} - g \hat{g}
\]

\[ g = 10 \text{ m/s}^2 \]

4. "Time of flight" \( \Delta t \) and \( \Delta x, \Delta y \) determined by either \( x \)- or \( y \)-motion.

\[
\Rightarrow \text{analyze horizontal and vertical motions separately!}
\]
**Horizontal Motion**

\[ a_x = 0 \quad \Rightarrow \quad V_x(t) = \text{const} = V_{0x} \text{ in } x \text{- direction} \]

\[ x(t) = x_0 + V_{0x} \cdot t \]
Vertical Motion:

\[ a_y = -g = -10 \text{ m/s}^2 = \text{const} \]

\[ y(t) = y_0 + v_{0y} t - \frac{1}{2} g t^2 \]

=> Free fall in y!
A student sits on a cart moving at constant speed, and tosses a ball upward.

If the student wants to catch the ball in his/her lap on the way down, in what direction \textit{relative to his body} should he toss the ball?

\[ \Rightarrow \text{x and y motions are independent!} \]

\[ \Rightarrow \text{think about "frame of reference" in moving cart!} \]

A. Throw behind.
B. Throw straight up.
C. Throw ahead.

\text{both, ball and cart are moving with the same horizontal speed.}
Special case:

1. Horizontal projectile motion from height $h$

\[ \vec{V}_0 = V_{0x} \hat{i} + V_0 \hat{j} \]

"time of flight" ($t_f$) determined by $y$-motion here

\[ h \rightarrow t_f \rightarrow \Delta x \]

\[ \Delta x = V_{0x} \cdot t_f \]

\[ -a \Delta x = \Delta y = -h = -\frac{1}{2} g t_f^2 \]

\[ \Rightarrow t_f = \sqrt{\frac{2h}{g}} \]

as in free fall from height $h$. \( \Box \)