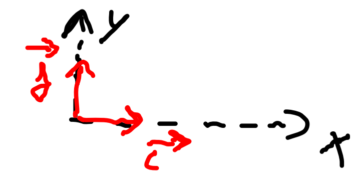
Recap: 2-D Motion Lecture 7

- $\vec{r}(t) = x(t)\,\vec{i} + y(t)\,\vec{j}$ $\qquad$ $\vec{v}(t) = V_x\,\vec{i} + V_y\,\vec{j}$



$$= \frac{dx}{dt}\,\vec{i} + \frac{dy}{dt}\,\vec{j}$$

slope of x - t graph $\qquad$ slope of y - t graph

$\vec{a}(t) = a_x\,\vec{i} + a_y\,\vec{j}$

$$= \frac{dv_x}{dt}\,\vec{i} + \frac{dv_y}{dt}\,\vec{j}$$

slope of V_x - t graph $\qquad$ slope of V_y - t graph

- $x(t) \longleftrightarrow V_x(t) \longleftrightarrow a_x(t)$
  $y(t) \longleftrightarrow V_y(t) \longleftrightarrow a_y(t)$

$\boxed{\text{x and y components of motion can be treated independently!}}$

- Projectile motion:

initial velocity: $\vec{V}_0 = V_{0x}\,\vec{i} + V_{0y}\,\vec{j}$ $\qquad$ $\uparrow$ +y up

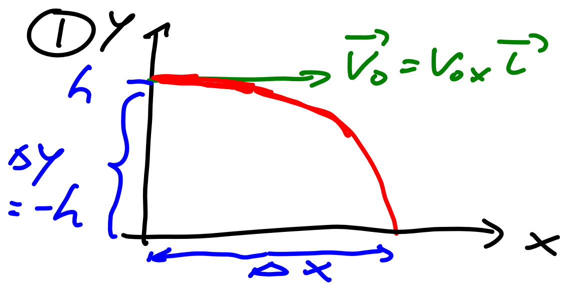
acceleration: $\vec{a} = 0\,\vec{i} + (-g)\,\vec{j}$

$\Rightarrow$ x motion: at const. speed $\quad V_x = V_{0x} = \text{const} \Rightarrow \Delta x = V_{0x}\,t$

$\Rightarrow$ y motion: free fall $\quad V_y(t) = V_{0y} - gt \Rightarrow \Delta y = V_{0y}\,t - \frac{1}{2}gt^2$

Special Cases:

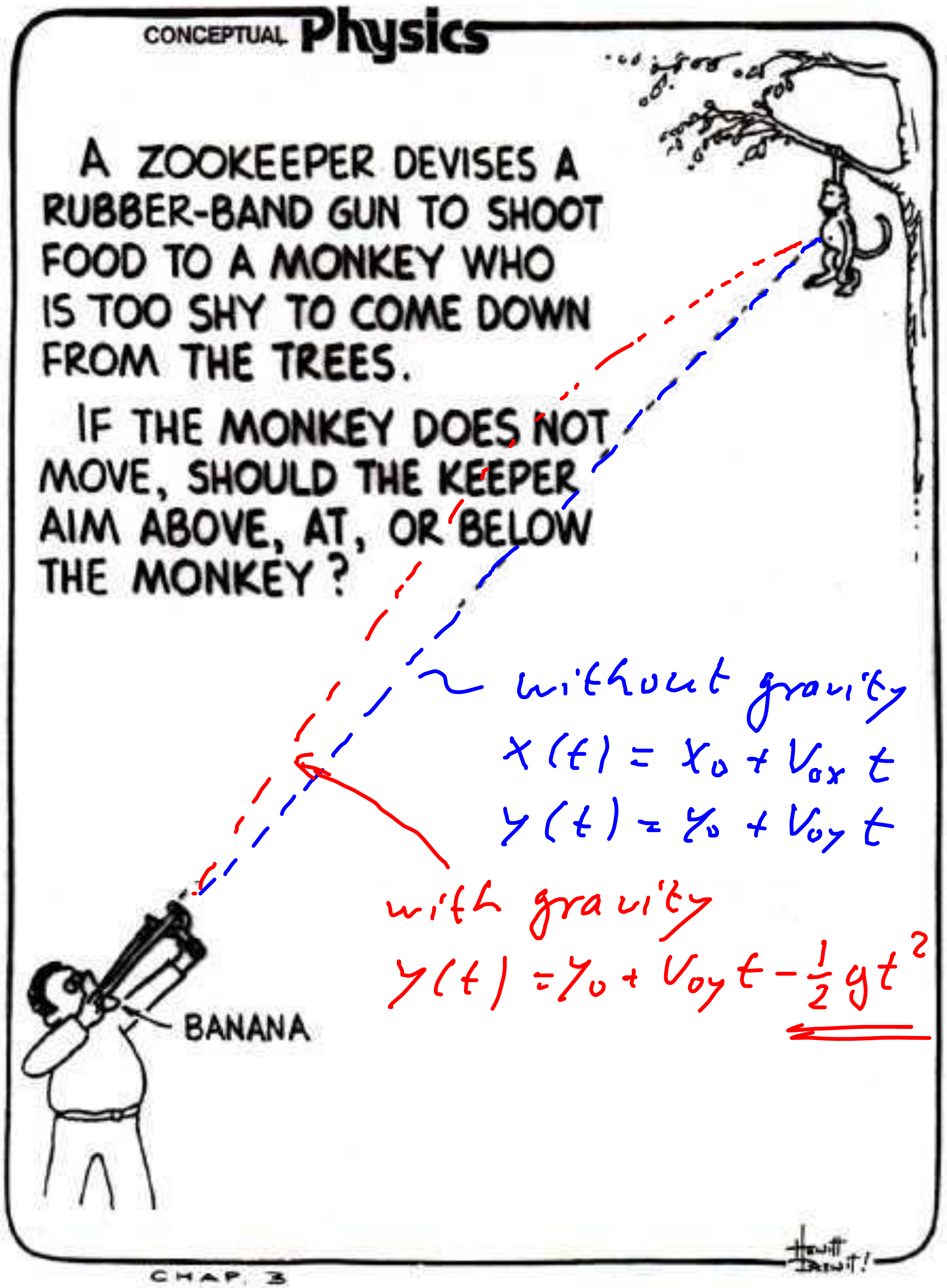
①



y motion determines t_f:

$$\Delta y = -h = -\frac{1}{2}g t_f^2 \Rightarrow t_f = \sqrt{\frac{2h}{g}}$$

$$\Delta x = V_{0x}\,t_f$$



without gravity

$x(t) = x_0 + V_{0x} t$

$y(t) = y_0 + V_{0y} t$

with gravity

$y(t) = y_0 + V_{0y} t - \underline{\frac{1}{2} g t^2}$

A. *Aim above*

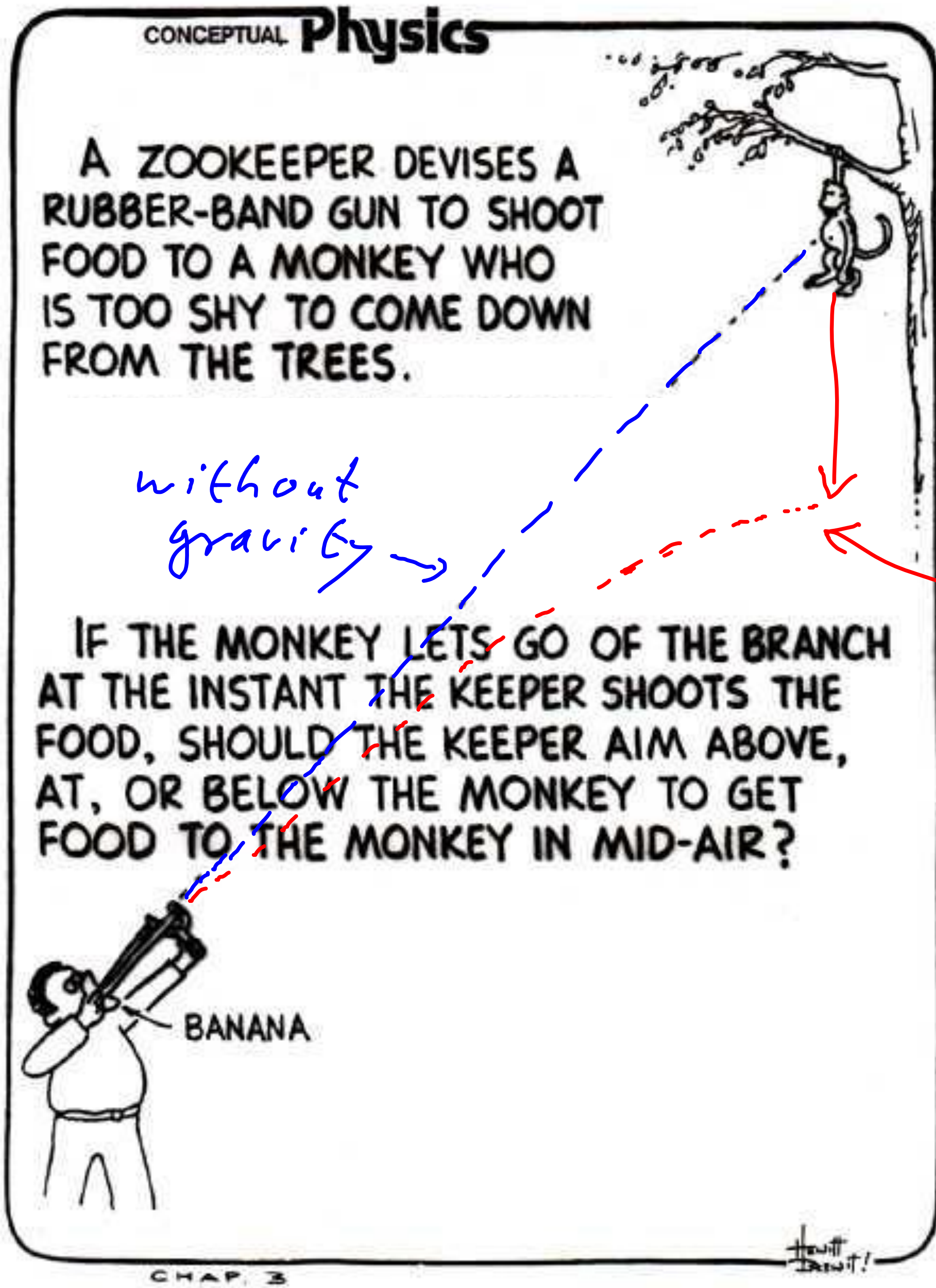
B. *Aim at*

C. *Aim below*

↑ +y up

$\Rightarrow$ need large V_{0y} as compared to $g=0$ case

$\Rightarrow$ aim above!



$\uparrow +y$

A. ***Aim above***

B. ***Aim at***

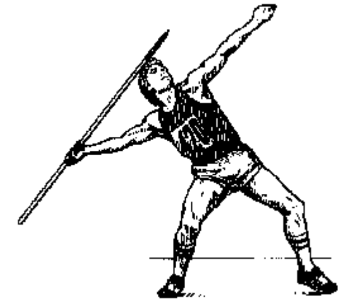
C. ***Aim below***

$$y_b(t) = y_{0,b} + v_{0,y,b}\, t - \underline{\tfrac{1}{2} g t^2}$$

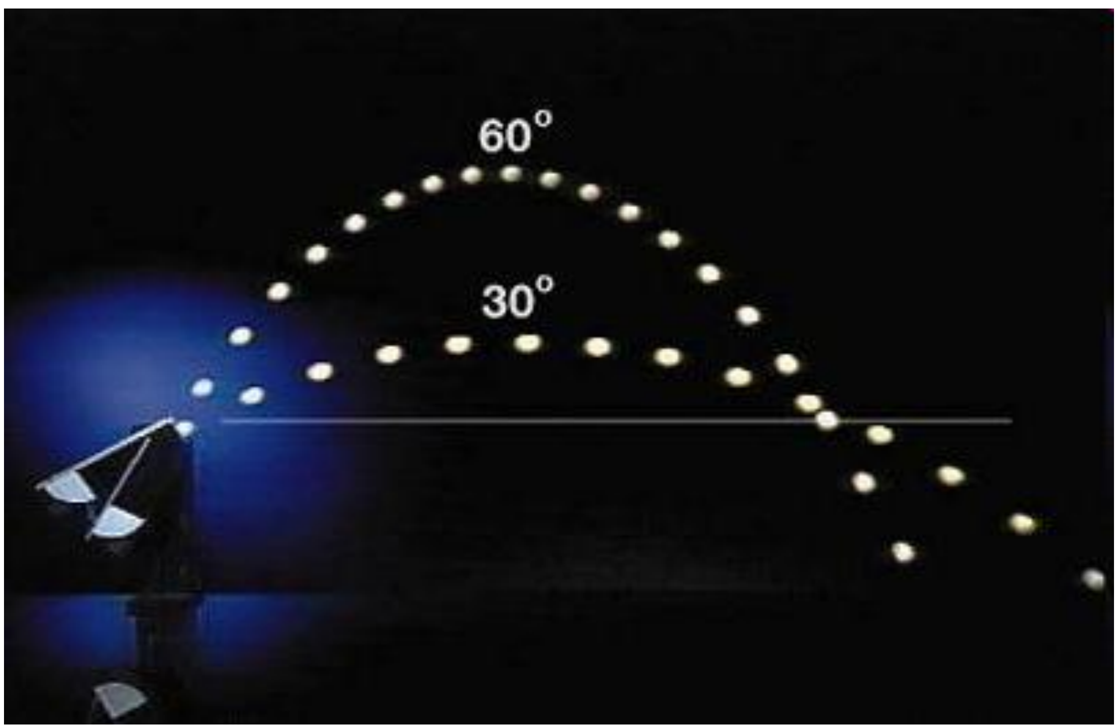
$$y_m(t) = y_{0,m} - \underline{\tfrac{1}{2} g t^2}$$

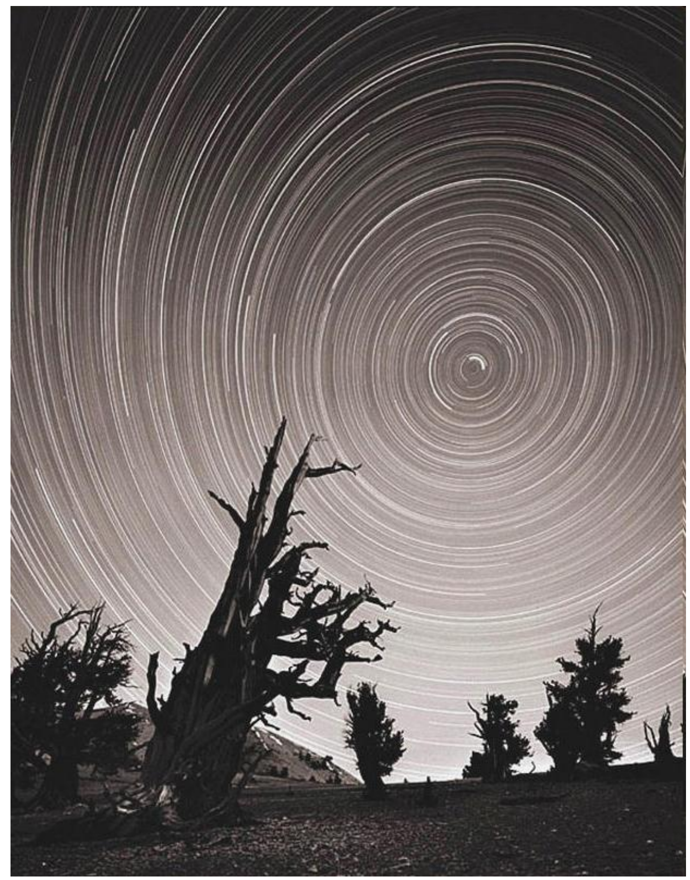
same "drop"

Today:

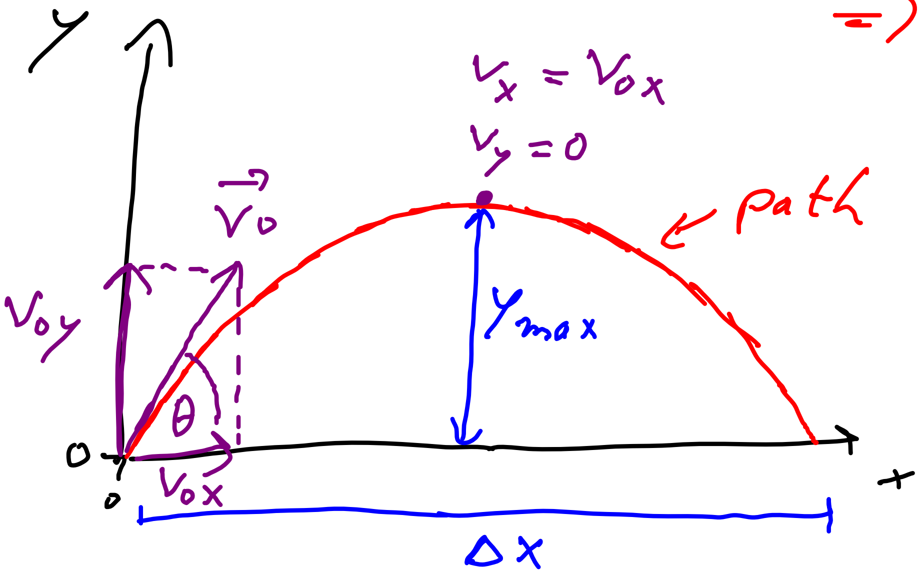


- Projectile motion
  - What is the best angle to throw a javelin at for maximum horizontal reach?
  - The zookeeper/hunter and the monkey…
- Relative motion





② Object projected and returns to its initial height



$\Rightarrow \Delta y = 0$

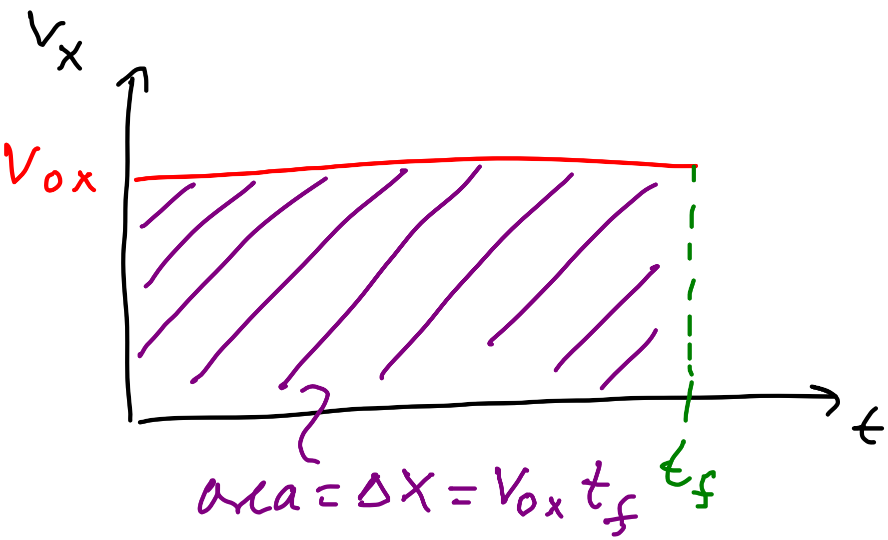
$$\vec{v}_0 = v_{0x}\,\vec{i} + v_{0y}\,\vec{j}$$

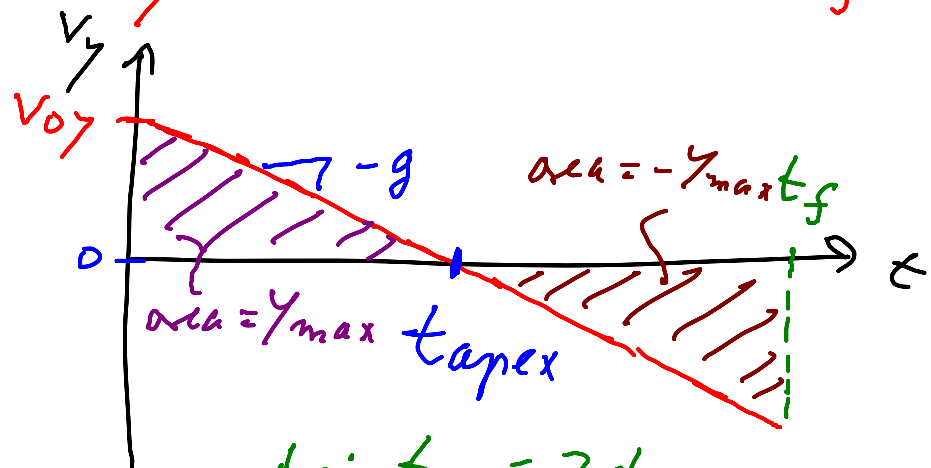
$$= v_0 \cos\theta\,\vec{i} + v_0 \sin\theta\,\vec{j}$$

$$v_x(t) = v_{0x} = v_0 \cos\theta = \text{const}$$

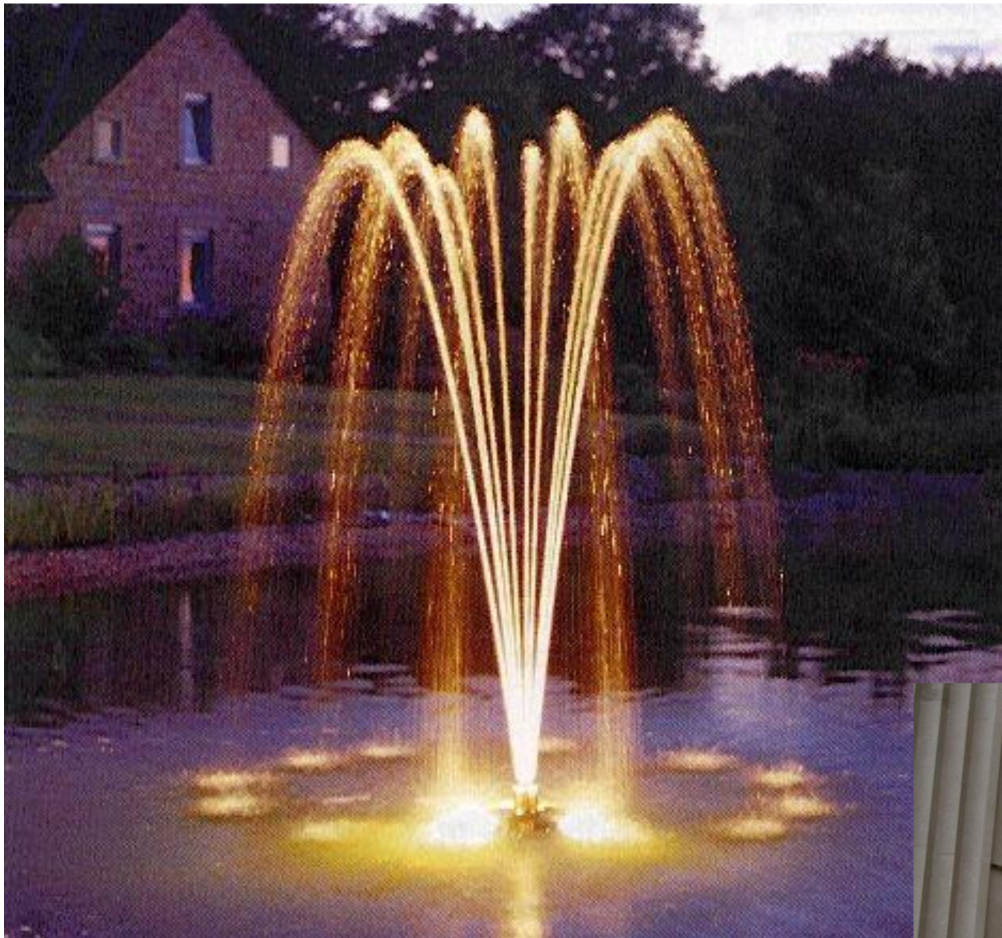
$$v_y(t) = v_{0y} - g\,t$$

y-motion determines t_f



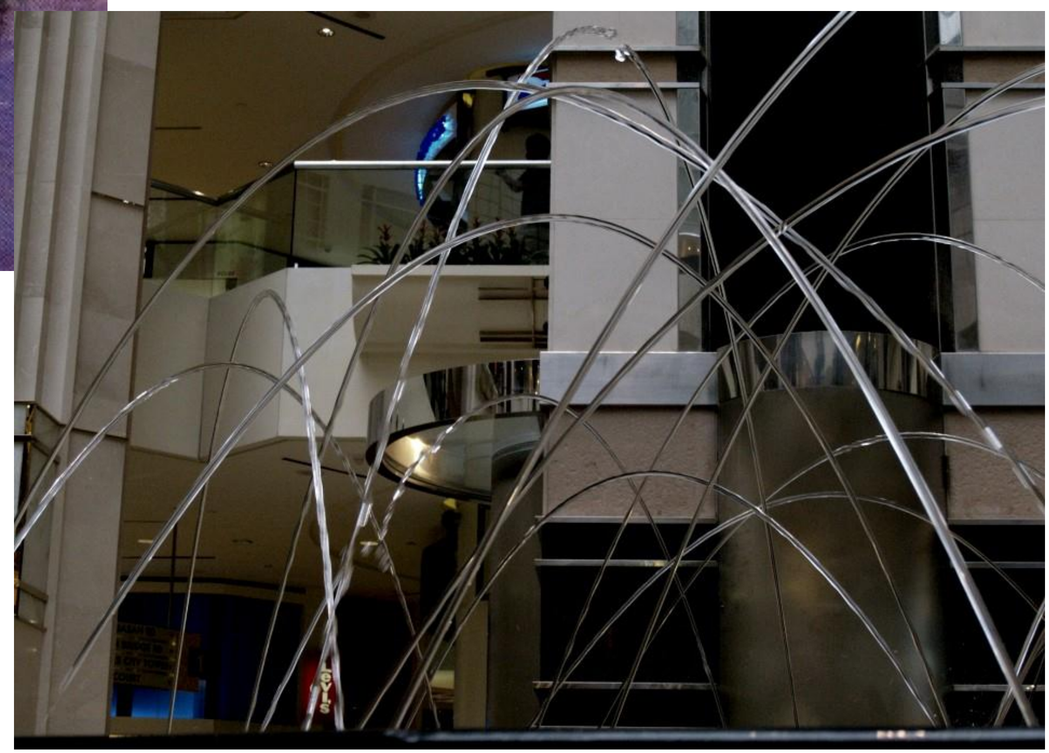


note: $t_f = 2\,t_{apex}$



Water Fountains: Projectile Motion

(not ideal)



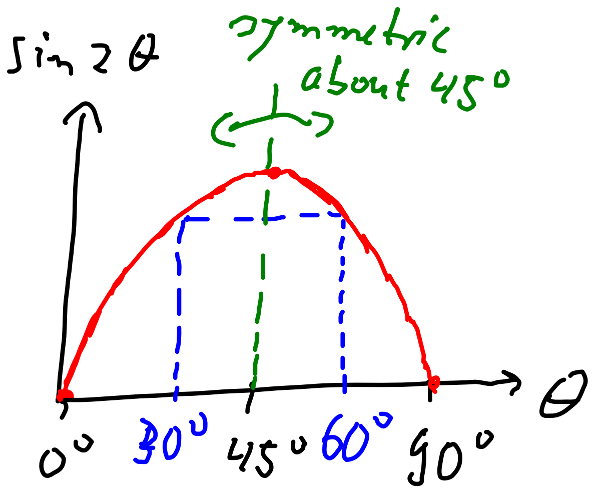
<u>time of flight:</u> $\Delta y = 0 = V_{0y} t_f - \frac{1}{2} g t_f^2$

$$\Rightarrow t_f = \frac{2 V_{0y}}{g} = \frac{2 V_0 \sin\theta}{g} = 2\, t_{apex}$$

$\Rightarrow$ max t_f for $\theta = 90°$

<u>range:</u> $\Delta x = V_{0x} t_f = V_{0x} \frac{2 V_{0y}}{g} = \frac{2 V_0 \cos\theta \; V_0 \sin\theta}{g}$

$$\Rightarrow \Delta x = \frac{V_0^2 \sin(2\theta)}{g}$$



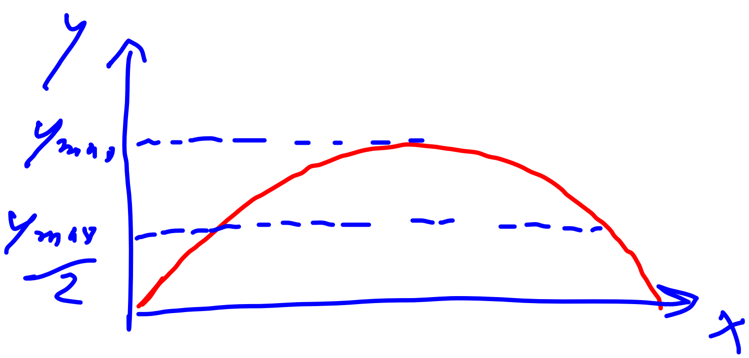
$\Rightarrow$ max Δx for $\theta = 45°$

$\Rightarrow$ symmetric:

e.g. $\Delta x(\theta = 30°) = \Delta x(\theta = 60°)$

An object is thrown on flat ground at some angle with respect to the horizontal. The object rises to a vertical height y_{max} before returning to the ground.

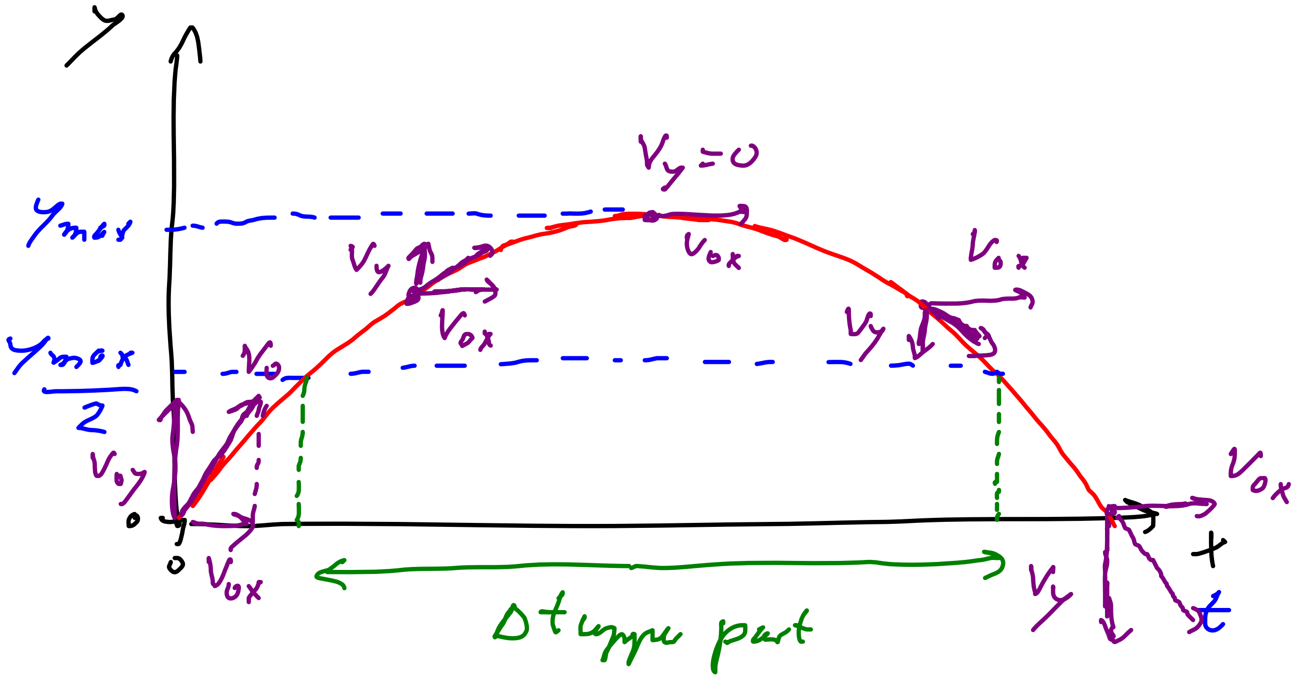
During its flight, how does the time the object spends with $y>y_{max}/2$ compare with the time it spends with $y<y_{max}/2$?



A. **More time in lower part.**

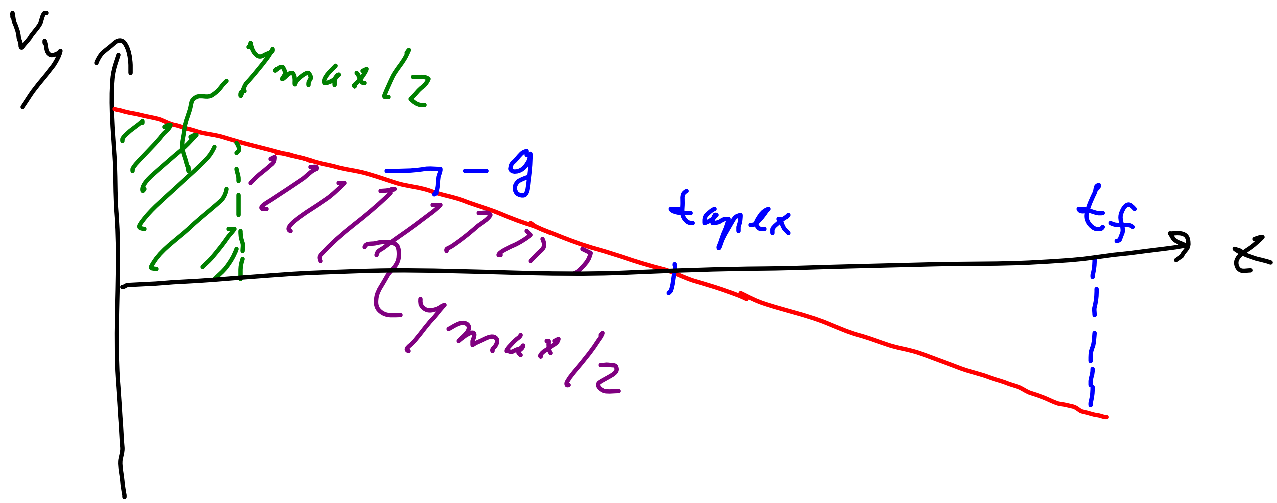
B. **The same**

C. **More time in upper part.**



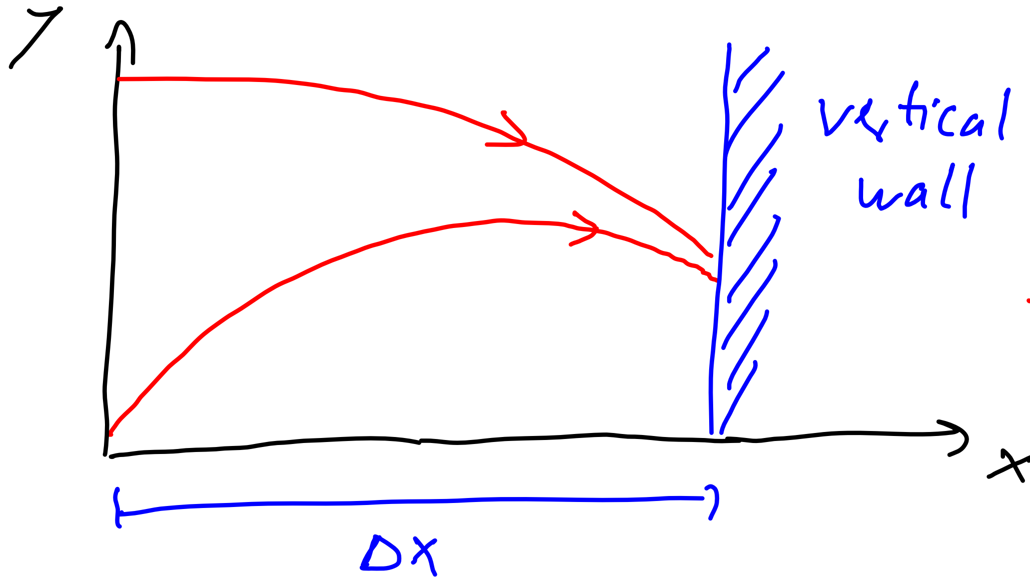
$\Delta x = V_{ox}\, t$

$\Delta x \propto t$



“He has great hang time…”

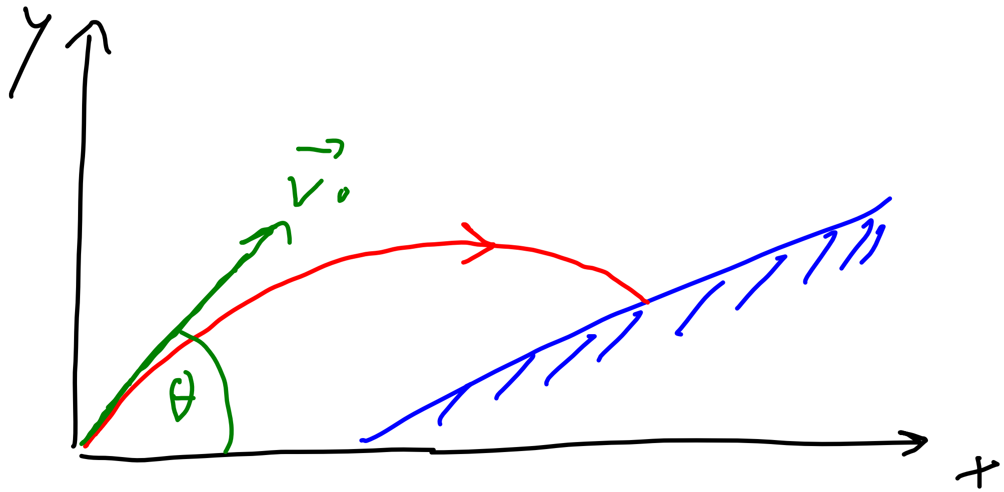
③



x - motion determines t_f here!

$$t_f = \frac{\Delta x}{v_{0x}}$$

④



t_f depends on θ