Recap: Lecture 8

· Projectile motion:

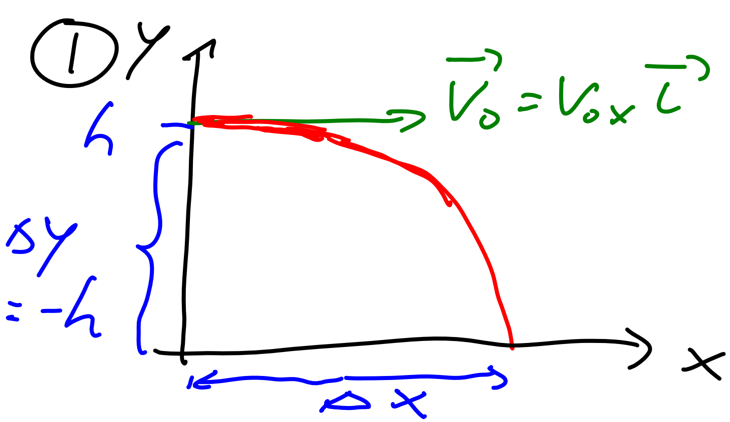
$\vec{v}_0 = V_{0x}\vec{i} + V_{0y}\vec{j}$ $\vec{a} = 0\vec{i} + (-g)\vec{j}$

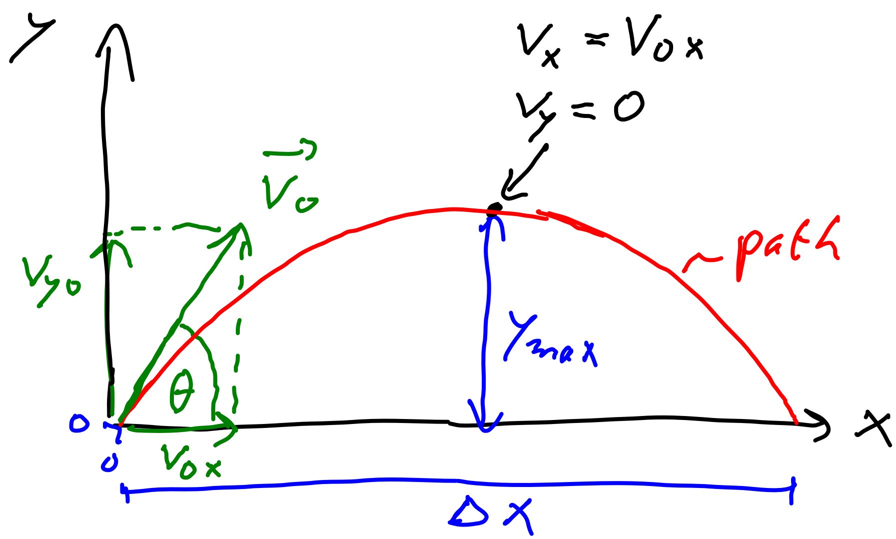
⇒ x-motion: at const speed $V_x(t) = V_{0x} \Rightarrow \Delta x = V_{0x} t$

⇒ y-motion: freefall $V_y(t) = V_{0y} - gt \Rightarrow \Delta y = V_{0y} t - \frac{1}{2} g t^2$

⇒ analyze components independently!!

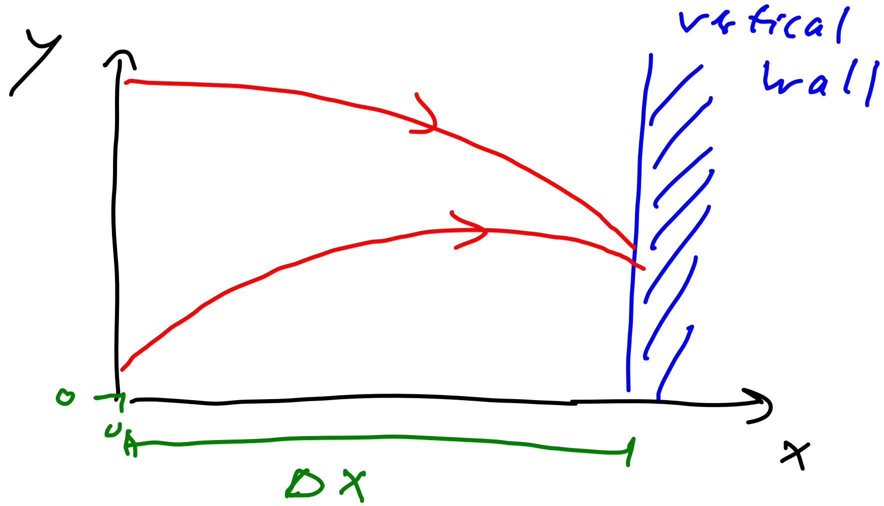
Special Cases:

① 

② 

Additional special cases of projectile motion:

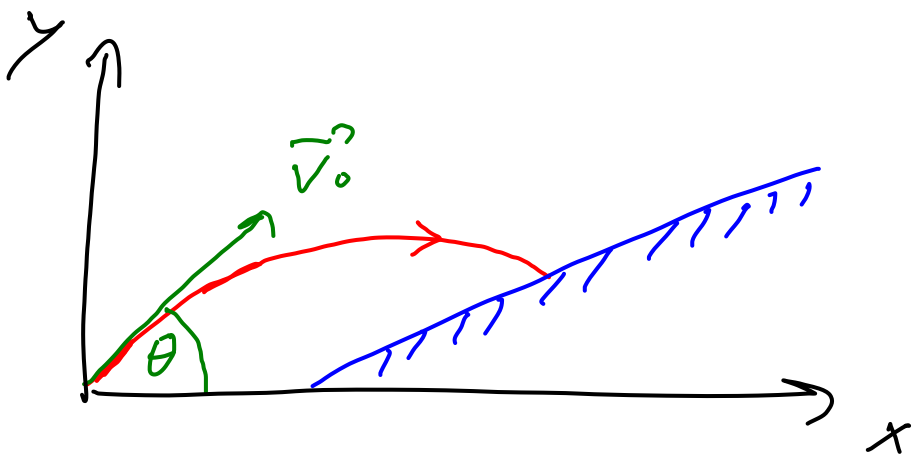
(3)



x-motion determines t_f

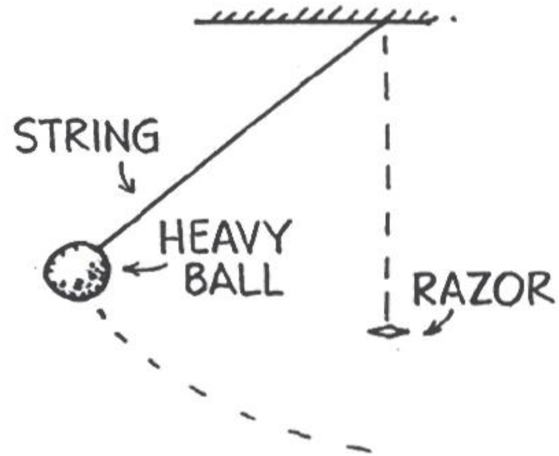
$$t_f = \frac{\Delta x}{V_{0x}}$$

(4)

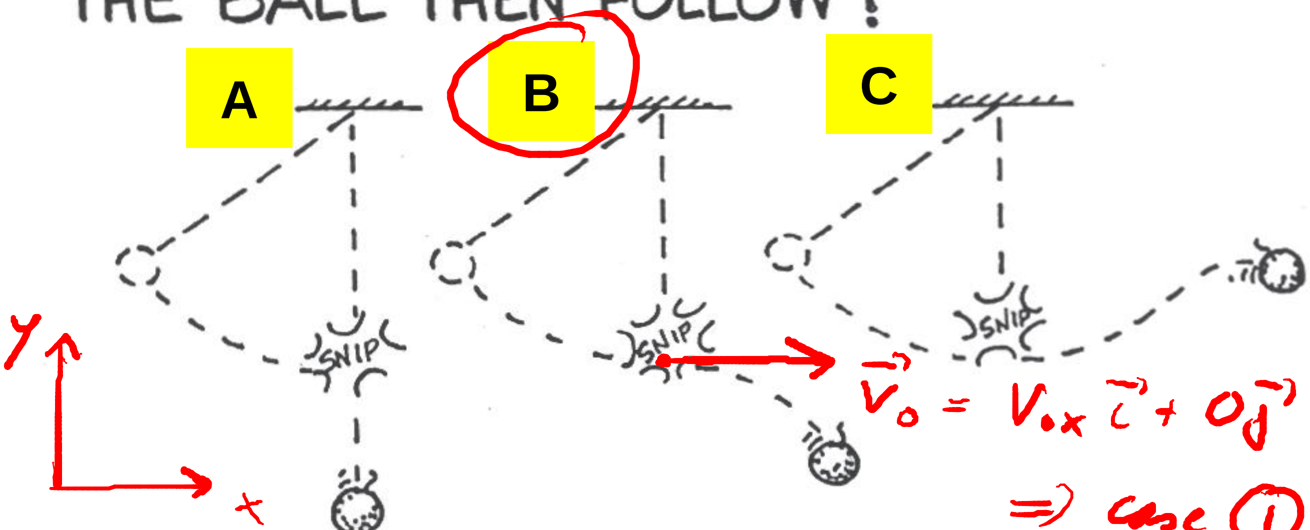


t_f depends on θ

WHEN THE BALL AT THE END OF THE STRING SWINGS TO ITS LOWEST POINT, THE STRING IS CUT BY A SHARP RAZOR.



WHICH PATH WILL THE BALL THEN FOLLOW?

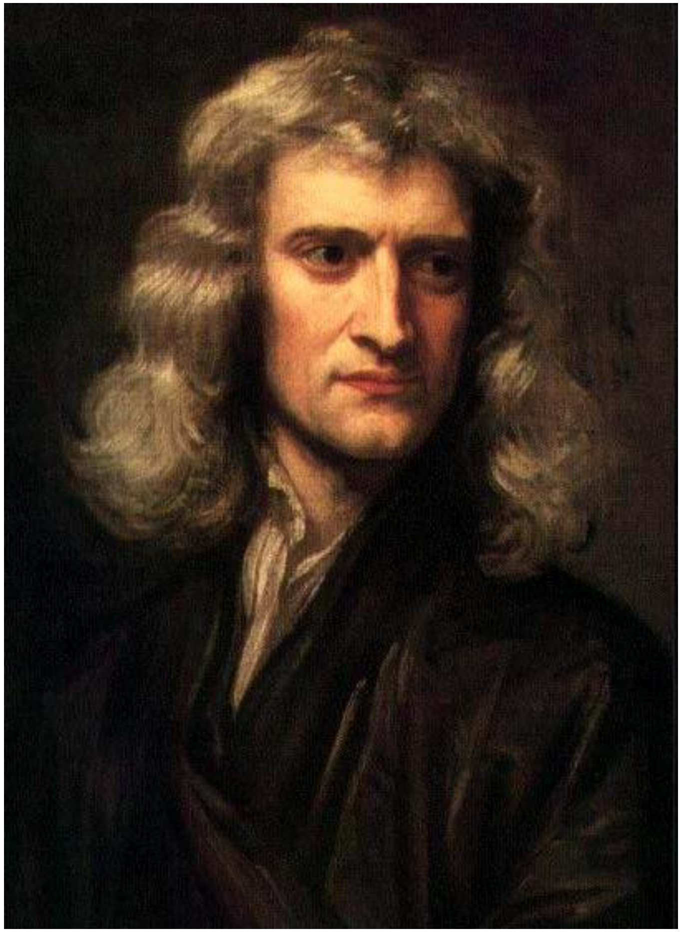


$\vec{V}_0 = V_{0x}\vec{i} + 0\vec{j}$ tangent to path

⇒ case ① of projectile motion

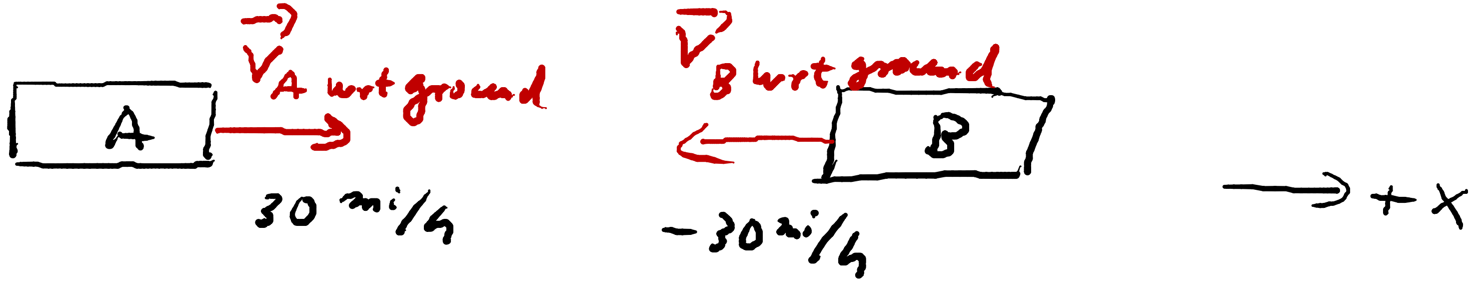
Today:

- Relative motion
- Uniform circular motion
- Forces: Intro



Relative Motion:

1-D:



V_B wrt A = V_B wrt ground − V_A wrt ground

B relative to A
B as seen by A
B in reference frame of A

⇒ V_B wrt ground = V_B wrt A + V_A wrt ground

here: V_B wrt A $= \left(-30 \frac{mi}{h}\right) - \left(30\ mi/h\right) = -60 \frac{mi}{h}$

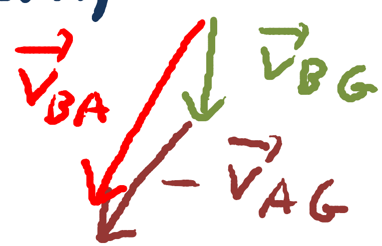
2-D:

$\vec{V}_B$ wrt A = $\vec{V}_B$ wrt ground − $\vec{V}_A$ wrt ground

⇒ $\vec{V}_B$ wrt ground = $\vec{V}_B$ wrt A + $\vec{V}_A$ wrt ground

⇒ separate out x- and y-components

⇒

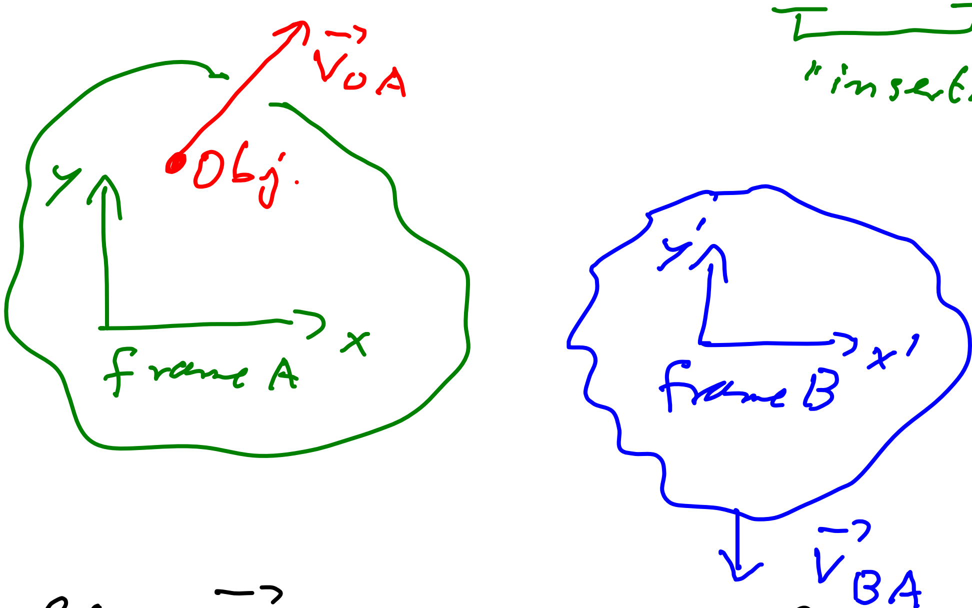


Frame of Reference:

$$\vec{V}_{obj \text{ in reference frame } A} = \vec{V}_{obj \text{ in ref. frame } B} + \vec{V}_{\text{frame } B \text{ wrt frame } A}$$

(short: $\vec{V}_{OA} = \vec{V}_{OB} + \vec{V}_{BA}$) ← note sequence for correct sign!

"inserting B" ← separate out x- and y-components



e.g.: $\vec{V}_{\text{moon wrt sun}} = \vec{V}_{\text{moon wrt. earth}} + \vec{V}_{\text{earth wrt. sun}}$

$$\underbrace{\vec{V}_{\text{bulldozer wrt table}}}_{O \;\; A} = \underbrace{\vec{V}_{\text{bulld. wrt platform}}}_{O \;\; B} + \underbrace{\vec{V}_{\text{platform wrt table}}}_{B \;\; A}$$

$$0 = >0 + <0$$

Two airplanes are flying along headings that form a right angle. **Plane 1** heads due **south** at **450 mi/h**. **Plane 2** heads due **west** at **150 mi/h**.

$\vec{V}_1 = -450 \frac{mi}{h} \vec{j}$ $\vec{V}_2 = -150 \frac{mi}{h} \vec{i}$

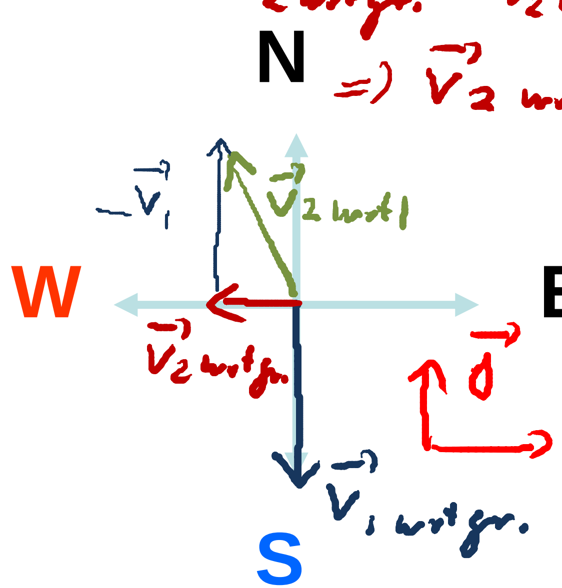
What is the velocity of Plane 2 relative to Plane 1? I.e., what is the ***apparent velocity*** of Plane 2 as viewed from the reference frame of Plane 1? (Take $\vec{i}$ pointing E and $\vec{j}$ pointing N.)

$\vec{V}_{2\ wrt\ gr.} = \vec{V}_{2\ wrt\ 1} + \vec{V}_{1\ wrt\ gr.}$

$\Rightarrow \vec{V}_{2\ wrt\ 1} = \vec{V}_{2\ wrt\ gr.} - \vec{V}_{1\ wrt\ gr.}$

$= -150 \frac{mi}{h} \vec{i} - (-450 \frac{mi}{h}) \vec{j}$

$= -150 \frac{mi}{h} \vec{i} + 450 \frac{mi}{h} \vec{j}$



$\vec{V}_{2\ \text{relative to}\ 1} = ?$

A. (150 mi/h) $\vec{i}$

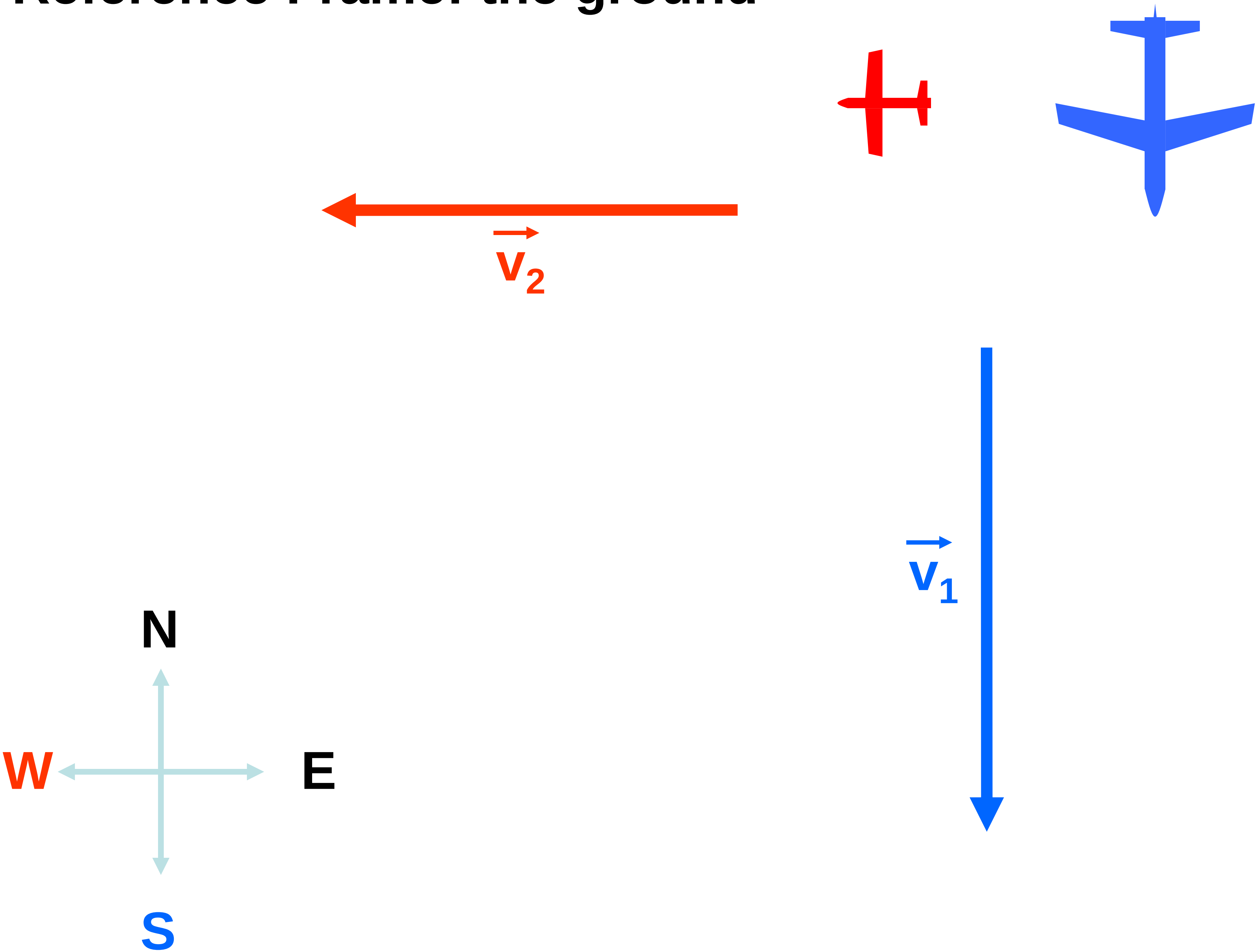
B. (-150 mi/h) $\vec{i}$

C. (-150 mi/h) $\vec{i}$ + (450 mi/h) $\vec{j}$

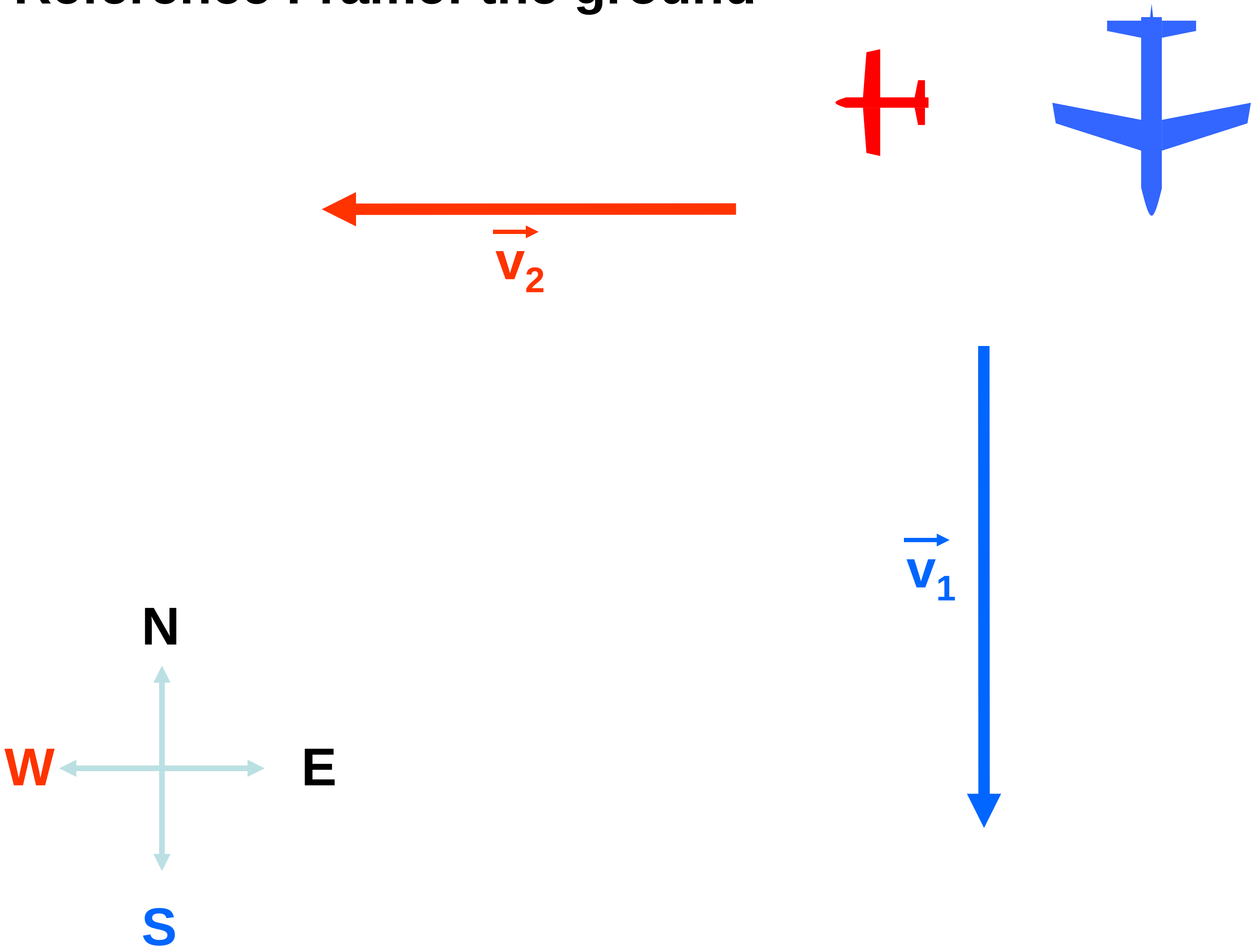
D. (150 mi/h) $\vec{i}$ + (- 450 mi/h) $\vec{j}$

E. (-150 mi/h) $\vec{i}$ + (- 450 mi/h) $\vec{j}$

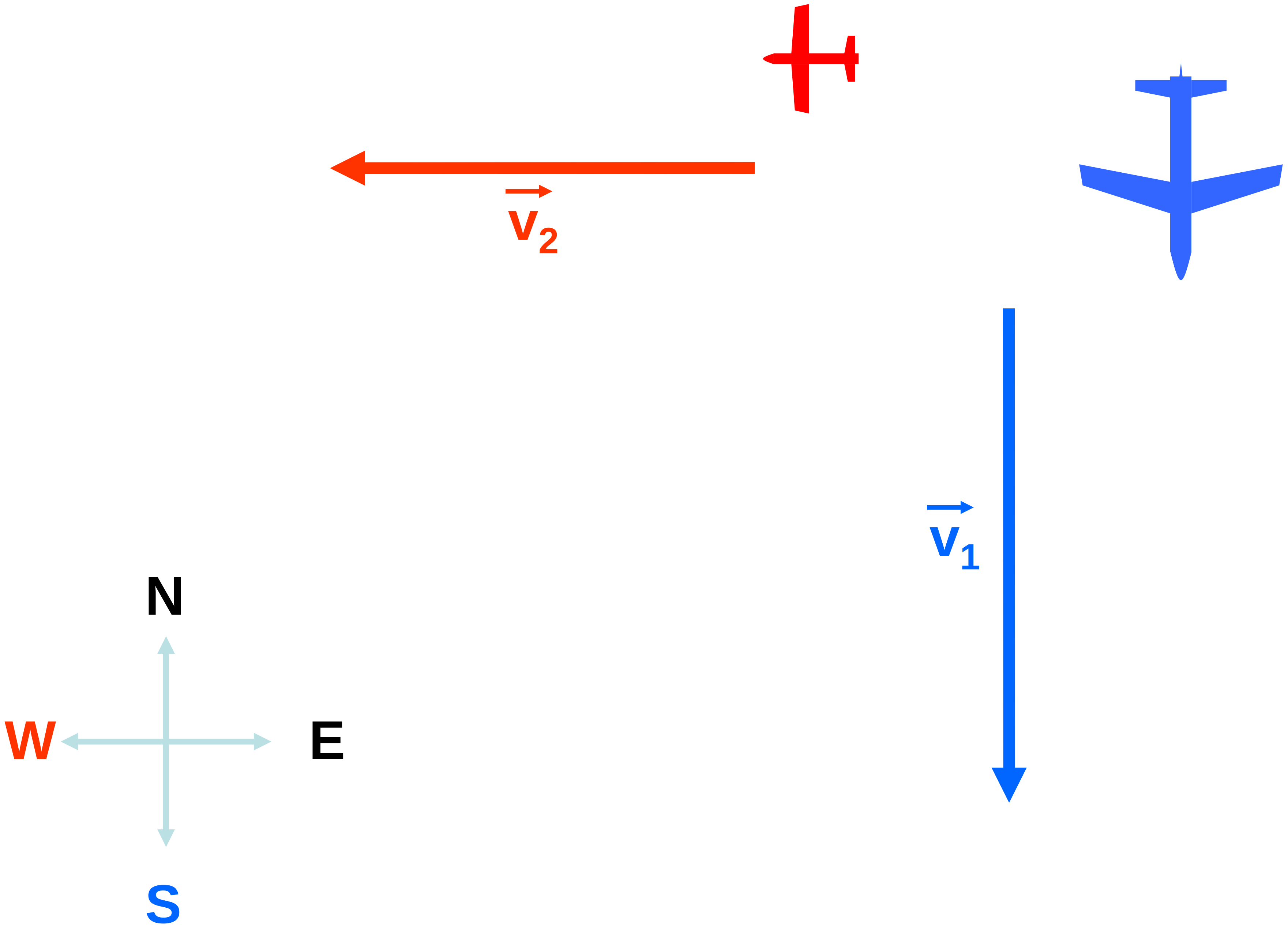
Reference Frame: the ground



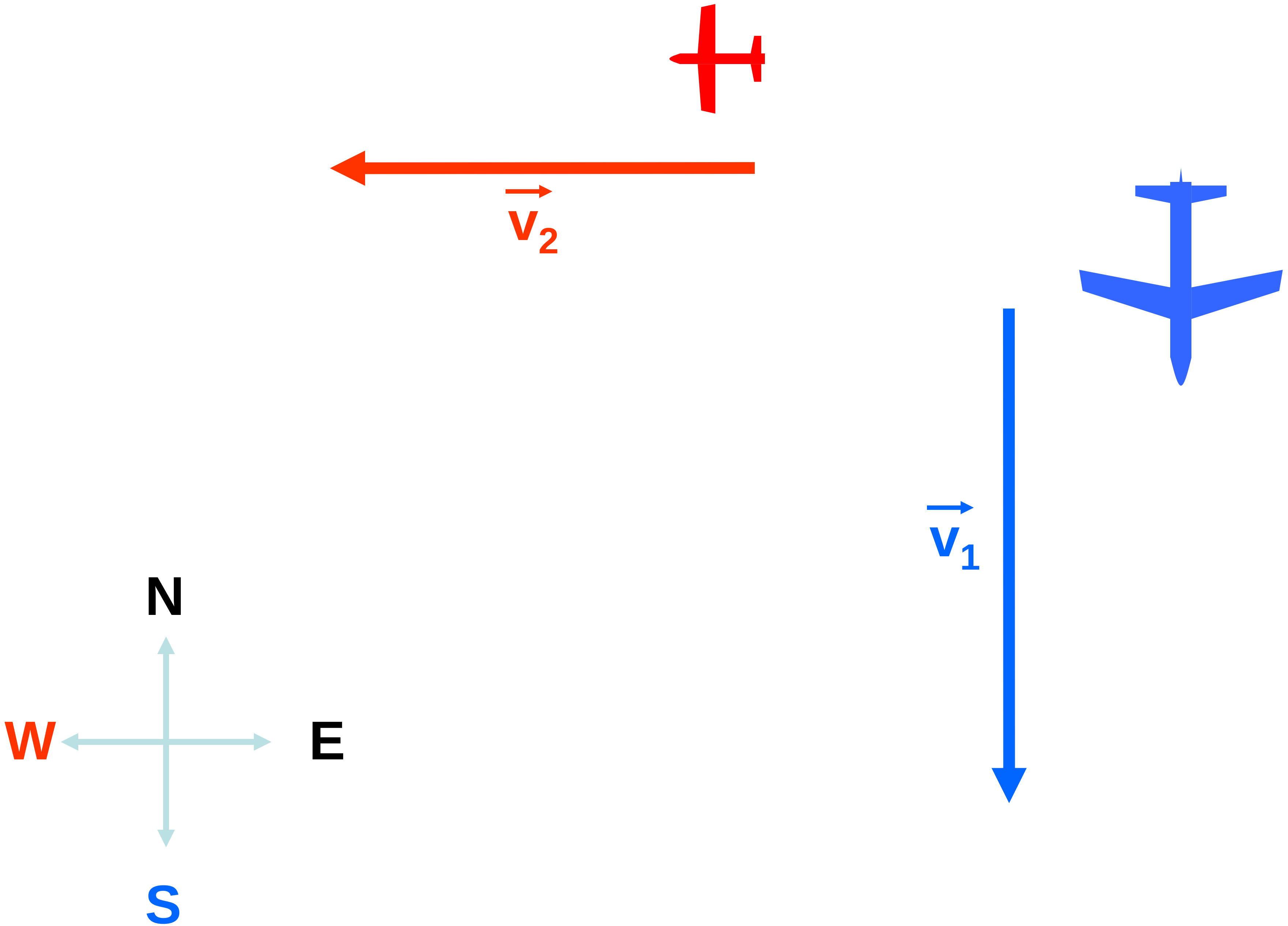
Reference Frame: the ground



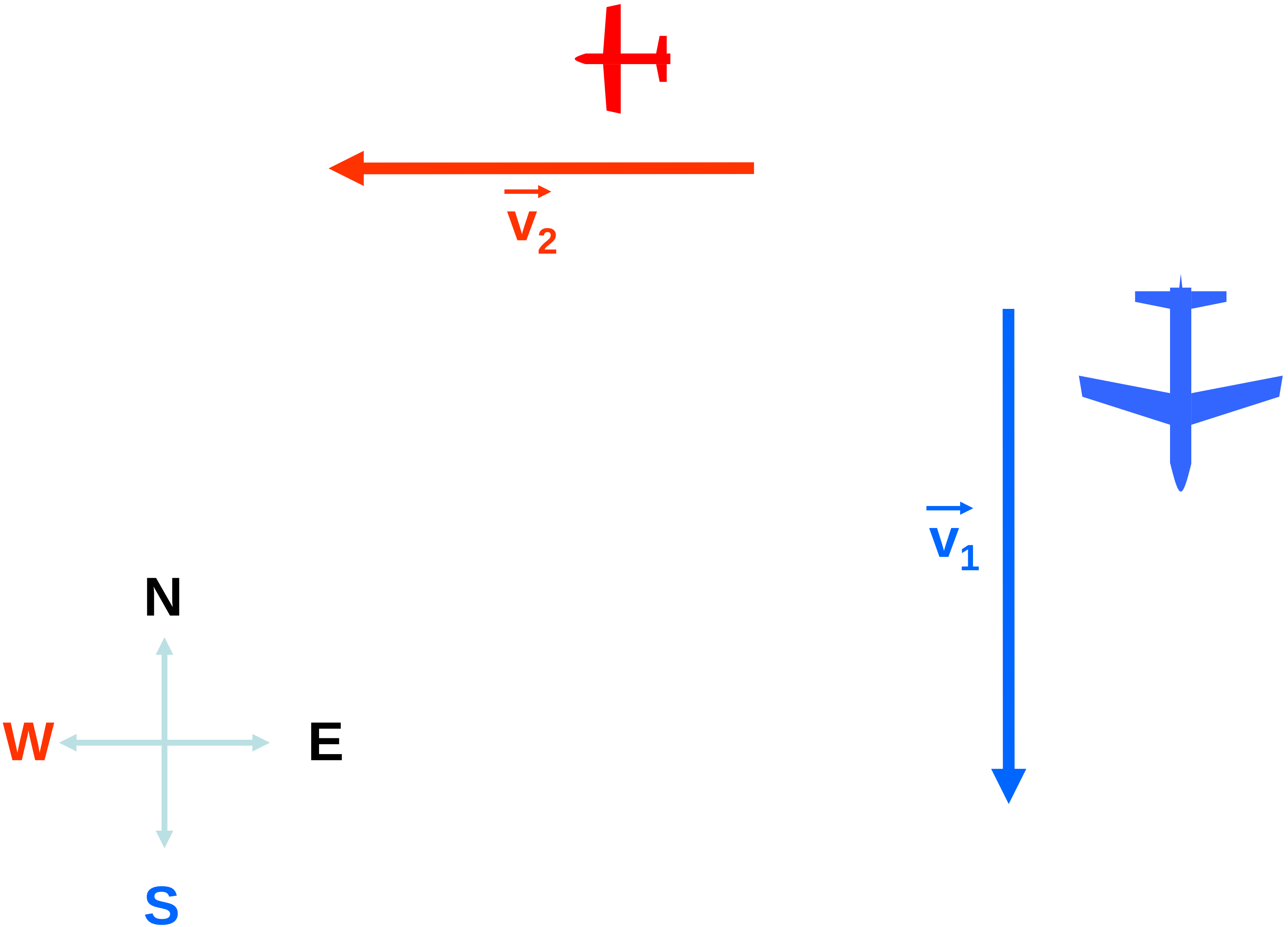
Reference Frame: the ground



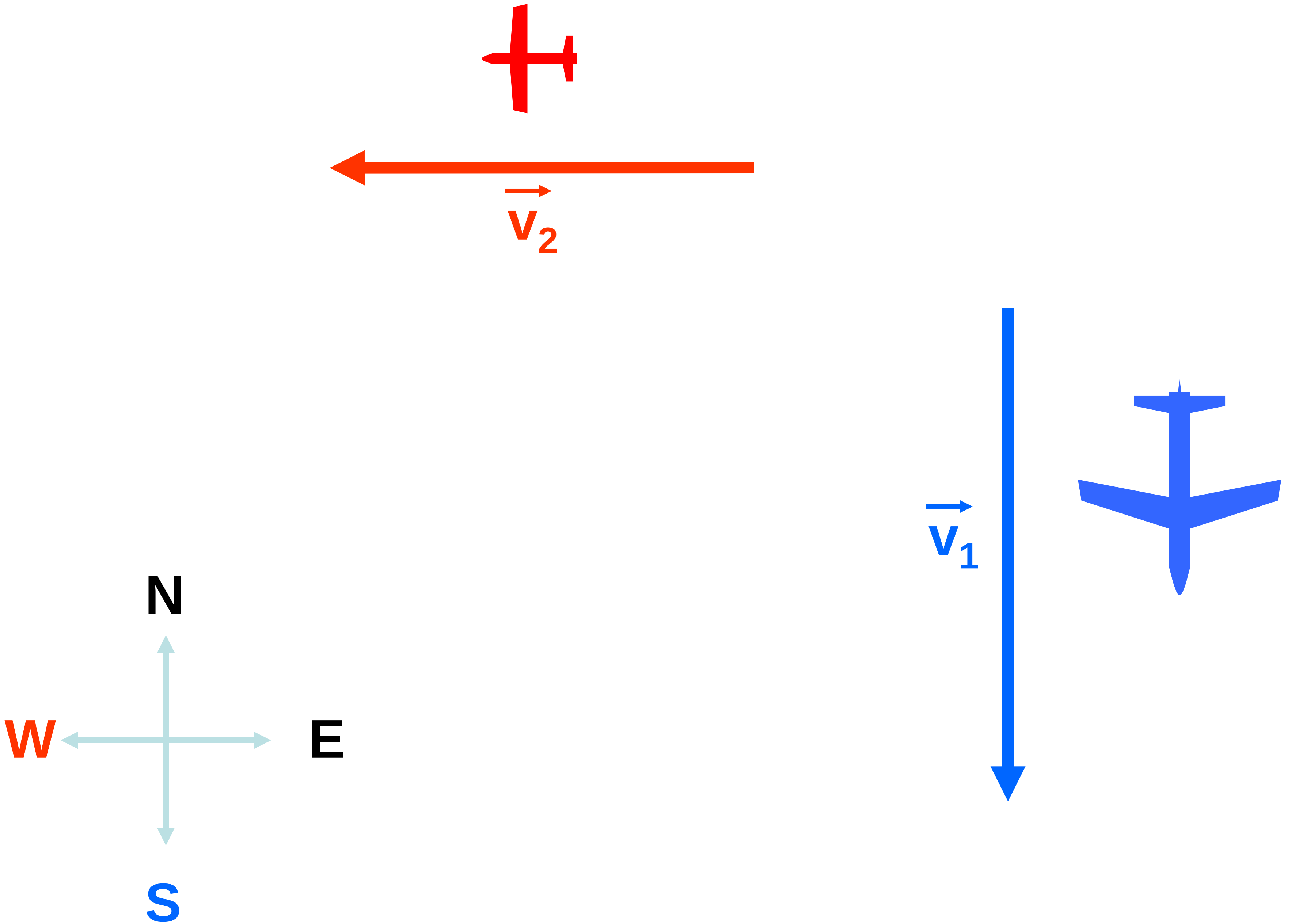
Reference Frame: the ground



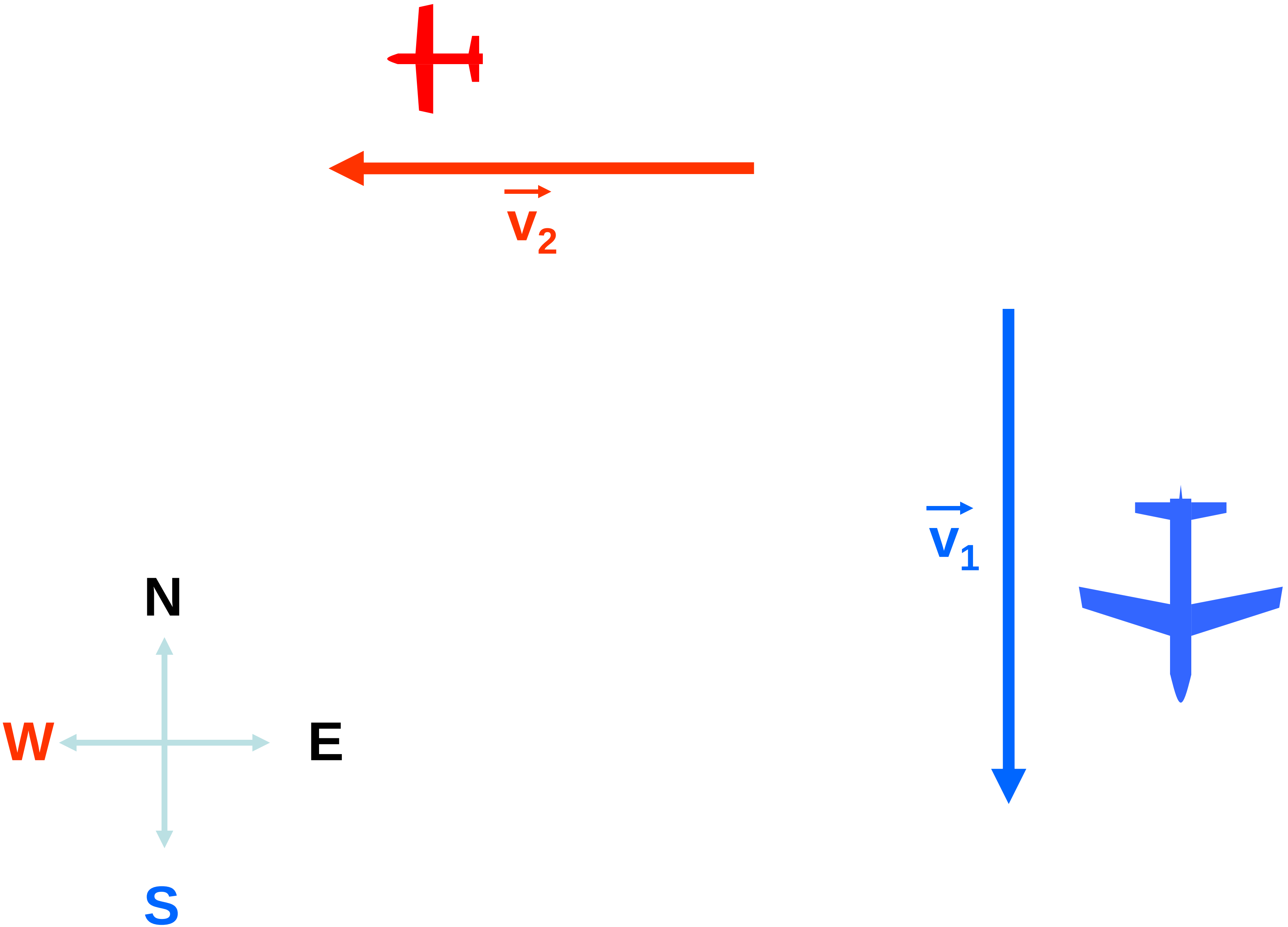
Reference Frame: the ground



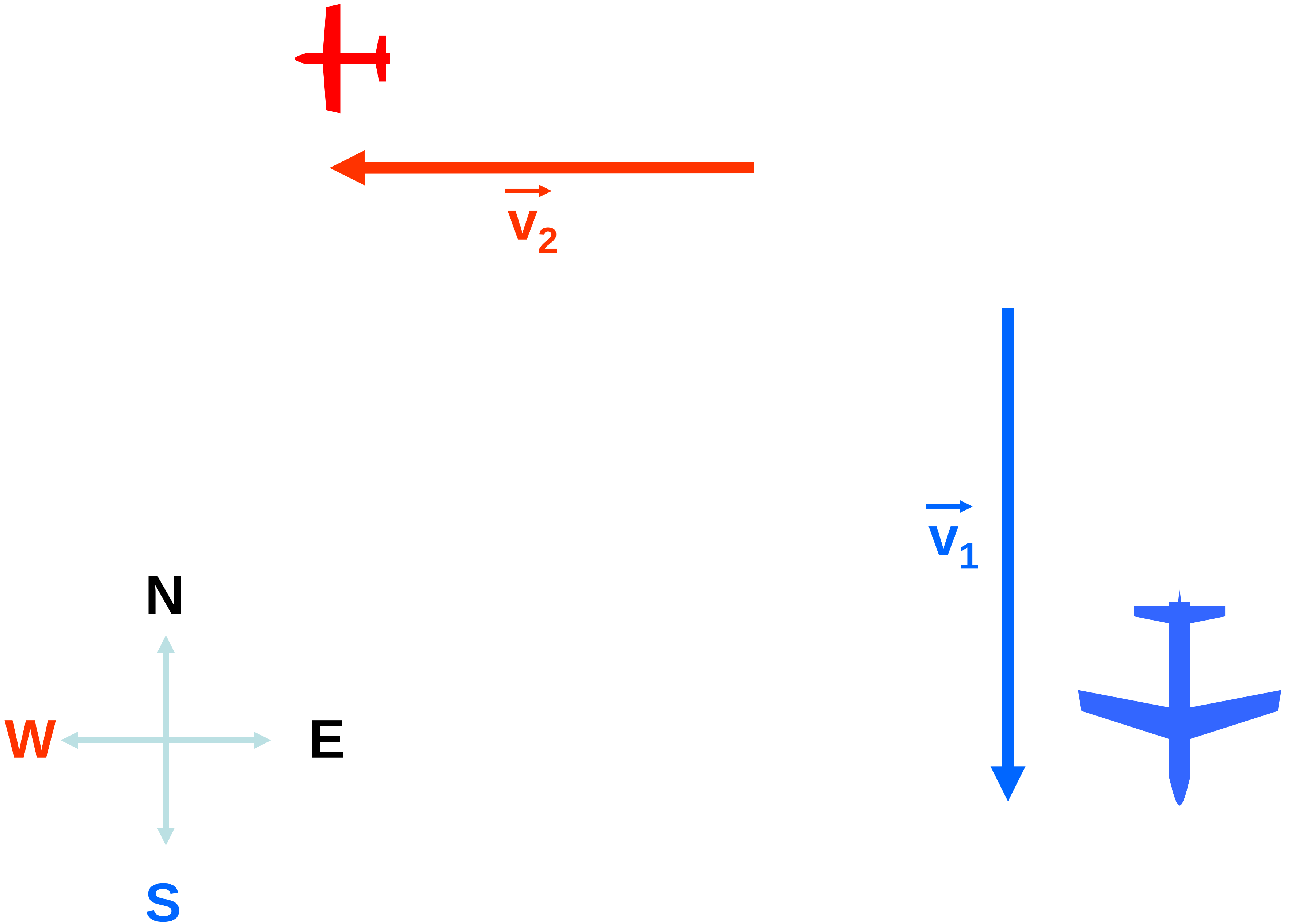
Reference Frame: the ground



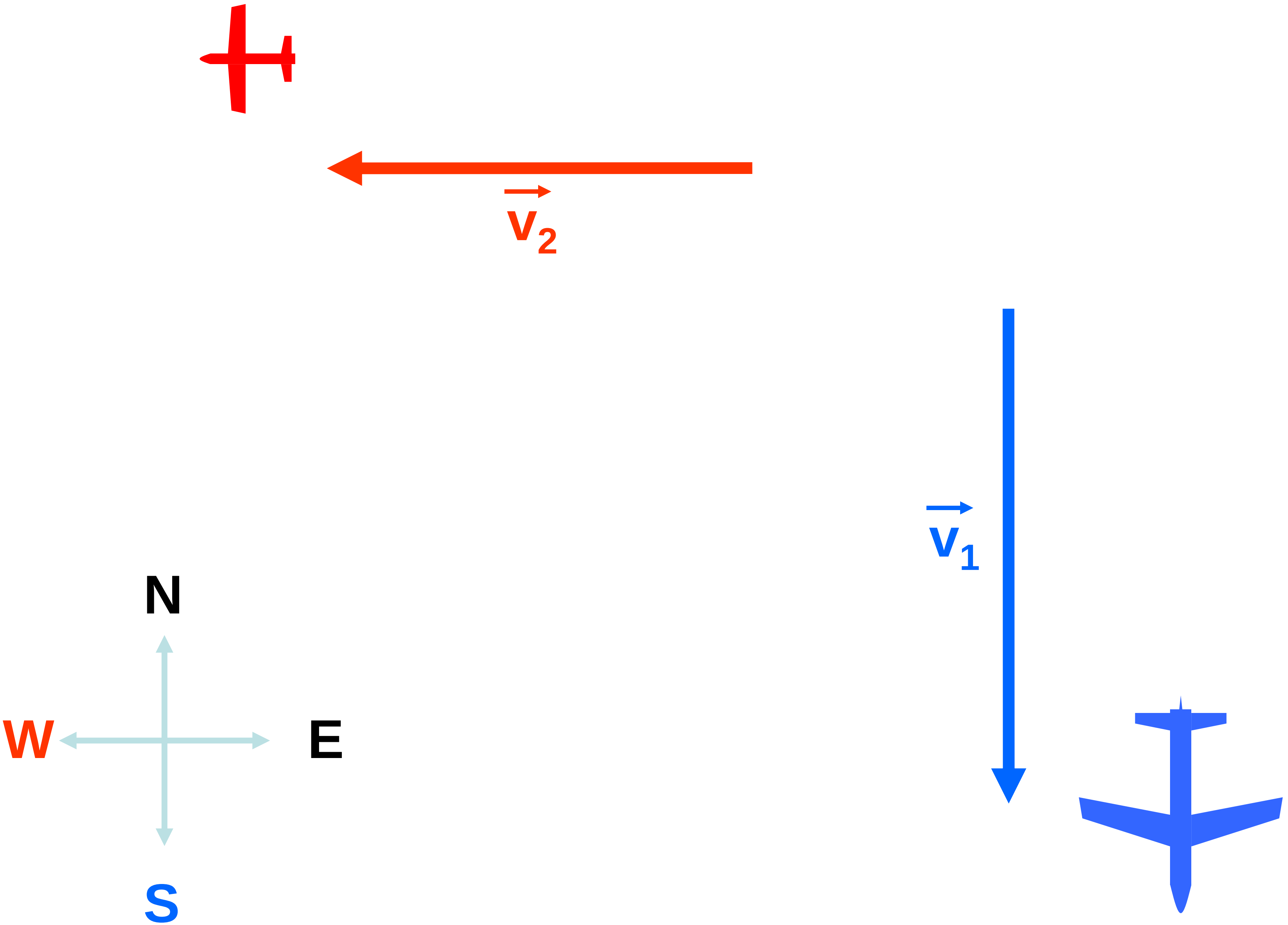
Reference Frame: the ground



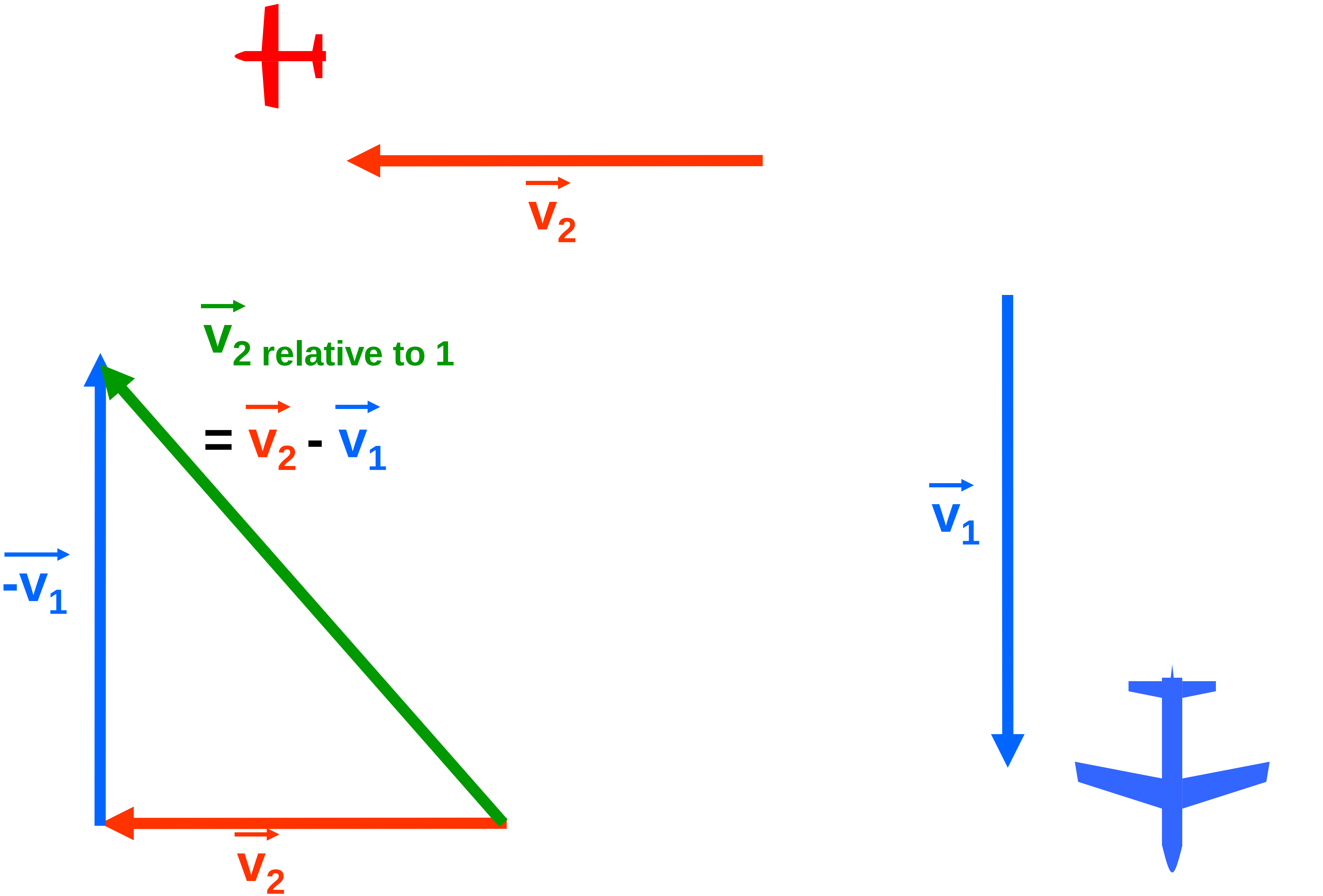
Reference Frame: the ground



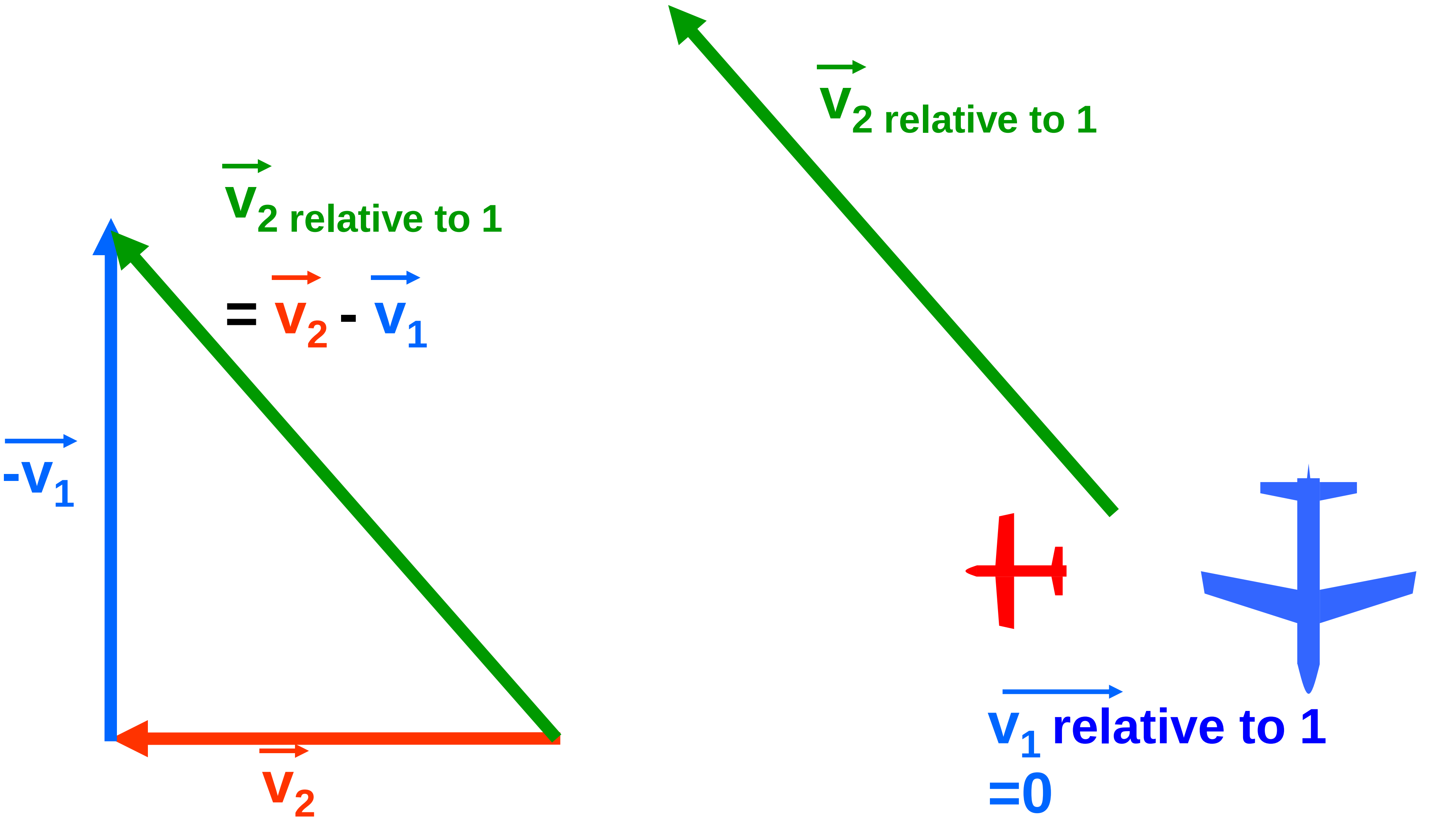
Reference Frame: the ground



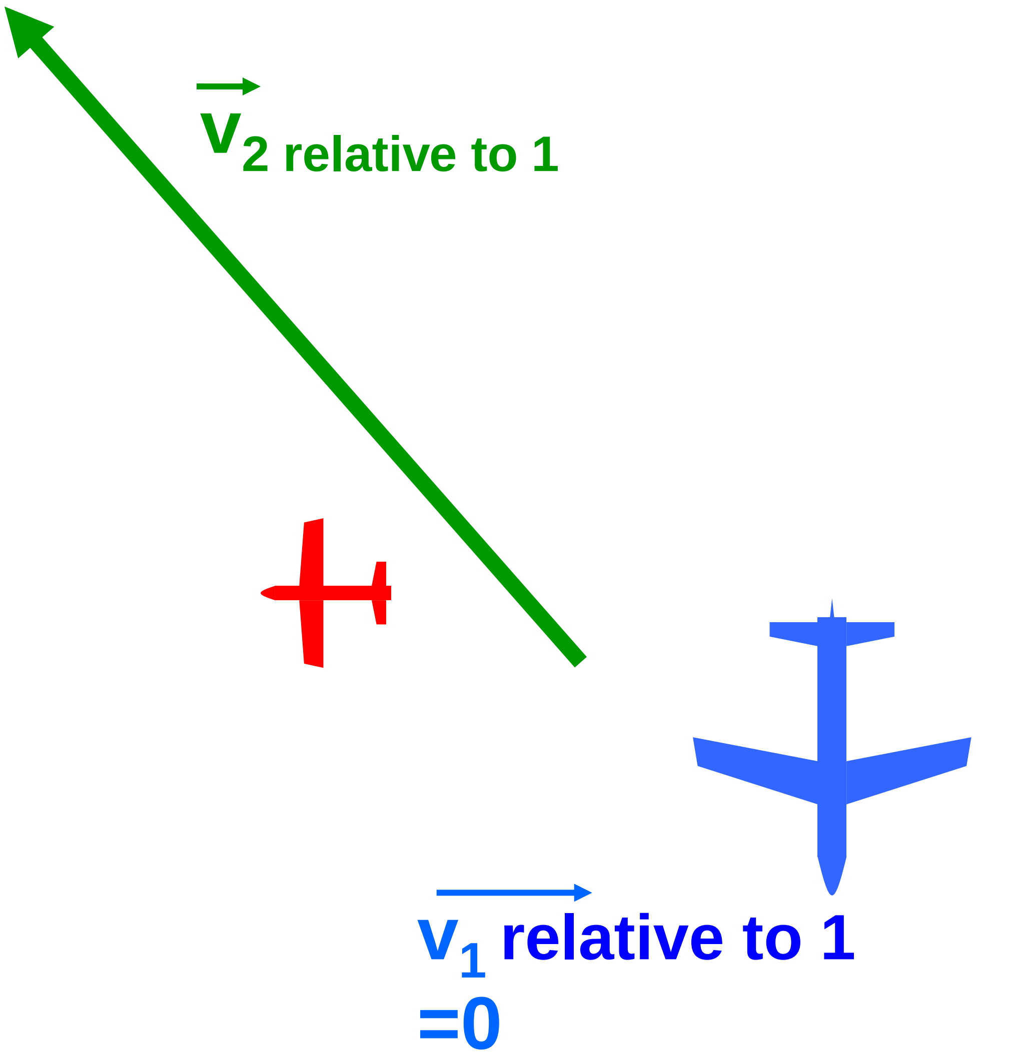
Reference Frame: the ground



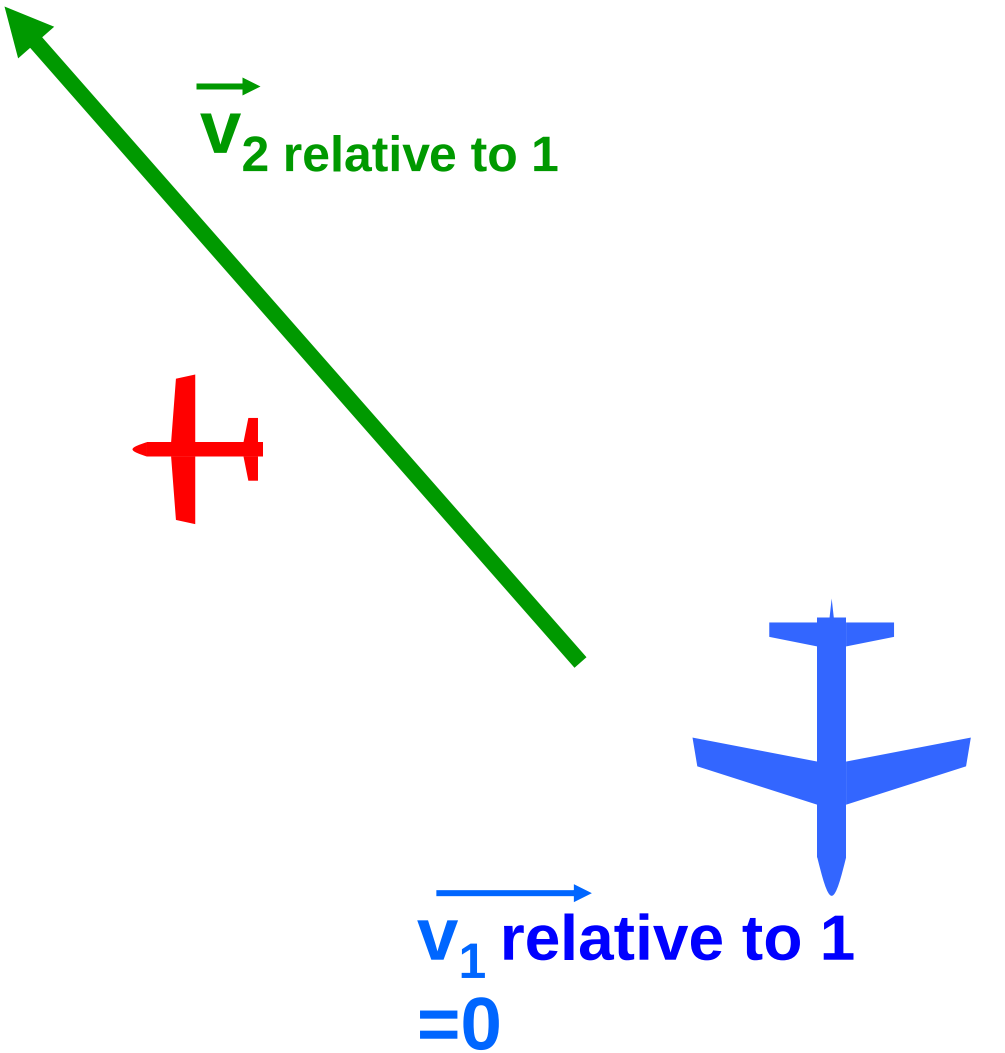
Reference Frame: Plane 1 (as viewed from plane 1)



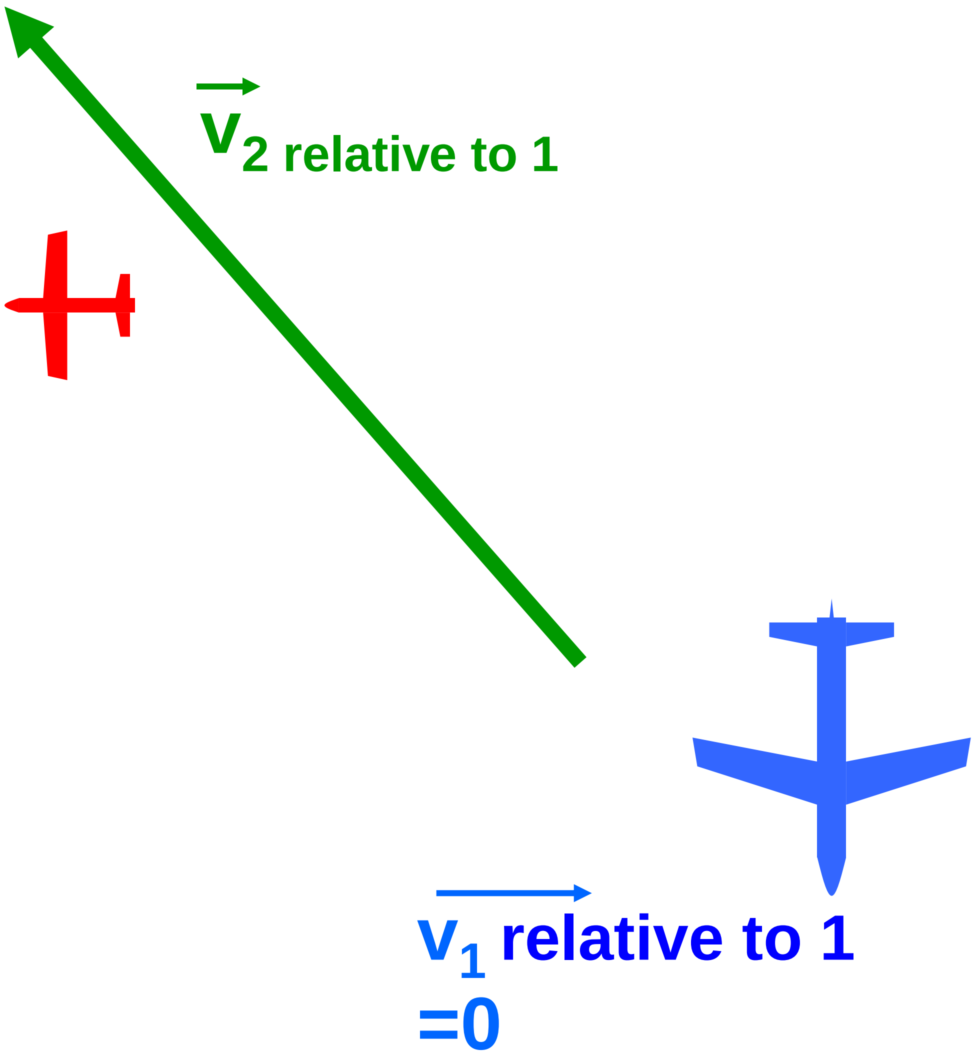
Reference Frame: Plane 1 (as viewed from plane 1)



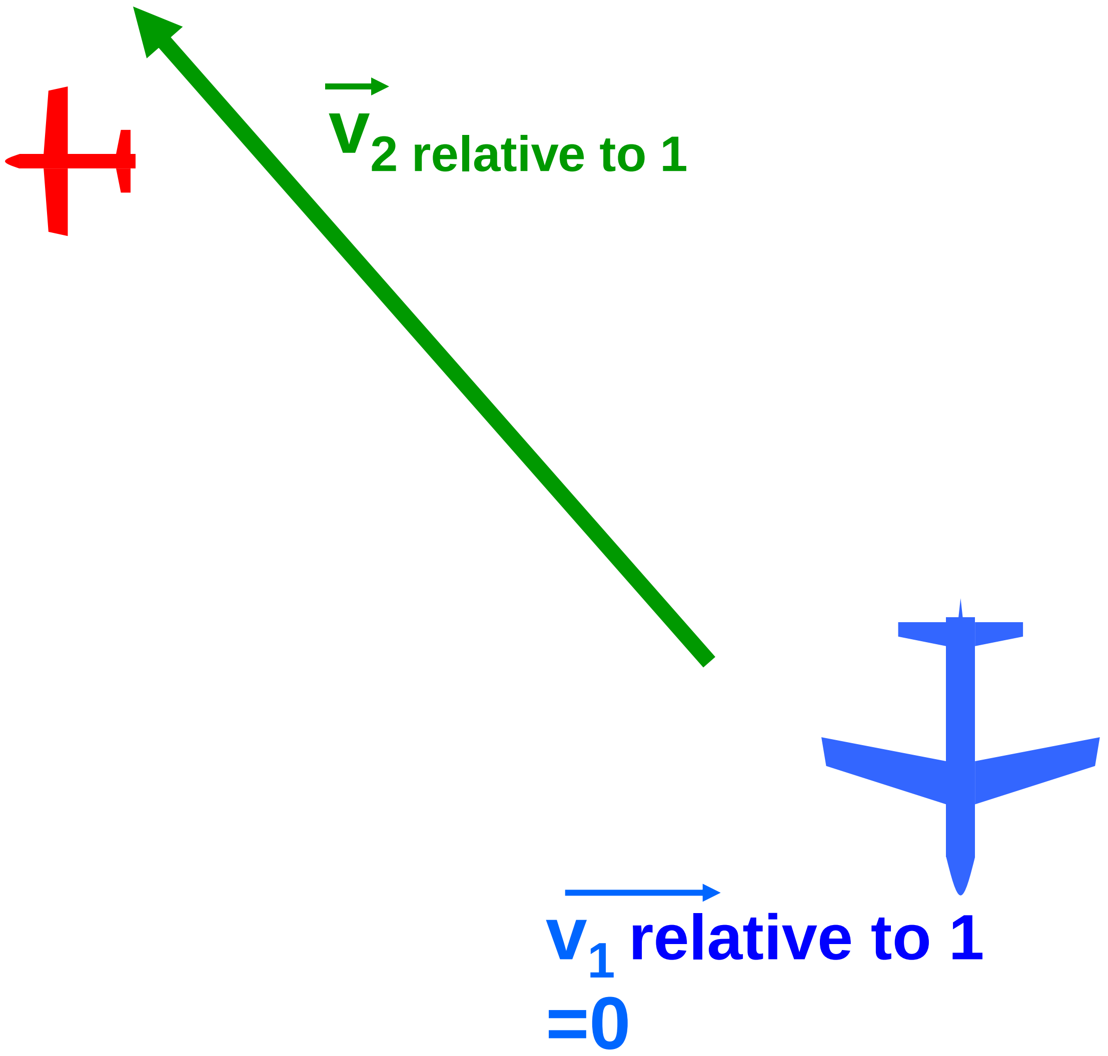
Reference Frame: Plane 1 (as viewed from plane 1)



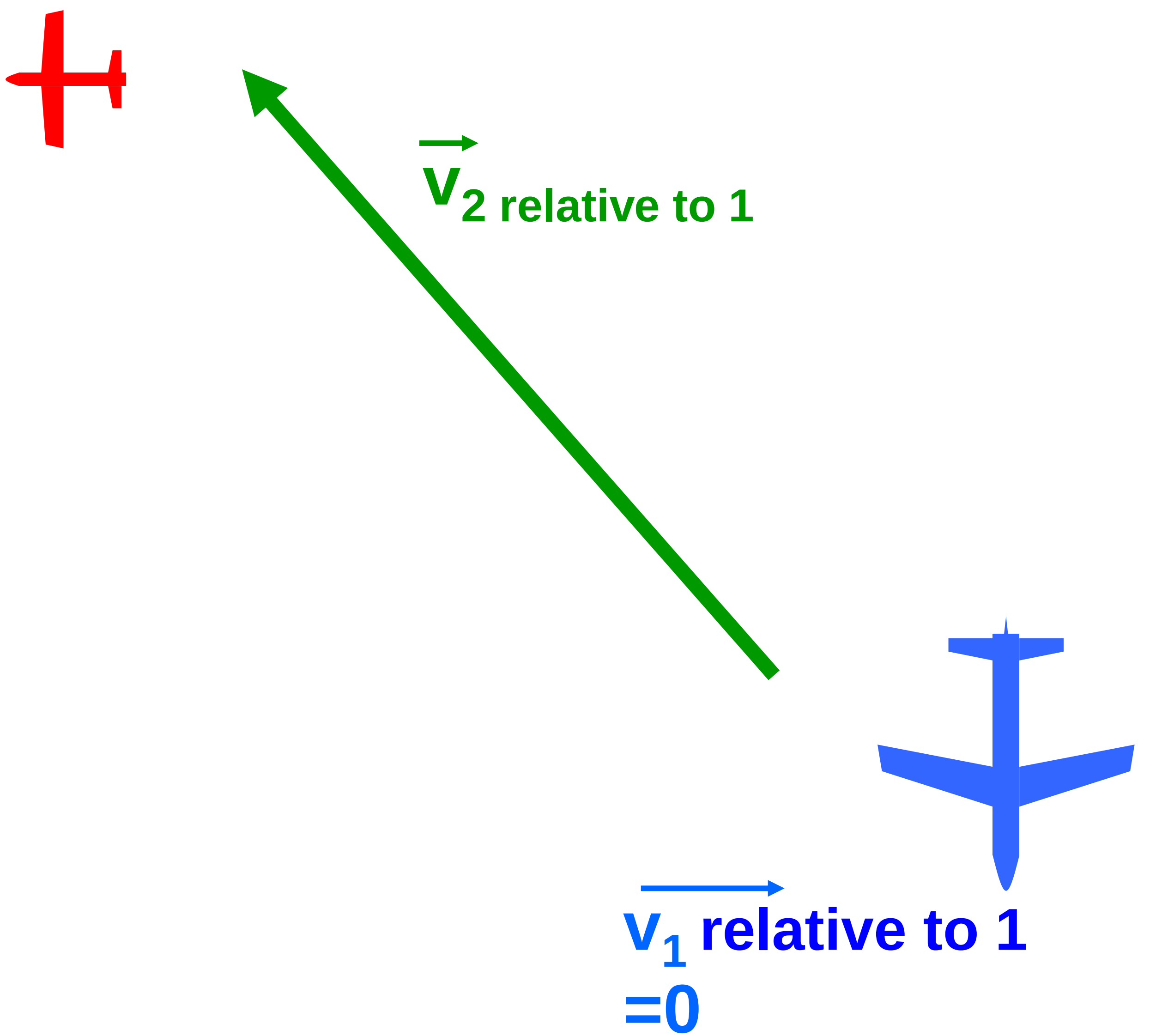
Reference Frame: Plane 1 (as viewed from plane 1)



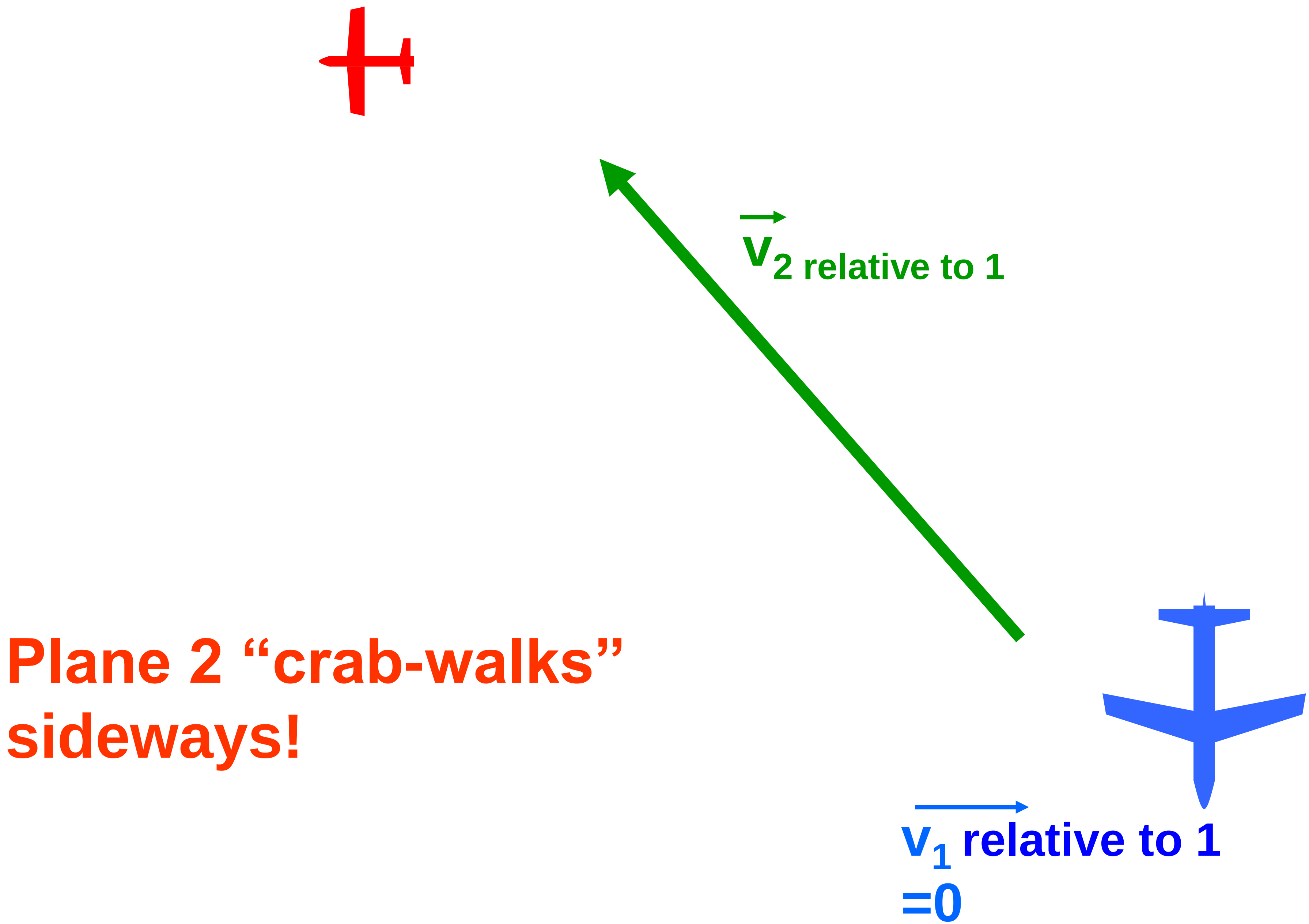
Reference Frame: Plane 1 (as viewed from plane 1)



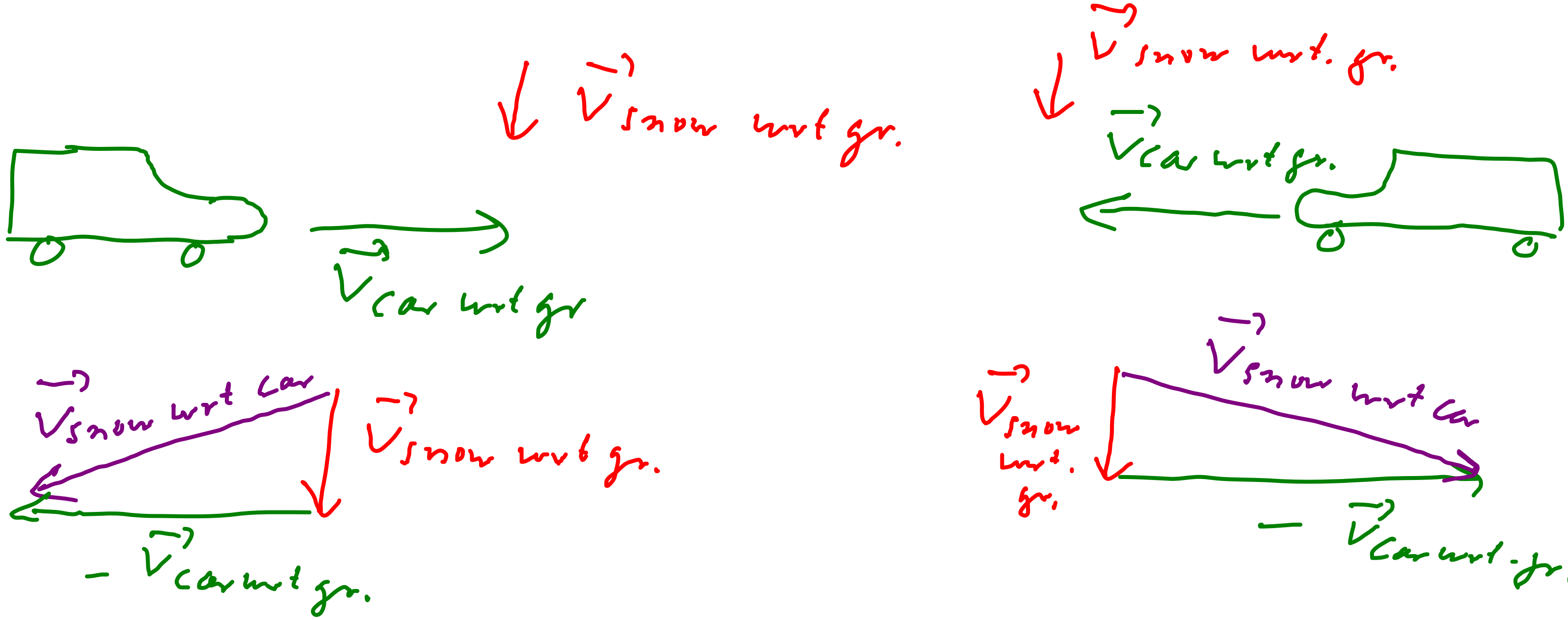
Reference Frame: Plane 1 (as viewed from plane 1)



Reference Frame: Plane 1 (as viewed from plane 1)

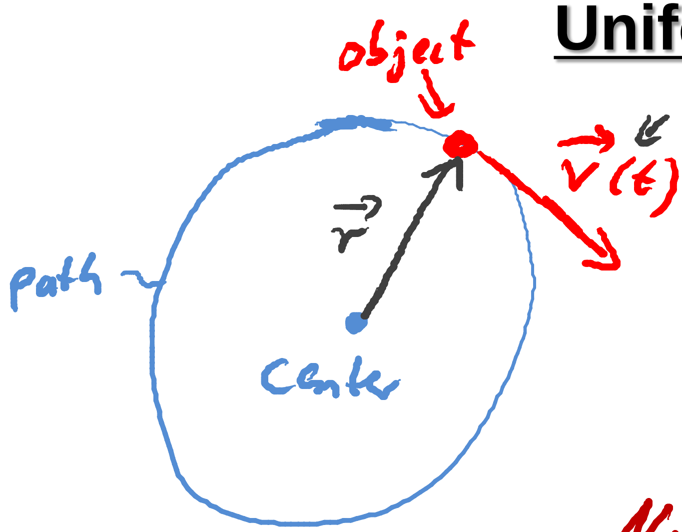


Snow on Windshield:



$$\vec{V}_{snow\ wrt.\ car} = \vec{V}_{snow\ wrt.\ gr.} - \vec{V}_{car\ wrt.\ gr.}$$

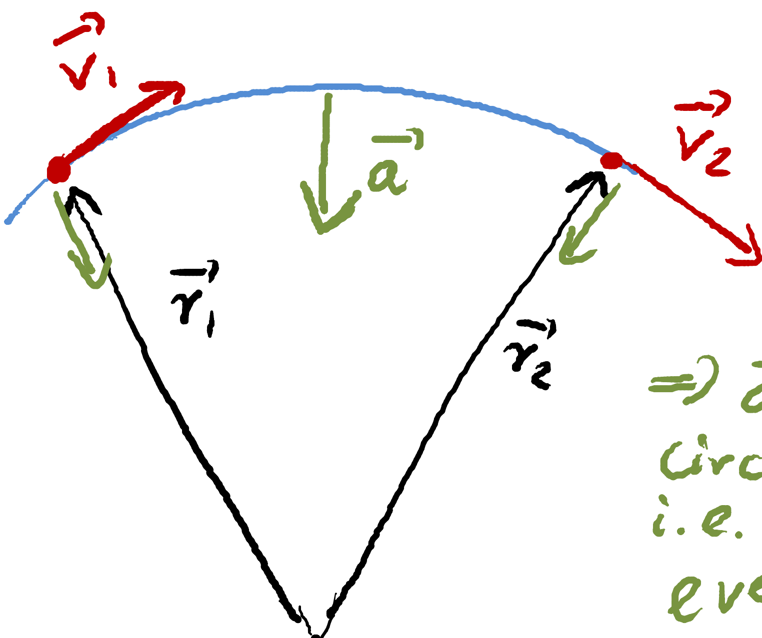
Uniform Circular Motion:



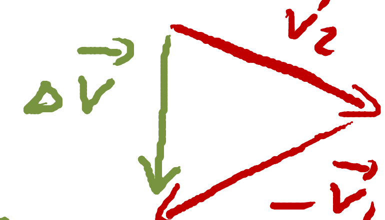
motion in a circle (or a circular arc) at constant speed $|v|$

Note: $\vec{v} \neq$ const

changes direction!



$$\Delta \vec{v} = \vec{v}_2 - \vec{v}_1 \neq 0$$



$$\Rightarrow \vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t} \neq 0$$

$\Rightarrow$ $\vec{a}(t)$ points to the center of the circle at each point in the motion, i.e. perpendicular ($\perp$) to motion everywhere!

An object of mass m moves in a circle of radius r at constant speed v.

What is the inward acceleration of the particle a in terms of m, r and v? *Use dimensional analysis*, i.e. $a \propto m^{\alpha} r^{\beta} v^{\gamma}$

a	m	r	v
$\frac{m}{s^2}$	kg	m	m/s

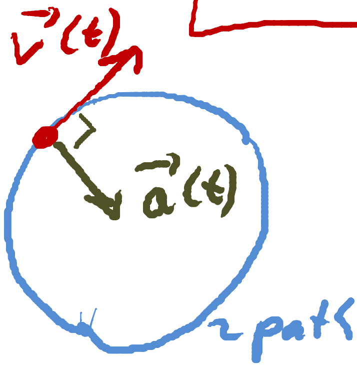
$\Rightarrow \alpha = 0, \ \gamma = 2, \ \beta = -1$

$\Rightarrow \ a \propto \frac{v^2}{r}$

- **A.** $a = rv$
- **B.** $a \propto mv^2/r$
- **C.** $a = mv^2/r$
- **D.** $a \propto v^2/r$
- **E.** $a = v^2/r$

detailed calculation gives:

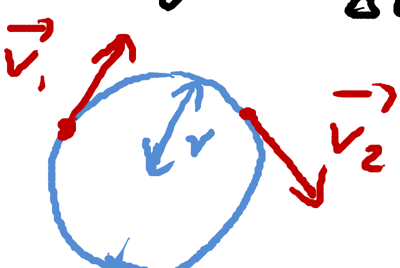
$$a = \frac{v^2}{r}$$ } centripetal acceleration in uniform circular motion



For an object to move in a circle at const. speed, it must have $|a| = \frac{v^2}{r} = \text{const}$, pointing to the centre of the circle, at each point in its path.

- why $a \propto v^2$, not v^1?

$$\vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t}$$



$|\Delta \vec{v}| \propto v$ (speed)

$\Delta t = \frac{\text{path length}}{v} \propto \frac{r}{v}$

$$\Bigg\} \; a \propto \frac{v}{\frac{r}{v}} = \frac{v^2}{r}$$

Analysis of a Salad Spinner

- diameter of basket d = 2r = 0.2 m
- basket revolutions/crank turn = 4
- crank turns/second during operation= 2
- Period of rotation

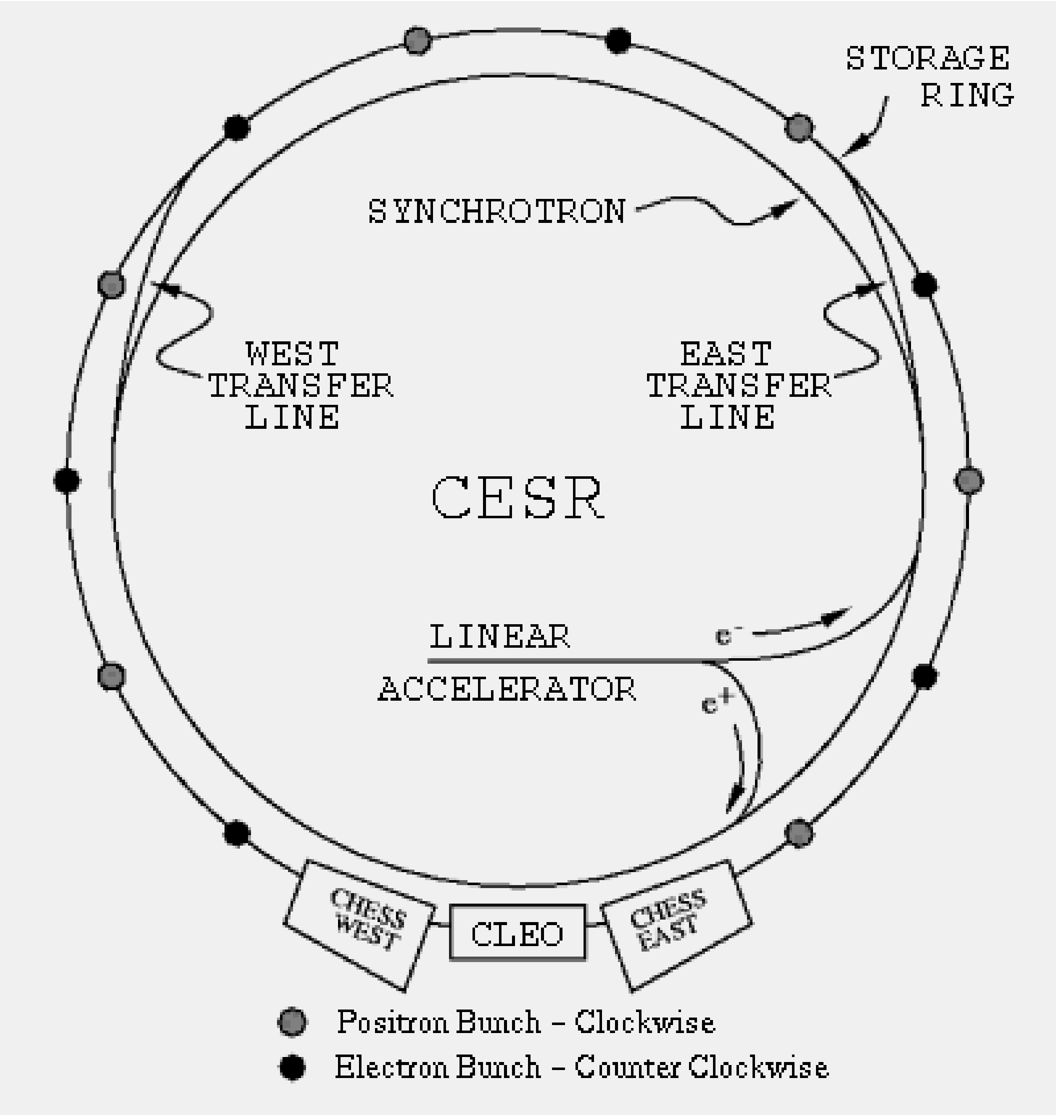
  = 1 / (crank turns × revs/turn) = 1/8 s
- speed of basket rim v = $2\pi r$ / T = $2\pi \cdot 0.1\,m / \frac{1}{8}\,s \approx 5\,m/s$
- inward acceleration of object on the rim

  a = v^2/r = $25\,\frac{m^2}{s^2} / 0.1\,m = 250\,m/s^2$
- Divide by g=10 m/s^2 to get accelerations in g's:

  a = 25g

The Cornell Electron Storage Ring (CESR)

Inside the Tunnel:



- Linear accelerator produces electrons and positrons.
- Synchrotron accelerates them to 99.9999995% of *c* (E=5 GeV).
- Electrons/positrons stored in storage ring, circulating there.

Ring circumference = 768 m

Ring radius = 122 m

Inward acceleration of electrons in storage ring?

$a = v^2/r$

$\sim (3 \times 10^8 \text{ m/s})^2 \;/\; 122 \text{ m}$

$\sim 7 \times 10^{14} \text{ m/s}^2$

$\sim$ 70 trillion g's

Force $\vec{F}$

so far: $\vec{r}(t) \rightleftarrows \vec{v}(t) \rightleftarrows \vec{a}(t)$ } completely describe motion

Next: What causes motion? ⇒ Force

What is a force:

- push or pull
- force acts on an object ("on")
- forces require an agent ("by")
- force is a vector ⇒ magnitude + direction
- can be contact forces, or long range forces (e.g. gravity)
- determines $\vec{a}(t)$:

$$\vec{a} \propto \vec{F}$$