Recap

- **Capacitor:**
  - Energy stored: \( U_{\text{cap}} = \frac{Q^2}{2C} = \frac{1}{2} Q \Delta V = \frac{1}{2} \varepsilon_0 \Delta V^2 \)
  - Capacitors in parallel: \( C_{\text{eff}} = \sum_{i=1}^{n} C_i \) \( \Rightarrow Q_{\text{total}} = \sum Q_i \)
  - Capacitors in series: \( \frac{1}{C_{\text{eff}}} = \sum_{i=1}^{n} \frac{1}{C_i} \) \( \Rightarrow Q_i = Q_2 = Q_3 \ldots \)

- **Energy density of an electric field:**
  \( u_{\text{el}} = \frac{\text{energy}}{\text{volume}} = \frac{1}{2} \varepsilon_0 E^2 \) \( \Rightarrow \) takes energy to create electric fields!

- **Dielectrics:** Insulator that can be polarized by an applied electric field \( \Rightarrow \) Electric field in dielectrics \( E_{\text{dielectric}} = \frac{E_{\text{applied}}}{K} \) \( K \) = dielectric constant

- **Electric current:**
  \( i = \frac{dQ}{dt} = \frac{\partial Q}{\partial t} = (\text{charge passing through}) \)
  \( \left[ i \right] = \text{coulombs per second} = \text{amps} = \text{ampere} = 1A \)
Today:

- Electric current
- Current density
- Electrical resistance
Notes:

1. Charge is conserved:
   \[ i_A = i_B \]
   The current is the same in any cross section of the wire ("continuity").

2. Average current density: \( J \)

   \[ J = \frac{i}{A_L} = \frac{\text{Current through } A_L}{\text{Area } A_L \text{ to current flow}} \]

   \[ [J] = \frac{A}{m^2} \]
Direction of current

Convention: Current arrow is drawn in direction in which positive charge carriers would move, if they would carry the current.

But: actual charge carriers can have positive or negative charge.

Moving + charge: $\Delta Q < 0 \quad \Delta Q > 0$

or

Moving - charge: $\Delta Q < 0 \quad \Delta Q > 0$

Current arrow points to right in both cases

Net effect same
Consider a beam of protons, all moving with constant velocity $\vec{V}$.

If $n$ is the number of protons per unit volume in the beam, how many protons pass through the cross sectional area $A$ in time $\Delta t$?

\[ \text{# of protons} = n \cdot \text{Volume passing through } A \text{ in } \Delta t = n \cdot A \cdot v \cdot \Delta t \]

A. $nA\Delta t$  
B. $n/(Av\Delta t)$  
C. $nAv\Delta t$  
D. $nAv/\Delta t$
Consider a beam of protons (charge $e$), all moving with constant velocity $\vec{V}$. $n$ is the number of protons per unit volume in the beam.

What is the electric current carried by the beam?

\[ i = nevA \]

\[ \Rightarrow \Delta Q = e \cdot (nA v ot) \text{ in time interval } ot \]

\[ \Rightarrow \frac{\Delta Q}{ot} = i = nevA \]

Average current density $\vec{J} = \frac{i}{A} = nev$

A. 0  B. $nevA$  C. $nev$  D. $evA$
Conclusion:

- magnitude of current density in conductor:

\[ \mathbf{J} = \frac{\mathbf{i}}{A_f} = n q \mathbf{v}_{drift} \]

- Define current density vector:

\[ \mathbf{J} = n q \mathbf{v}_{drift} \]

- "drift speed" of charge carriers along conductor

points in direction of "current arrow"
Electric Currents in Metals:

- Some of the electrons are the mobile charge carriers: \( q_{\text{electron}} = -e \)
- How many mobile (free) electrons are there?
  - Typically: 1 to 2 per atom
  - \[ n = \frac{\# \text{of charge carriers}}{\text{volume}} = (1\ldots 2) \cdot \frac{N_A \cdot e}{\text{atomic mass}} \times 10^{29} \text{ free } e^-/m^3 \]

- \( \text{few } 10^{-10} \text{m} \) microscopic view
- Electrons move randomly
- \( \sqrt{2} \) random motion speed comes from non-zero energy ("Fermi energy") of free electrons in metal

But: electrons collide constantly with each other and with atoms in metal (huge!)
- \( 10^{13} \text{ to } 10^{14} \text{ times/sec!} \)
- \( \vec{V}_{\text{average}} = 0 \)
How to get a current? =) Apply electric field! 

Note: battery maintains potential difference 

=) electric field along wire 

=) not in electrostatic equilibrium? 

(applied electric field) = \( \vec{E} \) 

electrons drift with average drift speed \( \vec{V}_{\text{drift}} \) in direction opposite to \( \vec{E} \), pine gection c0 

(in addition to fast, random motion) 

but constantly collide and loose energy 

=) average, constant drift speed 

\( V_{\text{drift}} \approx 10^{-5} \ldots 10^3 \text{ m/s} \) (very slow!!) 

Current density in metal: 

\( \vec{j} = n(-e) \vec{V}_{\text{drift}} \) \{ point in direction of \( \vec{E} \) \}
General case: current density $\mathbf{j} = \frac{i}{A}$ if both + and - charge carriers can move.

\[ \mathbf{j} = n_+ q_+ \mathbf{v}_{\text{drift}} + n_- q_- \mathbf{v}_{\text{drift}} \]

with $q_+ > 0$ and $q_- < 0$

Total current through area $A$:

\[ i = \int \mathbf{j} \cdot d\mathbf{A} = \mathbf{j} \cdot \mathbf{A} = \frac{\mathcal{C}}{A} \frac{q}{e} = n_+ q_+ \mathbf{v}_{\text{drift}} + n_- q_- \mathbf{v}_{\text{drift}} \frac{A}{A} \]

if current is uniform across the surface

$n$: number of + or - charge carriers/volume