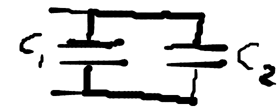
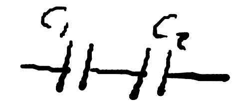
Recap

Lecture 10

- Capacitor:
  - Energy stored: $U_{cap} = \frac{Q^2}{2C} = \frac{1}{2} Q \Delta V = \frac{1}{2} C \Delta V^2$
  - Capacitors in parallel: $C_{eff} = \sum_{i=1}^{N} C_i \Big\} Q_{total} = \sum Q_i$



  - Capacitors in series: $\frac{1}{C_{eff}} = \sum_{i=1}^{N} \frac{1}{C_i} \Big\} Q_1 = Q_2 = Q_3 \ldots$



- Energy density of an electric field:

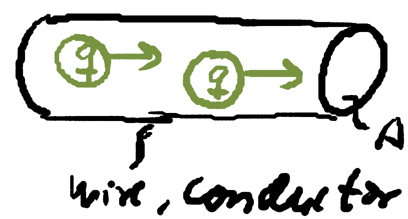
  $u_{el} = \frac{\text{energy}}{\text{volume}} = \frac{1}{2} \varepsilon_0 E^2 \Big\}$ takes energy to create electric fields!

- Dielectrics: Insulator that can be polarized by an applied electric field $\Rightarrow E_{\text{with dielectrics}} = \frac{E_{applied}}{\kappa}$ ← dielectric constant

  $\Rightarrow$ replace ε_0 by $\kappa\varepsilon_0$ in all equations if dielectric is present

- Electric current:

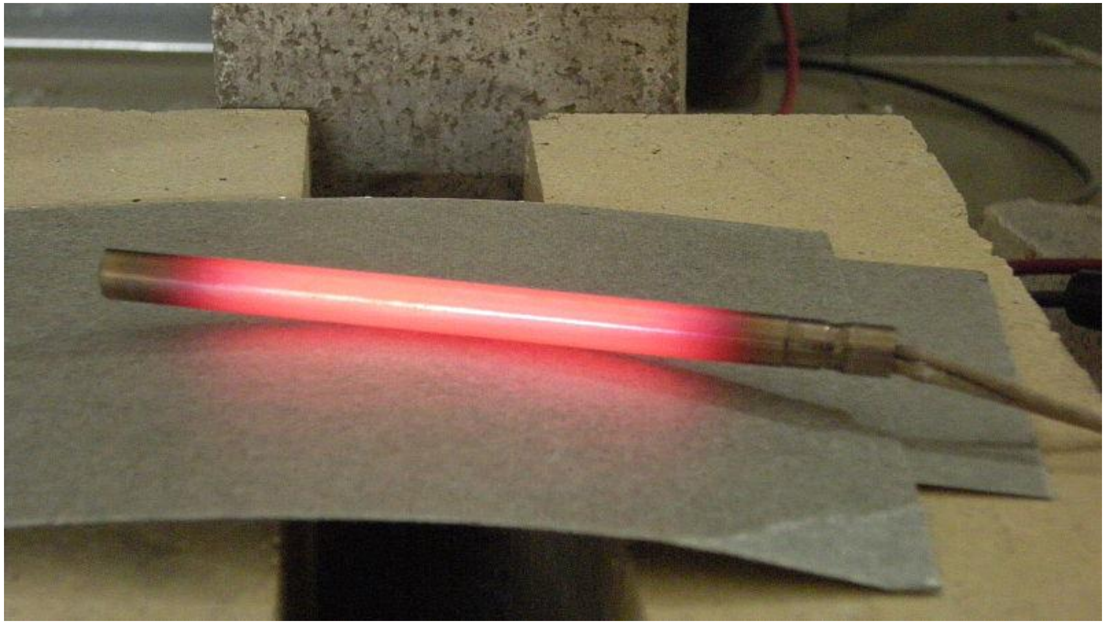
  $i = \frac{dq}{dt} = \frac{\Delta q}{\Delta t} = \left(\begin{array}{l}\text{charge passing trough} \\ \text{area A per time}\end{array}\right)$



  $[i] = C/s = \text{ampere} = 1A$

Today:

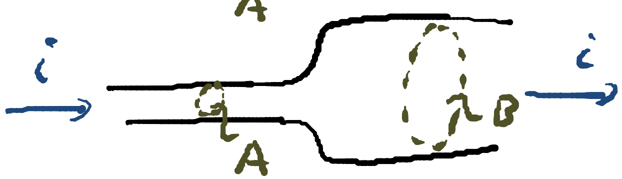
- Electric current
- Current density
- ~~Electrical resistance~~



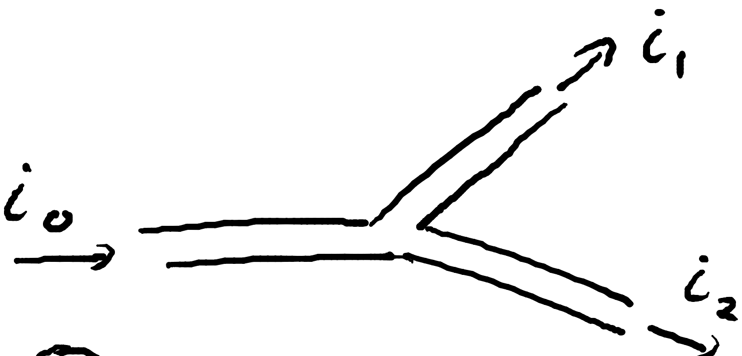
Notes:

① Charge is conserved:

$i_A = i_B$



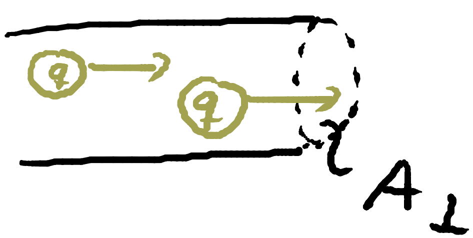
The current is the same in any cross section of the wire ("continuity")



for junction:

$$\boxed{i_0 = i_1 + i_2}$$

② Average current density: J



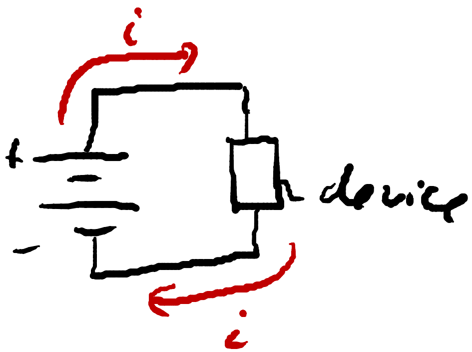
$$\boxed{J = \frac{i}{A_\perp} = \frac{\text{current through } A_\perp}{\text{Area} \perp \text{to current flow}}}$$

$[J] = A/m^2$

③ Direction of current

$$\underbrace{\xrightarrow{\;\;i\;\;}}_{\text{arrow indicating "direction" of current}}$$

Convention: Current arrow is drawn in direction in which positive charge carriers would move, if they would carry the current.



But: actual charge carriers can have positive or negative charge!

$$\xrightarrow{\qquad i \qquad}$$

moving + charges: $\Delta Q < 0 \xrightarrow{\quad +Q \quad} \Delta Q > 0$

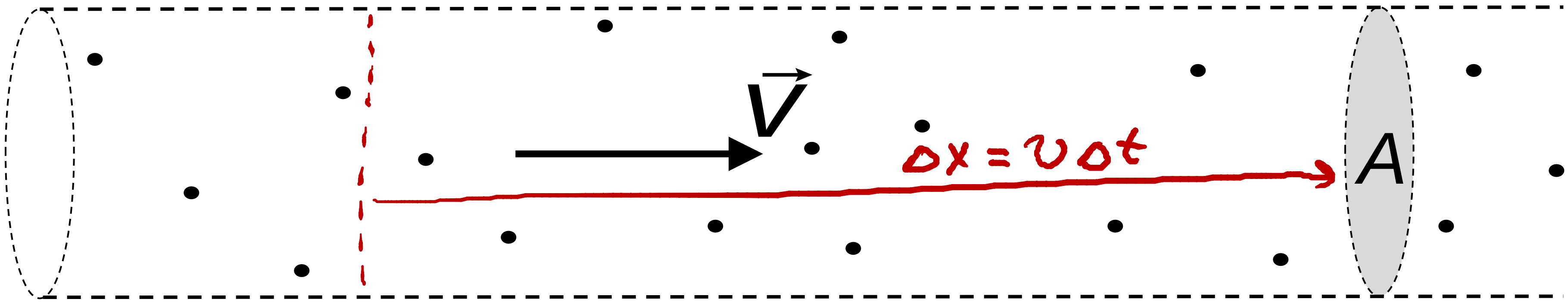
or

moving − charges: $\Delta Q < 0 \xleftarrow{\quad -Q \quad} \Delta Q > 0$

"current arrow" points to right in both cases

$$\underbrace{\qquad\qquad\qquad\qquad\qquad\qquad}_{\text{net effect same}}$$

Consider a beam of protons, all moving with constant velocity $\vec{V}$.
If n is the number of protons per unit volume in the beam, how many protons pass through the cross sectional area A in time Δt?



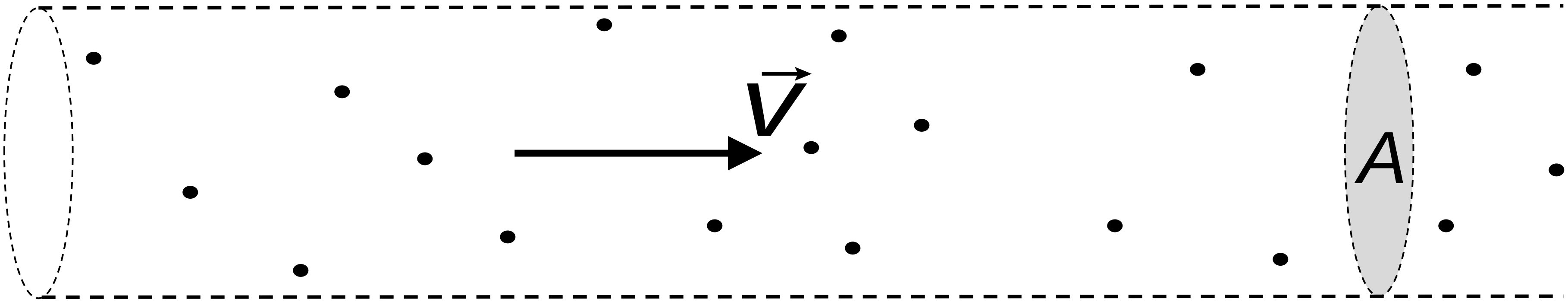
\# of protons $= n \cdot$ Volume passing through A in Δt

$= n\,\Delta x\, A = n A v \Delta t$

A. $nA\Delta t$ B. $n/(Av\Delta t)$ C. $nAv\Delta t$ D. $\cancel{\text{D}}$. $nAv/\Delta t$

Consider a beam of protons (charge e), all moving with constant velocity $\vec{V}$. n is the number of protons per unit volume in the beam.

What is the electric current carried by the beam?



in Δt: $nAv\Delta t$ protons cross A, each with charge e

$\Rightarrow \Delta Q = e \cdot (nAv\Delta t)$ in time interval Δt

$\Rightarrow \frac{\Delta Q}{\Delta t} = \boxed{i = nevA}$ $\Rightarrow$ average current density $\boxed{= J = \frac{i}{A_\perp} = nev}$

A. 0 B. *nevA* C. *nev* D. *evA*

Conclusion:

- magnitude of current density in conductor:

$$\boxed{|\vec{J}| = \frac{i}{A_\perp} = n \; q \; v_{drift}}$$

number of charge carriers per volume

electric charge per charge carrier

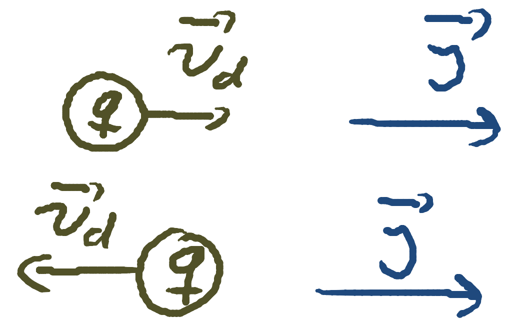
"drift speed" of charge carriers along conductor

- Define current density vector:

$$\boxed{\vec{J} = n\, q\, \vec{v}_{drift}}$$

$q > 0$

$q < 0$



points in direction of "current arrow" $\xrightarrow[i]{}$

Electric Currents in Metals:

no field applied

area A

few 10 nm

microscopic view

$\Rightarrow$ electrons move randomly

- some of the electrons are the mobile charge carriers: $q_{electron} = -e$
- How many mobile (free) electrons are there?

Avogadros number

typically: 1 to 2 per atom

density

$$\Rightarrow n = \frac{\#\text{ of charge carriers}}{\text{volume}} = (1\ldots2)\cdot\frac{N_A\cdot\rho}{\text{atomic mass}}$$

$\approx 10^{29}$ free e^-/m^3

$\underbrace{\text{atomic mass}}_{A\ (g/mole)}$

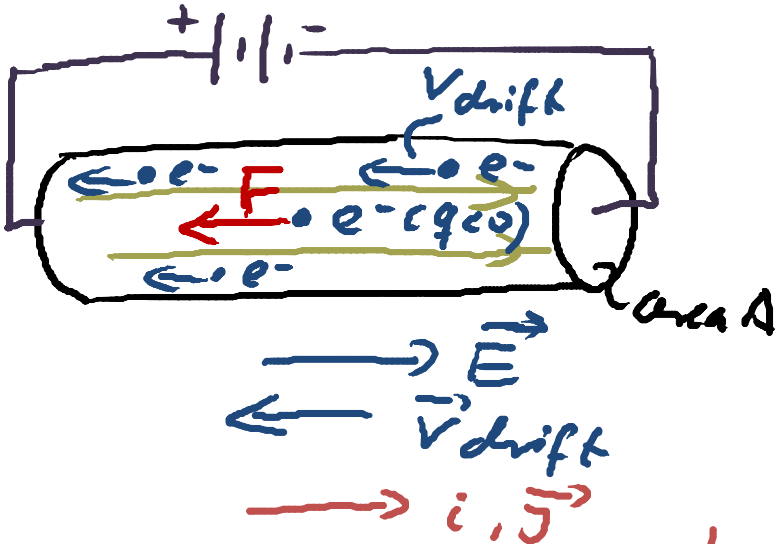
__why?__ random motion speed comes from non-zero energy ("Fermi energy") of free electrons in metal $\Rightarrow V_{random} = \sqrt{\frac{2E_{fermi}}{m}} \approx 10^6 \frac{m}{s}$

(huge!)

__But:__ electrons collide constantly with each other and with atoms in metal (10^{13} to 10^{14} times/sec!) $\Rightarrow$ random motion

$\Rightarrow \vec{V}_{average} = 0$

How to get a current? $\Rightarrow$ Apply electric field!



} <u>Note</u>: battery maintains potential difference
$\Rightarrow$ electric field along wire
$\Rightarrow$ <u>not</u> in electrostatic equilibrium!

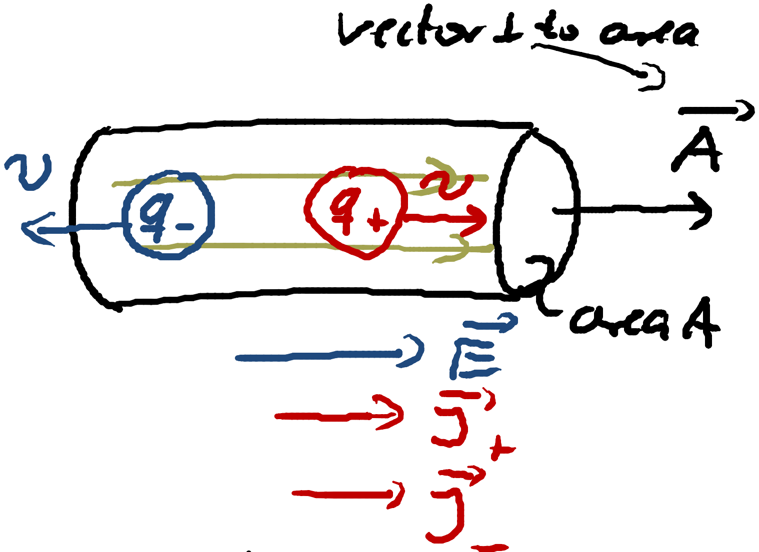
$\left(\text{applied electric field } \vec{E}\right) \Rightarrow$ (electrons drift with average drift speed v_{drift} in direction opposite to $\vec{E}$, since $q_{electron} < 0$ (in addition to fast, random motion))

$\left(\vec{F}_{el} \text{ on electron accelerates them}\right) \Rightarrow$ (but constantly collide and loose energy) $\Rightarrow$ (average, constant drift speed $v_{drift} \approx 10^{-5} \ldots 10^{-3}$ m/s (very slow!!))

$\Rightarrow$ <u>Current density in metal</u>:

$\vec{J} = n(-e)\,\vec{v}_{drift}$ } points in direction of $\vec{E}$!

General case: current density $J = i/A_\perp$ if both + and − charge carriers can move



$$\boxed{\vec{J} = \underbrace{n_+ q_+ \vec{v}_{drift,q_+}}_{\vec{J}_+} + \underbrace{n_- q_- \vec{v}_{drift,q_-}}_{\vec{J}_-}}$$

with $q_+ > 0$ and $q_- < 0$

⇒ total current through area A:

$$i = \int_{\text{area } A} \vec{J} \cdot d\vec{A} = \vec{J} \cdot \vec{A} = J A_{\perp(\text{to } \vec{J})} = n_+ q_+ |v_{drift,q_+}| A_\perp + n_- |q_-| |v_{drift,q_-}| A_\perp$$

if current is uniform across the surface

n: number of + or − charge carriers / volume