Recap

- **Resistor in Series and Parallel**

\[
\frac{1}{R_{\text{eff, parallel}}} = \sum_{i=1}^{N} \frac{1}{R_i}
\]

- **Emf device**:

\[
\text{emf} = \mathcal{E} = \frac{dW}{dt} = V_+ - V_- = \int \mathcal{E} \, dt = \text{volt}
\]

Power delivered by emf device = \( P_{\text{emf}} = i \mathcal{E} \)

Power "used" by circuit device = \( P_{\text{device}} = i \Delta V \) over device

- **Kirchhoff's rules**:

  - for closed loop: \( \sum_{l=1}^{N} \Delta V_e = 0 \)
  - for junction: \( \sum i_{\text{in}} = \sum i_{\text{out}} \)
Today:

- More on RC circuits
- Magnets and magnetic field
RC circuit: Charging

At time $t = 0$ the switch is moved to position $a$.

Use loop rule: $\mathcal{E} + \Delta V_R + \Delta V_C = 0$

$$= \mathcal{E} - Ri - \frac{q}{C} = 0$$

Use: $i = \frac{dq}{dt}$

$$= \mathcal{E} - R \frac{dq}{dt} - \frac{q}{C} = 0$$

"differential equation" for charging

At $t = 0$: $q(t=0) = 0$

"initial condition"

Solution: $q(t) = \mathcal{E} \left[ 1 - e^{-t/RC} \right]$ exponential form

Check: $q(t=0) = 0 \quad \frac{dq}{dt} = \frac{\mathcal{E}}{R} e^{-t/RC} \quad e = 2.718...$
Charging of a Capacitor:

Charge: \[ q_C(t) = CV \left[1 - e^{-t/\tau}\right] \]

with time constant \[ \tau = RC \]

of exponential charging

recall: \[ e^{-1} = 0.37 = 37\% \]
\[ \Rightarrow (1 - e^{-1}) = 0.63 = 63\% \]

Current during charging

\[ i(t) = \frac{dq}{dt} = \frac{V}{R} e^{-t/\tau} \]

at \( t = 0 \): \[ i(0) = \frac{V}{R} \]

Potential change across capacitor

\[ \Delta V_C = \frac{q}{C} = V \left[1 - e^{-t/\tau}\right] \]

at \( t = 0 \): \[ \Delta V_C(0) = V \]

Graph showing charge over time with a maximum charge of \( CV \). The current is shown as a function of time with an initial current of \( \frac{V}{R} \). The potential change across the capacitor is also shown, starting at \( V \) and decaying exponentially over time.
The switch has been at position $a$ for a very long time.

At time $t = 0$ move the switch to position $b$.

Current $i$ begins to flow to discharge the capacitor.
At time $t = 0$ the switch is moved to position $b$. 

**RC circuit: Discharging**

\[ \Delta V_c = \frac{q}{C} \]

\[ \Delta V_c = \frac{q_{\text{final}}}{C} = \frac{q(t)}{C} \]

\[ q_{\text{initial}} = q(t=0) = q_0 \]

**Solution:**

\[ q(t) = q_0 e^{-t/RC} \]

**Check:**

\[ q(t=0) = q_0 \quad \text{and} \quad \frac{dq}{dt} = -\frac{q_0}{RC} e^{-t/RC} \]
Discharging of a capacitor:

Charge:

\[ q(t) = q_0 e^{-t/\tau} \]

With time constant \( \tau = RC \) of exponential decay

Current during discharging:

\[ i(t) = -\frac{dq}{dt} = \frac{q_0}{RC} e^{-t/\tau} \]

\[ i(t=0) = \frac{q_0}{RC} \]

Potential change across capacitor:

\[ \Delta V_c = \frac{q}{C} = \frac{q_0}{C} e^{-t/\tau} \]

\(~\text{exp. decay}~\)

~ exp. decay

\(~\text{exp. decay}~\)
What is the approximate value of the time constant $\tau$ for this decay of electric current from a discharging capacitor in a simple $RC$ circuit?

$\tau = \ ?$

A. $\sim 25 \text{ s}$
B. $\sim 35 \text{ s}$
C. $\sim 50 \text{ s}$
D. $\sim 100 \text{ s}$
E. $\sim 250 \text{ s}$

\[ i(t) = \frac{i_0}{q_0/RC} e^{-t/\tau} \]

\[ i(t=\tau) = i_0 e^{-1} = 120 \text{mA} \cdot 0.37 \]

\[ = 44 \text{mA} \]

\[ \Rightarrow \tau = 50 \text{s} \]
Approximately, what was the discharging capacitor’s initial charge at time $t = 0$?

$q_o = ?$

A. 1.2 C  
B. 3.0 C  
C. 6.0 C  
D. 12 C  
E. 18 C

\[
q_o = \frac{q_0}{RC} = \frac{q_0}{C} \Rightarrow q_0 = \dot{q}_0 \cdot C
\]

$$= 120 \times 10^{-3} \times 50 \text{ s} = 6 \text{ C}$$
The graph shows the electric charge on a charging capacitor in a simple $RC$ circuit.

At time $t = 2\tau$, how much charge is on the capacitor?

\[ q(t) = q_{\text{final}} \left( 1 - e^{-t/\tau} \right) \]

\[ q(t = 2\tau) = q_{\text{final}} \left( 1 - e^{-2\tau/\tau} \right) \]

\[ = q_{\text{final}} \left( 1 - e^{-2} \right) \]

\[ = q_{\text{final}} \cdot 0.86 \]

A. $0.14 \, q_f$
B. $0.37 \, q_f$
C. $0.63 \, q_f$
D. $0.79 \, q_f$
E. $0.86 \, q_f$
The Earth’s magnetic field near the surface can be approximated by the field of a bar magnet. In which direction would the magnetic north pole of Earth’s magnet point?

A. To the geographic north pole
B. To a point near the geographic north pole
C. To the geographic south pole
D. To a point near the geographic south pole
Magnetic Fields and Forces

- What produces magnetic fields $\mathbf{B}$?

(a) magnetic charges? (magnetic monopole)

$\text{No! Never have been found (nobody knows why they do not exist...)}$

$\Rightarrow$ no individual "north" or "south" pole, always come in north-south pairs!

(b) Electromagnet:

$\Rightarrow$ electric currents (moving charges) produce a magnetic field around them!

(c) Permanent magnet:

$\Rightarrow$ elementary particles have an intrinsic magnetic field around them $\Rightarrow$ magnetic fields of particles add up in certain materials $\Rightarrow$ net magnetic field around the material
How can we detect a magnetic field $\vec{B}$?

- Recall: for electric fields $\vec{E}$: generates force on test charge $q_e$: $\vec{F} = q_e \vec{E}$

- for magnetic fields $\vec{B}$:
  
  (a) Torque $\tau$ on compass needle (bar magnet)

$\Rightarrow$ In a magnetic field, the torque on a bar magnet tends to align the magnet with the direction of the $\vec{B}$-field!
(b) by the magnetic force $\vec{F}_B$ exerted on a moving electric charge:

\[ q \rightarrow \vec{v} \rightarrow \vec{B} \rightarrow \quad \vec{v} = 0 \quad \Rightarrow \quad \vec{F}_B = 0 \quad \text{change needs to move!} \]

\[ q \rightarrow \vec{v} \rightarrow \vec{B} \rightarrow \quad \vec{v} \rightarrow \vec{B} \rightarrow \quad \vec{F}_B = 0 \quad \text{still no force if charge is moving in direction of } \vec{B} \]

\[ q \rightarrow \vec{v} \rightarrow \vec{B} \rightarrow \quad \vec{v} \rightarrow \vec{B} \rightarrow \quad \Rightarrow \left| \vec{F}_B \right| = 1 q v \perp \mathbf{B} \]

component of velocity $\vec{v} \perp \vec{B}$
Magnetic Force on a moving charge \( q \):

\[
|F_B| = \frac{q |\vec{v}| \times |\vec{B}|}{c} \quad \vec{B} = \frac{\vec{v} \times \vec{B}}{v} \quad B \sin \phi = \frac{|\vec{v}| B \sin \phi}{v}
\]

with \( \phi \) : smallest angle between \( \vec{v} \) and \( \vec{B} \) (\( 0 \leq \phi \leq 180^\circ \))

\[
\text{Unit of } B = \frac{|F|}{q |\vec{v}|} = \frac{N}{C \cdot m} = \frac{N}{A \cdot m} = 1 \text{ tesla} = 1 \text{T}
\]

\[
= 10^4 \text{ gauss}
\]