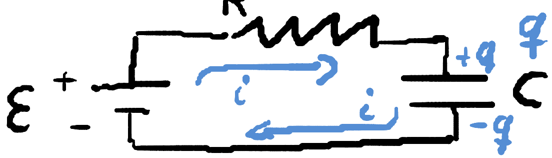
Recap

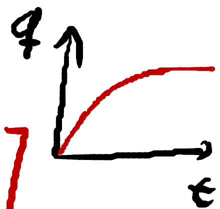
Lecture 14

- RC circuit:

Charging:



$q(t=0) = 0$

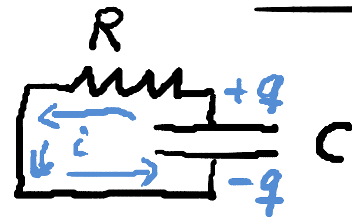


$$q_c(t) = C\varepsilon\left[1 - e^{-t/\tau}\right]$$

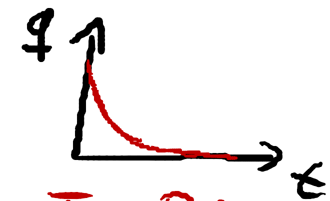
with time constant $\tau = RC$

$$\Rightarrow i(t) = \frac{dq}{dt} = \frac{\varepsilon}{R} e^{-t/\tau}$$

Discharging:



$q(t=0) = q_0$



$$q_c(t) = q_0 e^{-t/\tau}$$

with time constant $\tau = RC$

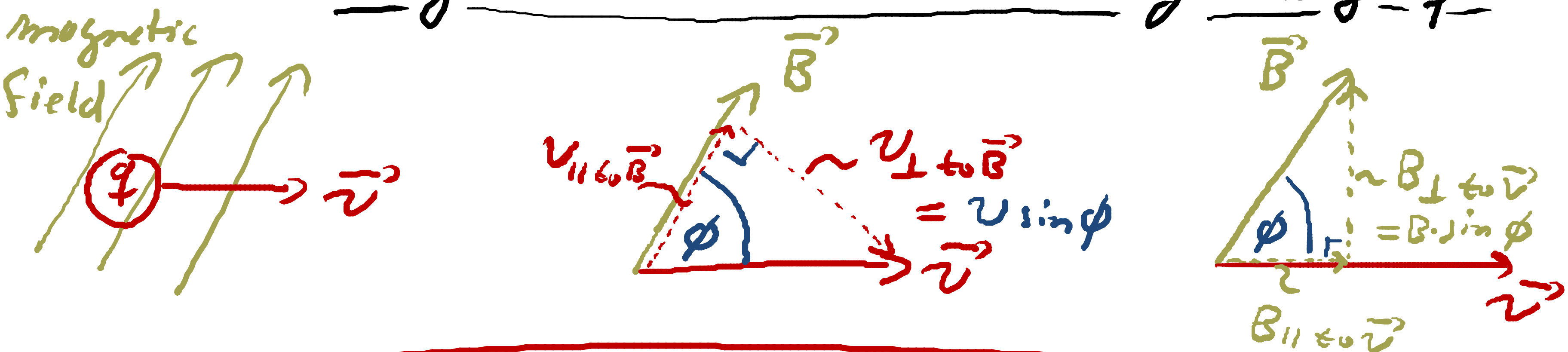
$$\Rightarrow i(t) = -\frac{dq}{dt} = \frac{q_0}{RC} e^{-t/\tau}$$

- Magnetic Fields $\vec{B}$:

  - are produced by electromagnets (i.e. by moving charges) and by permanent magnets.

  No magnetic monopoles (charges)!

⇒ Magnetic Force on a moving charge q:



$$\boxed{|F_B| = |\overset{\text{charge}}{q}|\, v_{\perp \text{ to } \vec{B}}\, B = |q|\, \overset{\text{speed of moving charge}}{v}\, B \sin\phi = |q|\, v\, B_{\perp \text{ to } \vec{v}}}$$

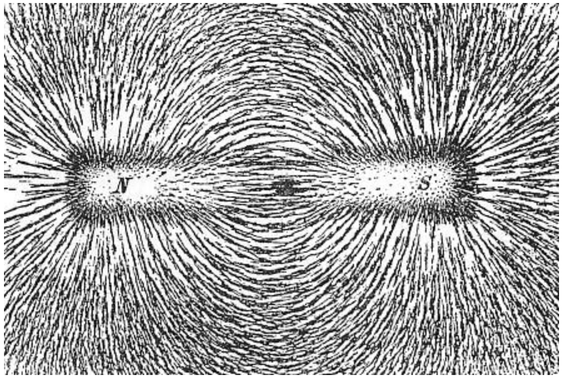
with ϕ: smallest angle between $\vec{v}$ and $\vec{B}$ $(0 \leq \phi \leq 180^\circ)$

⇒ this equation defines the magnetic field B

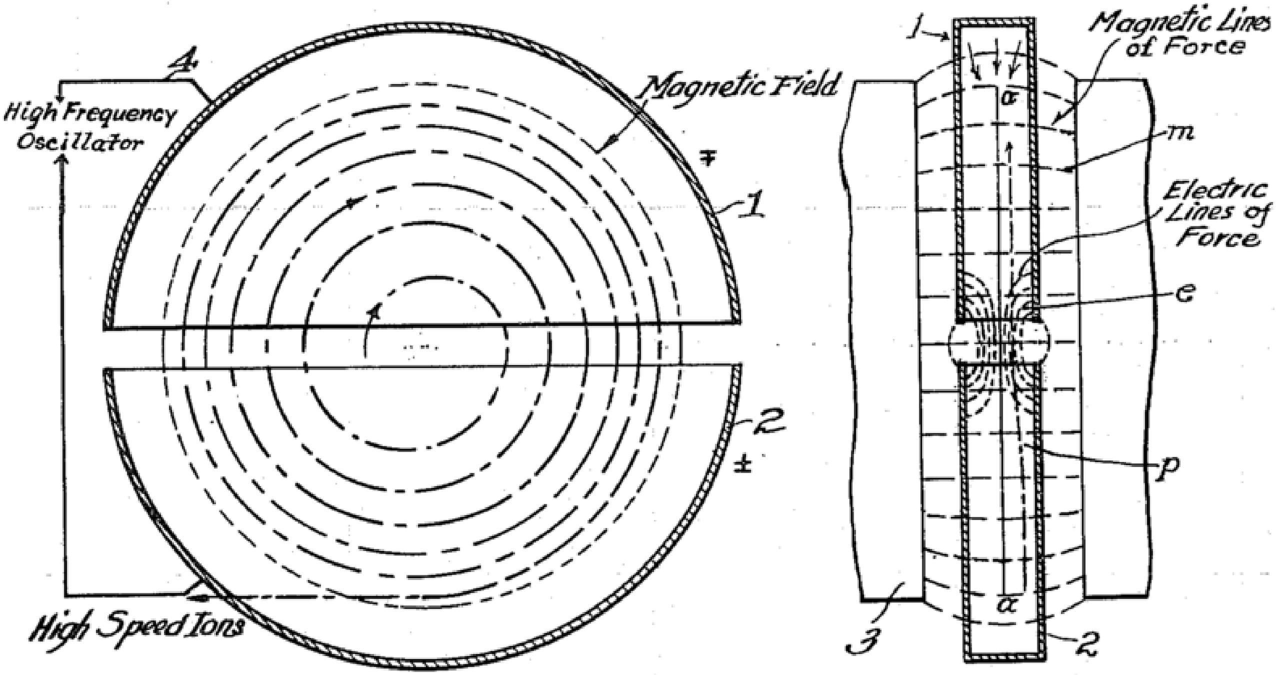
Units: $[B] = \frac{[F]}{[q][v]} = \frac{N}{C\,\frac{m}{s}} = \frac{N}{A\,m} \equiv$ 1 tesla $=$ 1 T

$= 10^4$ gauss

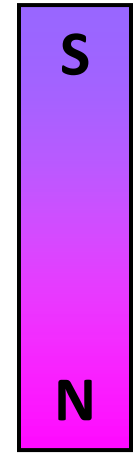
Today:

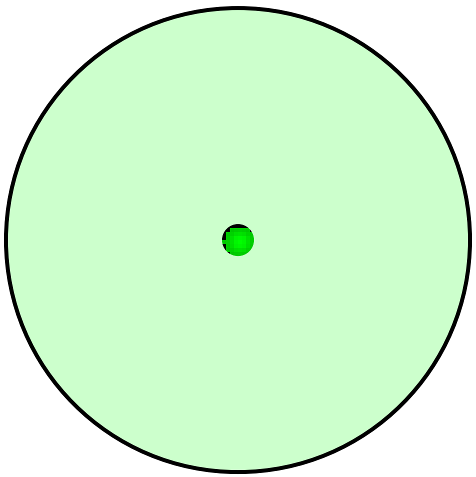


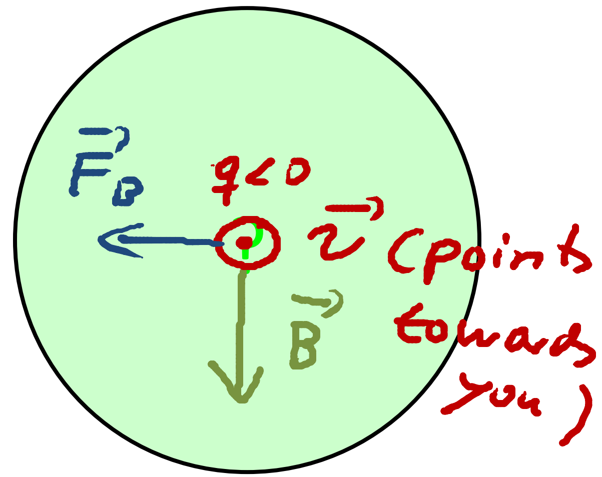
- Magnetic field
- Magnetic field lines
- Charge moving in a uniform B-field
  - Particle accelerators: The cyclotron and synchrotron



A beam of electrons traveling directly towards you produces a bright spot when it hits a CRT screen.







If a magnet with its north pole facing down is brought near the beam from above, which way will the spot on the screen move?

A. ↑ B. ↓ C. ← D. → E. It won't move.

Which way does $\vec{F}_B$ point?

- magnitude of force: $|F_B| = |q|\, v\, B \sin\phi$
- <u>Direction:</u> $\vec{F}_B$ always point perpendicular to the velocity $\vec{v}$ <u>and</u> magnetic field $\vec{B}$, i.e. $\perp$ to plane defined by $\vec{v}$ and $\vec{B}$, in direction shown below:

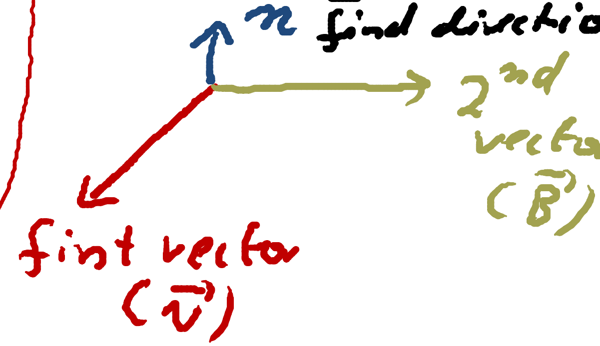
- Mathematical shorthand: <u>Cross-product</u> of 2 vectors

$$\boxed{\vec{F}_B = q\, \vec{v} \times \vec{B} = q\, v B \sin\phi\, \vec{n}}$$

cross product

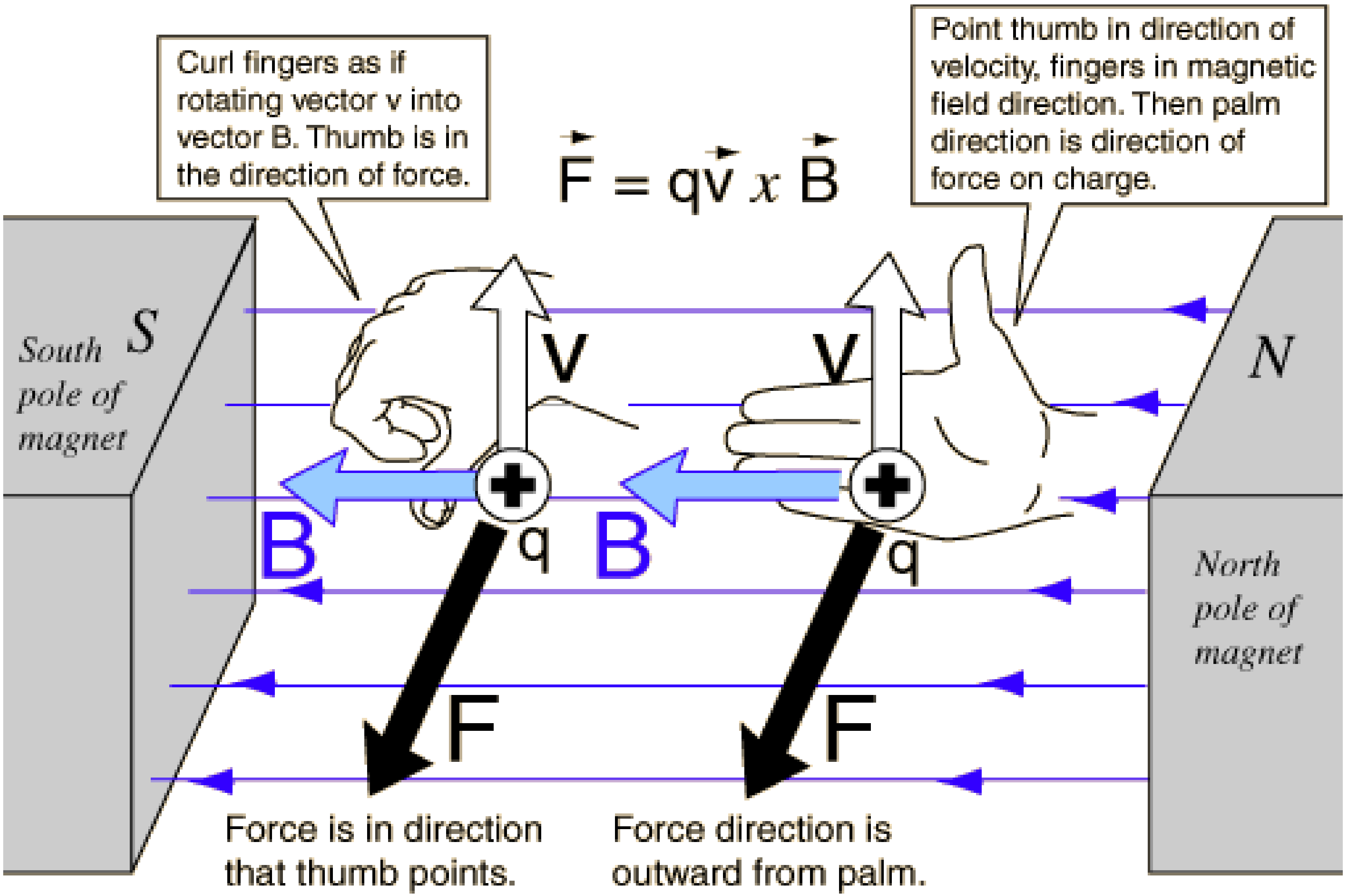
unit vector $|\vec{n}| = 1$

use "right hand rule" to find direction

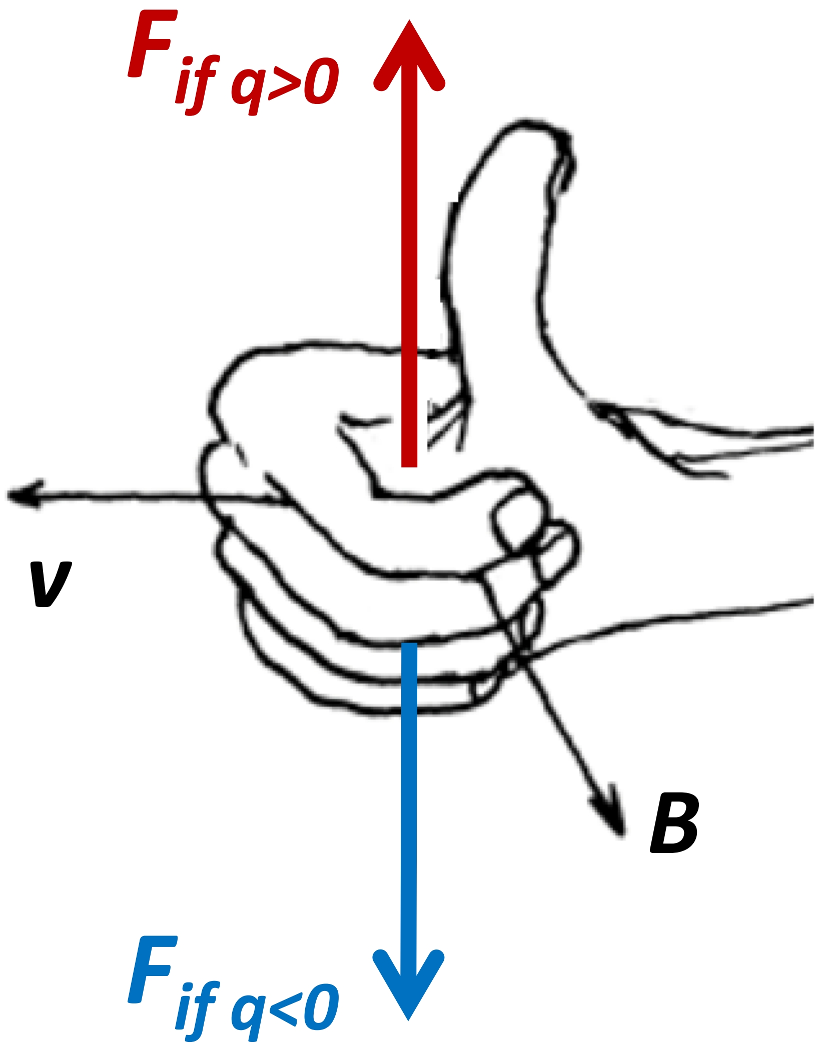


"Right Hand Rule":

Must use your right hand! The figure below shows the force for a positive charge, i.e. q>0!!

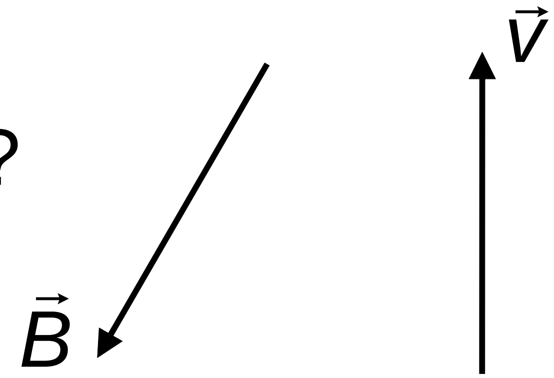


"Right Hand Rule":
Must use your right hand!!!



- *FINGERS* of the right hand point in the direction of the *FIRST* vector (**v**) in the cross product,
- then adjust your wrist so that you can bend your fingers (at the knuckles!) toward the direction of the second vector (**B**);
- extend the thumb. If charge is **positive** the **force is in direction that the thump points**!
- If charge is **negative**, the force is **opposite to direction that the thump points**!

What is the direction of $\vec{v} \times \vec{B}$?



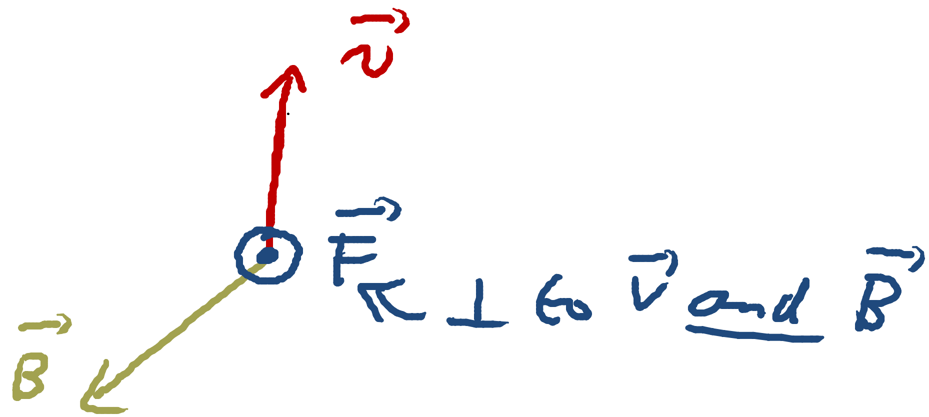
A. ←

B. →

C. ⊙(out of)

D. ⊗ (into)

E. Other



What is the direction of the magnetic force on the negative charge?

A. ←

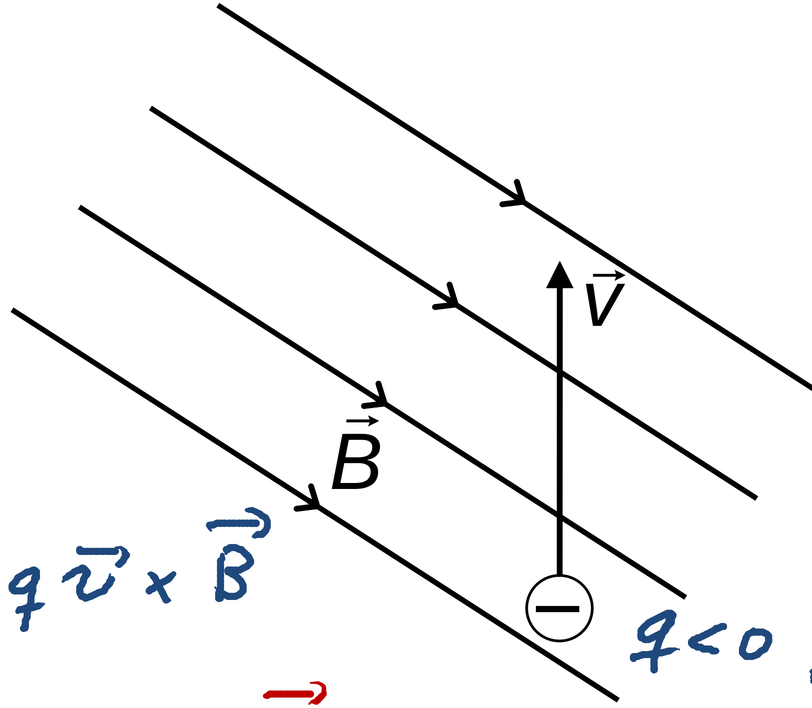
B. →

C. ⊙(out of)

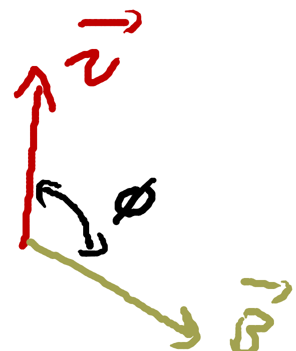
D. ⊗ (into)

E. Other

$\vec{F} = q\vec{v} \times \vec{B}$



$q<0$!

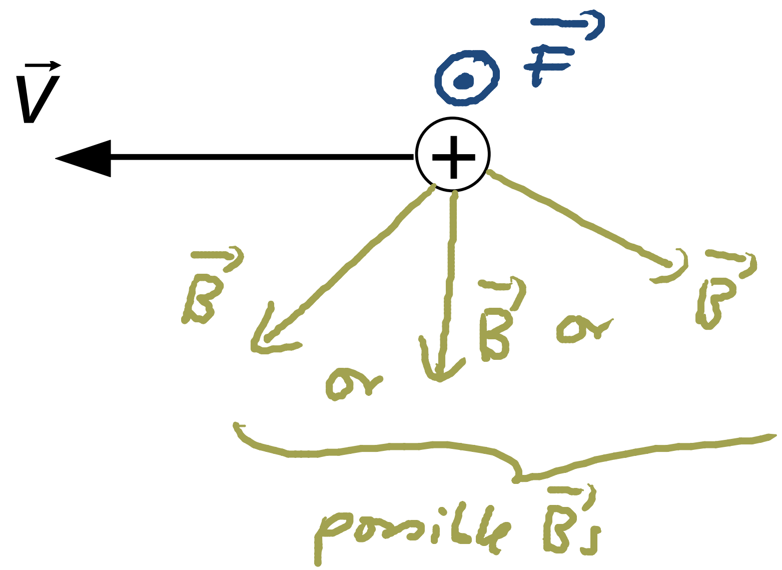


$\vec{v} \times \vec{B}$ points into page!

$\Rightarrow \underbrace{q}_{<0}\, \vec{v} \times \vec{B}$ points out of page

The magnetic force on the positive charge is directed out of the picture (⊙). What is the direction of the magnetic field?

- ~~A.~~ ←
- ~~B.~~ →
- ~~C.~~ ⊙(out of)
- ~~D.~~ ⊗ (into)
- E. Can't tell for sure



Magnetic Field Line Model

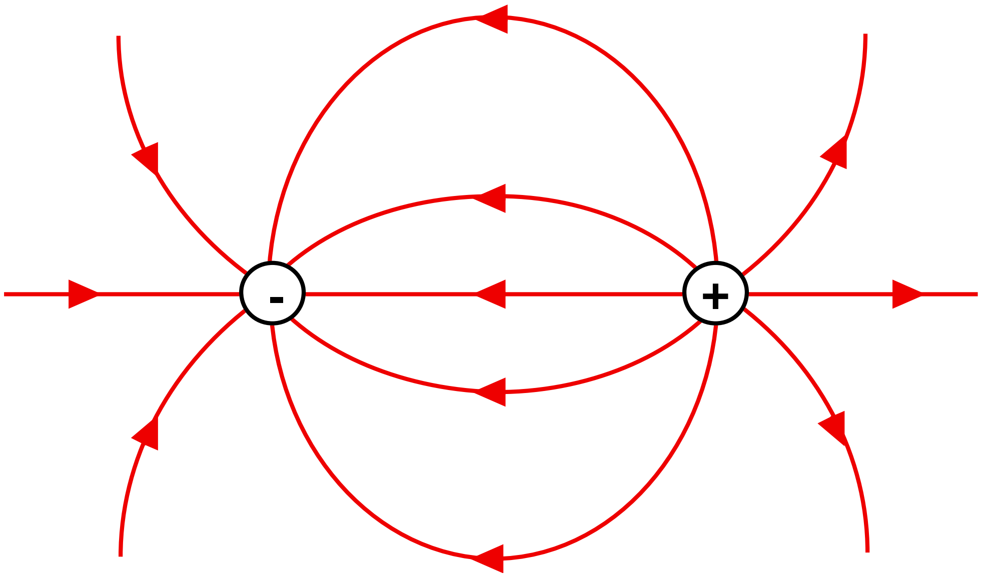
A way of visually representing a magnetic field (lines are not real!).

1. Magnetic field lines **point in the direction of the (total) magnetic field** at each point in space.
2. Magnetic field lines **cannot cross**.
3. The **strength (magnitude) of the magnetic field at any place is proportional to the density of field lines** there, i.e.,

$$B \propto \frac{(\#\ \text{of field lines})}{(\text{area} \perp \text{lines})}$$

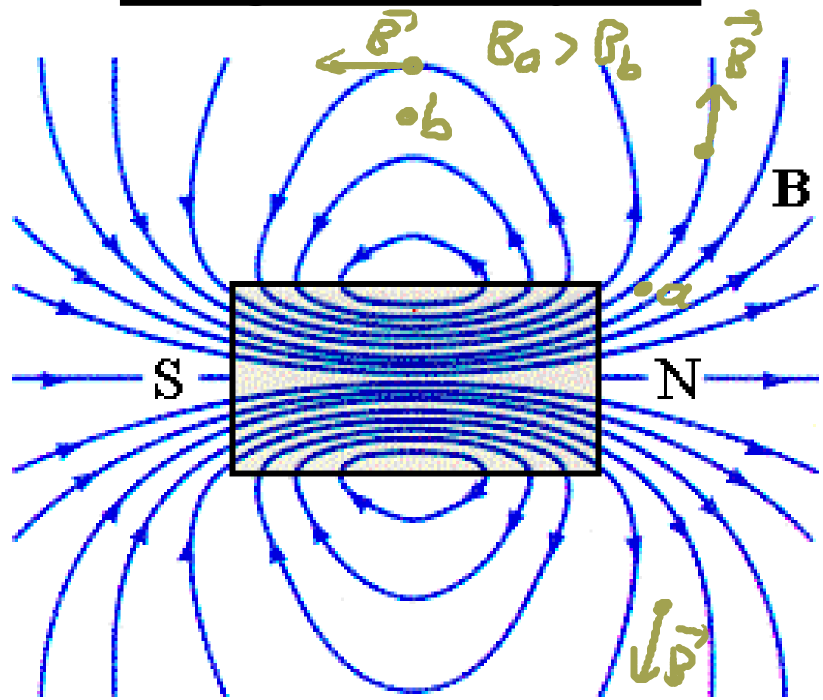
4. **Magnetic field lines never start nor end.** They always form closed loops. This means that there are no isolated magnetic "charges" (monopoles). Magnetic "poles" always occur in N-S pairs.

Electric Dipole



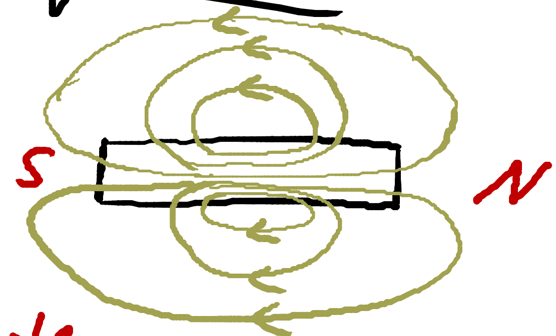
Electric field lines go from positive to negative electric charge.

Magnetic Dipole



Magnetic field lines never start nor end. They always form closed loops.

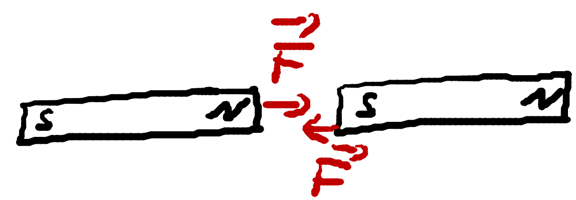
For Permanent Magnets:



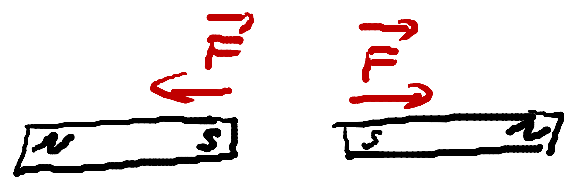
- "field lines" emerge from one end: North pole
- "field lines" enter other end of magnet: South pole

$\Rightarrow$ Magnet has two poles $\Rightarrow$ magnetic dipole

- for multiple magnets

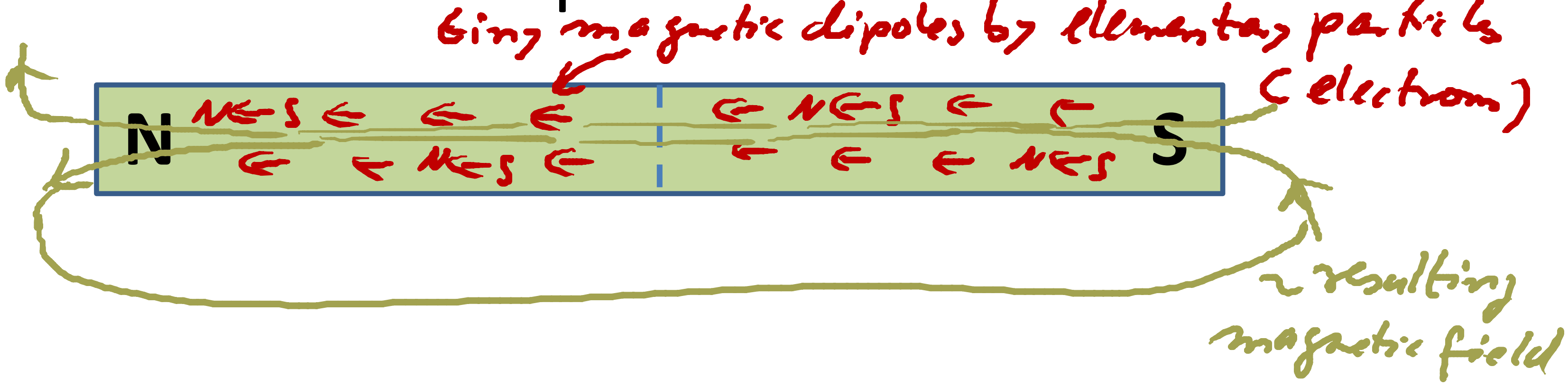


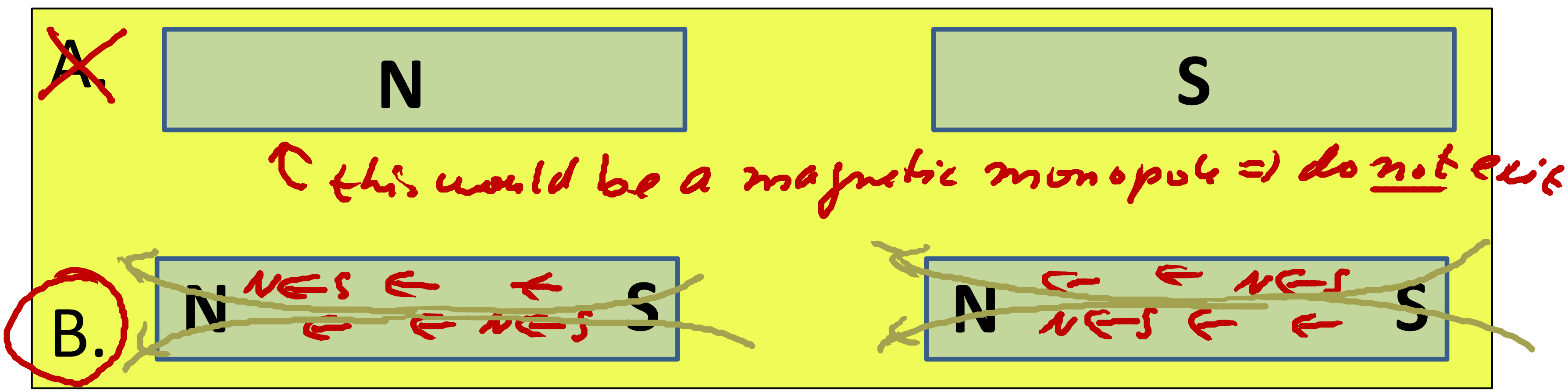
- Opposite magnetic poles (N and S) attract each other



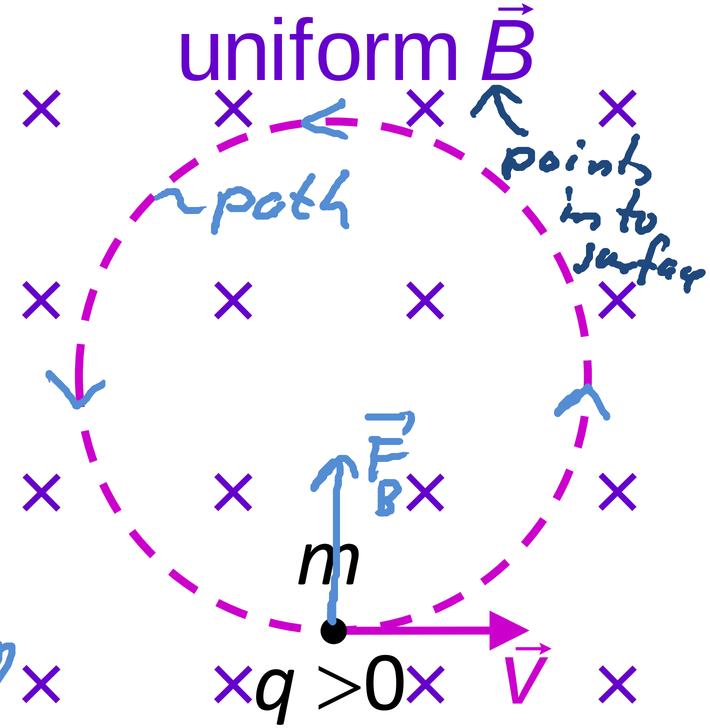
- Like magnetic poles (N and N or S and S) repel each other

What do you get if you cut a bar magnet right in the middle into two parts?





A particle of mass m and charge $q > 0$ is moving with speed $v \perp$ to a **uniform magnetic field** B.



The particle follows a circular path in the field. $\vec{F}_B \perp \vec{v}$ always!

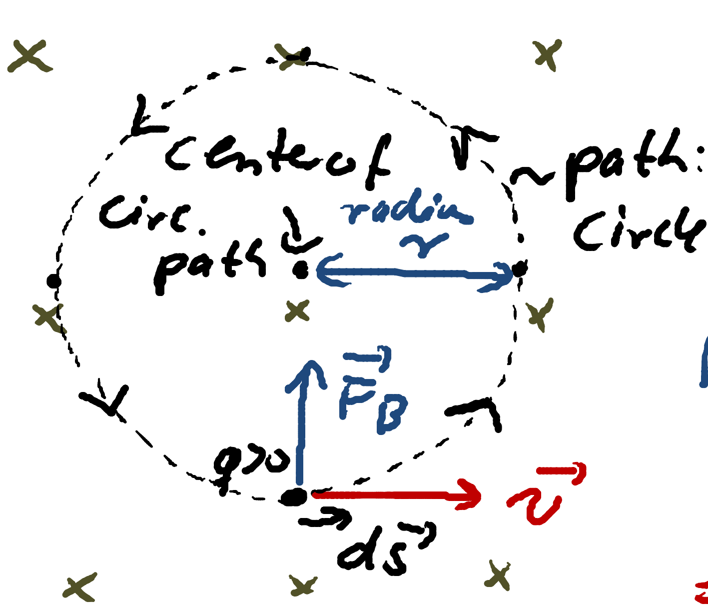
How is the **radius *r*** of the particle's path related to its speed v? $\sum F_{on\,q} = F_B = ma = \underline{qvB} = m\frac{v^2}{r} \Rightarrow r = \frac{mv}{qB} \propto v$

$a = v^2/r$ for circ. motion

A. $r \propto v$ ~~B.~~ $r \propto v^2$ C. $r \propto v^{-1}$ D. $r \propto v^{1/2}$

E. r does not depend on v

Circulating Charged Particles



uniform $\vec{B}$-field;
$\vec{B}$ points into surface

$\vec{F}_B$ is $\perp$ to $\vec{v}$ and $\vec{B}$ always

$\Rightarrow$ charge is moving on circular path here

$|F_B| = m|a| \Rightarrow |q|vB = m\frac{v^2}{r}$

$a = \frac{v^2}{r}$ for circ. motion

$\Rightarrow$ radius of circular path:

$$\boxed{r = \frac{mv}{|q|B} = \frac{\text{momentum } p}{|q|\,B}}$$

<u>Note:</u> $\vec{F}_B \perp \vec{v}$ always! $\Rightarrow \vec{F}_B \perp$ to path always!

$dW = \vec{F}_B \cdot d\vec{s}$
$= F_B\, ds \cos 90°$
$= \underline{\underline{0}}$

$\Rightarrow$ **Magnetic force $\vec{F}_B$ <u>never</u> does any work on moving charge!**

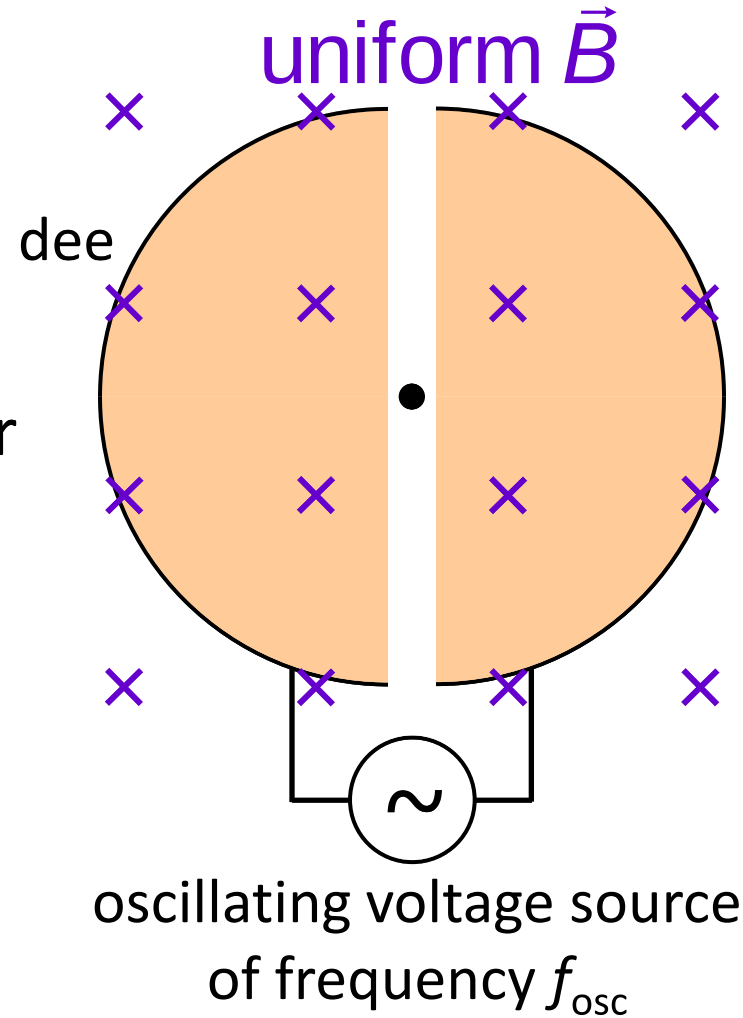
$\Rightarrow$ here: $\Delta K = 0$

$\Rightarrow |v| = \text{const}$

Application: Particle Accelerators

The cyclotron:

- **Fixed magnetic field; changing orbit radius**
- composed of two hollow copper dees that are immersed in a uniform magnetic field & connected to an oscillating voltage source.
- Particles (e.g., protons), each of charge *q* & mass *m*, start at a source near the center of the dees.



The Cornell Cyclotron

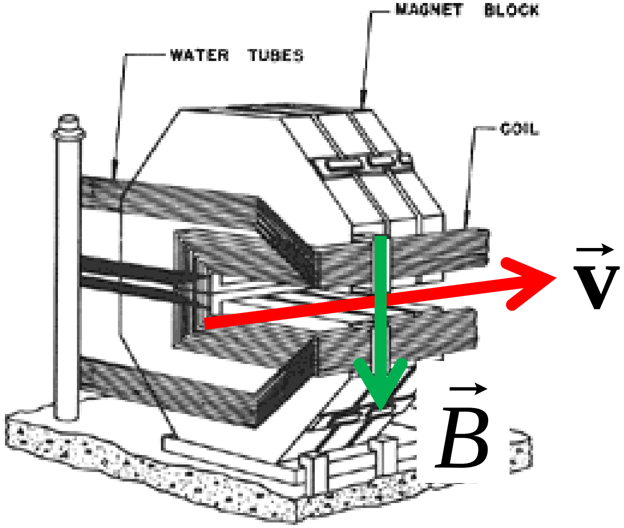
The Cornell cyclotron (2 MeV protons) was built about 1935 and decommissioned in 1956.

This photo with Assistant Professor Boyce D. McDaniel was taken in 1955.

Application: Particle Accelerators

The synchrotron:

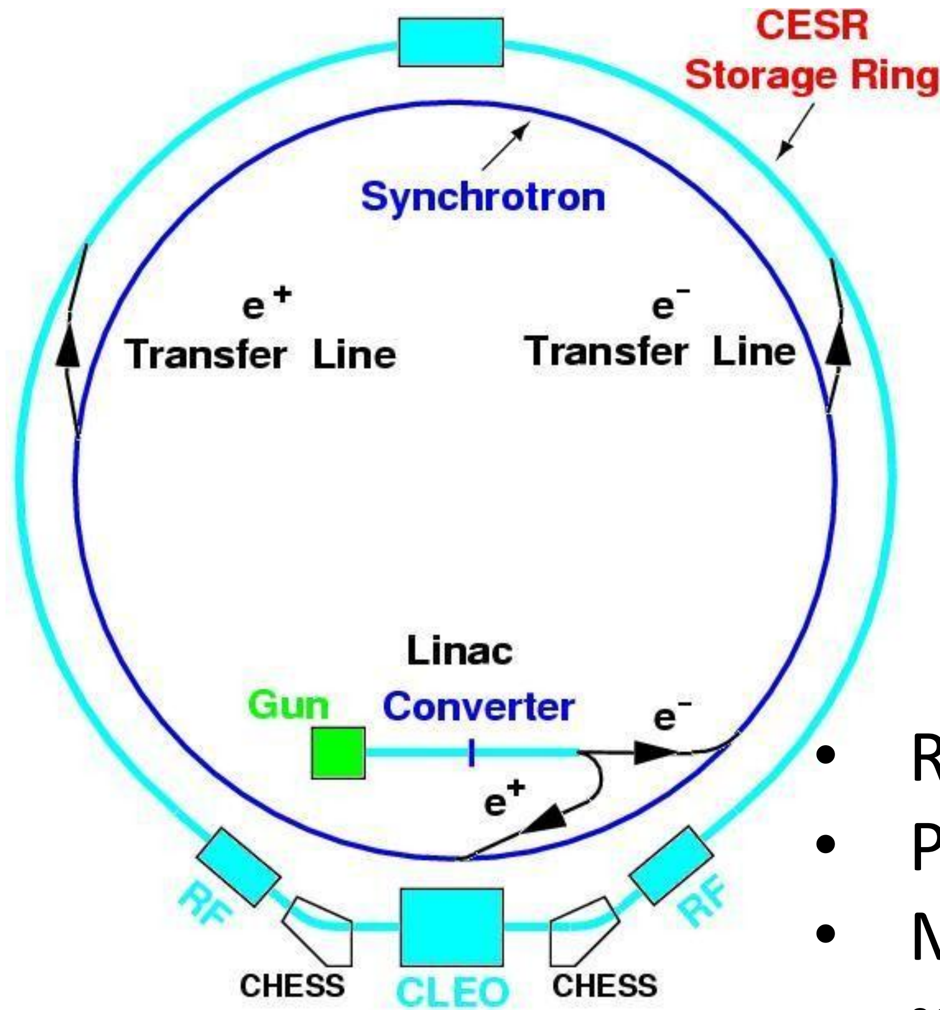
- **Fixed orbit radius; magnetic field adjusted for particle momentum/energy**



- "Dipole magnets" keep particles on fixed orbit.

$$Radius = \frac{p(t)}{qB(t)} = \text{const.}$$

The Cornell Synchrotron



- Radius = 122 m
- Particle energy: up to 5 GeV
- Magnetic bending fields: up to ~0.2 T (~3000* Earth's magnetic field)