Recap

- **Crossed Electric and Magnetic Fields:**
  - **Velocity selector:**
    - Charge moves along straight line, if \( F_B = F_E \) \( \Rightarrow \) \( \lim_{\text{straight}} \frac{E}{B} = \frac{\lim_{\text{straight}} E}{B} \)
  - **Hall effect:**
    - Production of a potential difference \( \Delta V_H \) across an electric conductor by a magnetic field \( \perp \) to the current direction
    \[ V_{\text{drift}} = \frac{E \mu_i}{B} = \frac{\Delta V_H}{B d} \]
    \[ n = \frac{i B d}{e \Delta V_H A} \]

- **Magnetic force on a current-carrying wire:**
  - Angle between \( \vec{L} \) and \( \vec{B} \)
  - \( \vec{F} \) by \( \vec{B} \)-field on wire of length \( l \)
    \[ \vec{F} = l \vec{L} \times \vec{B} \quad \Rightarrow \quad |F| = l L B \sin \phi \]
    - Length vector points in direction of current.
Today:

- Magnetic force on a current carrying wire
  - Torque on a current loop
- Magnetic field due to a current
  - Field due to a circular arc
  - Field due to a straight wire
Current loop in a uniform magnetic field:

Side view:

What is the direction of the net magnetic force on the current loop?

A. ⊗ (out of)  B. ⊘ (into)  C. ↑  D. ↓  E. The net magnetic force on the loop is zero.
Current loop in a uniform magnetic field:

Top view:

\[ \vec{F}_i = i \vec{L} \times \vec{B} \]

\[ \vec{F}_3 = i \vec{L}_3 \times \vec{B} \]

\[ \vec{F}_1, \vec{F}_2 = i \vec{L}, \vec{B} \]

\[ \text{net torque:} \]

\[ \vec{L}_1 + \vec{L}_2 + \vec{L}_3 = \frac{\vec{L}_2}{2} \vec{F}_1 + \frac{\vec{L}_3}{2} \vec{F}_3 \]

\[ \Rightarrow \vec{L}_\text{net} = \vec{L}_1 \vec{L}_2 i \vec{B} = A i \vec{B} \]

\[ A = L_1 \cdot L_2 = \text{area enclosed by the loop} \]
For general orientation of the loop relative to $\vec{B}$:

Top view:

Unit vector normal to current loop

Angle between $\hat{n}$ and $\vec{B}$

$F_1 = |F_1| = i L_1 B$

$F_3 = |F_3| = i L_3 B$

$\vec{F} = \frac{L}{2} \sin \theta F_1 + \frac{L}{2} \sin \theta F_3$

$|\vec{F}| = L \sin \theta i L_1 B = A \cdot B \sin \theta$

Note: in this case, there are also forces on wire sections #2 and #4, but they create no net force and no net torque!
Conclusion: Torque on current loop:

Side view:

\[ \mathbf{\tau} = A \mathbf{i} \times \mathbf{B} \sin \theta \]

Area enclosed by the loop \( \mathcal{A} \)

Angle between \( \mathbf{B} \) and unit vector \( \mathbf{n} \) normal to the loop \( \mathbf{n} \)

\[
\begin{align*}
\mathcal{A} & \text{ is the area enclosed by the loop.} \\
\mathbf{B} & \text{ is the magnetic field.} \\
\mathbf{n} & \text{ is a unit vector normal to the loop.} 
\end{align*}
\]

\( \mathcal{A} = \pi \mathcal{R}^2 \)

\( \mathbf{n} = \mathbf{\hat{z}} \)

\( \mathbf{n} \times \mathbf{i} \)

\( \mathbf{\tau} = (A \mathcal{R} i) \mathbf{B} \sin \theta = \mu \mathbf{B} \sin \theta \)

\( \mu = A \mathcal{R} i \) of coil.

Top view:

\( \mathcal{A} \equiv \mathbf{\hat{z}} \times \mathbf{i} \)

\( \mathbf{n} \equiv \mathbf{\hat{z}} \)

\( \mathbf{n} \times \mathbf{i} \)

\( \mathbf{\tau} = (A \mathcal{R} i) \mathbf{B} \sin \theta = \mu \mathbf{B} \sin \theta \)

\( \mu = A \mathcal{R} i \) of coil.

\( \Rightarrow \text{This equation is valid for all flat current loops, no matter what the shape; e.g. } \mathcal{A} \mathcal{R} \)

\( \Rightarrow \text{for coil with } N \text{ loops, or turns: } \)

\( \mathbf{\tau} = (A \mathcal{R} i N) \mathbf{B} \sin \theta = \mu \mathbf{B} \sin \theta \)

with "magnetic dipole moment" \( \mu = A \mathcal{R} i \) of coil.
Electric Motor: How it works

An electric current in a magnetic field will experience a force.

The pair of forces creates a turning influence or torque to rotate the coil.

If the current-carrying wire is bent into a loop, then the two sides of the loop which are at right angles to the magnetic field will experience forces in opposite directions.

Practical motors have several loops on an armature to provide a more uniform torque and the magnetic field is produced by an electromagnet arrangement called the field coils.
Electric Motor: How it works

When electric current passes through a coil in a magnetic field, the magnetic force produces a torque which turns the DC motor.

Electric current supplied externally through a commutator.

Magnetic force acts perpendicular to both wire and magnetic field.

\[ F = ILB \]
Electric Motor: How it works
Magnetic Fields due to Currents:

\( \vec{F}_B = q \vec{v} \times \vec{B} \)

\( B \) produced by moving charge

\( I \times B \propto |q| \times (\text{speed} \, \nu) \)

Use Right-Hand Rule to find direction of magnetic field around moving charge.
Right-hand rule: Point the thumb of your right hand in direction of the current. The fingers then reveal the B-field vector's direction.
Magnetic Fields due to a Current:

~ Break current path into small sections of length $ds$

~ Define length vector: $ds$; point in direction of current

~ Each section produces some magnetic field $dB_p$ at point $P$

~ Total field at point $P$ is sum of contributions $dB_p$ from all sections of wire:

$$\vec{B}_p = \sum dB_p = \int dB_p$$

all contributions along current path from all wire sections
What is the magnetic field $dB_p$ at point $P$ produced by the current in a very short section $ds$ of the current path?

From above:

$$dB_p \propto dQ_{in} \cdot \vec{v} = i \, dt \, v = i \, ds$$

$$dB_p = \frac{\mu_0}{4\pi} \frac{i \, ds}{r^2} \sin \theta$$

$\mu_0$: permeability constant

$\mu_0 = 4\pi \cdot 10^{-7}$ T m/A

$r$: distance from current path section to point $P$

$\theta$: angle between $ds$ and $\vec{r}$

Law of Biot and Savart

$$dB_p = \frac{\mu_0}{4\pi} \frac{i \, ds \times \hat{r}}{r^2}$$

with $\hat{r} = \frac{\vec{r}}{r}$

Unit vector; points from current path element $ds$ to point $P$.

Note: $dB_p$ points perpendicular to $ds$ and $\vec{r}$ always perpendicular.
Consider a current carrying circular wire loop:

What is the direction of the magnetic field at the center of the loop due to the current at $ds$?

A. $\uparrow$  
B. $\downarrow$  
C. $\leftarrow$  
D. $\bigcirc$ (out of)  
E. $\otimes$ (into)