Recap

- Magnetic field at center of circular arc of wire:
  \[ B_p = \frac{\mu_0 i}{2\pi R} \phi \]
  \( \phi \) = angle in rad!

- Magnetic field by an infinite straight wire:
  \[ B_p = \frac{\mu_0 i}{2\pi R} \]

- Forces between two parallel wires:
  \[ |F_{1 \text{ on } 2}| = |F_{2 \text{ on } 1}| = |i_1 L^2 \times \vec{B}^'| \]
  \[ = \frac{\mu_0 L i_1 i_2}{2\pi d} \]

- Parallel currents attract each other
- Anti-parallel currents repel each other

Arc length = \( \phi R \)
Today:

- Ampere’s law
- Applications of Ampere’s law
  - Straight wire
  - Solenoid

\[ \sum B_\parallel \Delta l = \mu_0 I \]
Next: Ampère’s Law:

1st: Need to define circulation $\Gamma$ of a $\vec{B}$-field:

\[ \Gamma = \oint \vec{B} \cdot d\vec{s} \]

\( \sim \) consider some imaginary closed path in a given magnetic field

\( \sim \) Then "walk" along the closed path and integrate over (sum up) the magnetic field component $B_{||}$ pointing along the direction of the path, for one full turn.

Break path into small path length elements $d\vec{s}_i$, with $\vec{B}_i \cdot d\vec{s}_i$ count over given path section.
Define:

\[ \Gamma = \sum_i B_{\| \text{to path}} \cdot ds_i = \sum_i B_i \cos \Theta_i \cdot ds_i \]

angle between \( \vec{B}_i \) and \( ds_i \)

\[ = \sum_i \vec{B}_i \cdot ds_i \]

\[ \Rightarrow \text{get integral around closed path} \]

\[ \Gamma = \oint \vec{B} \cdot ds = \oint \vec{B} \cdot \cos \Theta ds = \oint B_{\|} ds \]

"closed path integral"

Component \( \parallel \) to path

\[ S = \text{area "inside" contour} \]
Ex.: Calculate $\Gamma$ for a circular path centered around a long straight wire:

<table>
<thead>
<tr>
<th>What is the component of $\vec{B}$ along the direction of $d\vec{s}$?</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. $B_s = \mu_0 i/(2\pi R)$.</td>
</tr>
<tr>
<td>B. $B_s = -\mu_0 i/(2\pi R)$.</td>
</tr>
<tr>
<td>C. 0.</td>
</tr>
<tr>
<td>D. It depends on where $d\vec{s}$ is along the path.</td>
</tr>
<tr>
<td>E. Not enough information.</td>
</tr>
</tbody>
</table>
Ex.: Calculate $\Gamma$ for a circular path centered around a long straight wire:

$$\Gamma = \oint \vec{B} \cdot d\vec{s} = \oint B_{||} \text{topath} \, ds$$

$$= \oint \frac{\mu_0 i}{2\pi R} \, ds = \frac{\mu_0 i}{2\pi R} \oint ds$$

$$= \Gamma = \left( \frac{\mu_0 i}{2\pi R} \right) (2\pi R) = \mu_0 i \text{ here}$$

$\Gamma = \mu_0 i \text{ turns out to be true for any given magnetic field and any closed path!}$

$\Rightarrow$ Ampère's Law!
Ampere’s law:

\[ \oint \mathbf{B} \cdot d\mathbf{s} = \int B \cos \theta ds = \int B_{\parallel} ds = \mu_0 i_{\text{enc, net}} \]

where \( i_{\text{enc, net}} \) is the net current enclosed by the closed path of integration and \( \theta \) is the angle between \( \mathbf{B} \) and \( ds \).

Use a right-hand rule to assign + or – signs to enclosed currents.
• “current enclosed by the closed path”:
  • current must pierce through imaginary surface that is completely bounded by the closed integration path

• right-hand rule to find sign of current:
  • Curl fingers of your right hand along the direction of the closed integration path. Then a positive current will run in the general direction of your thumb, while a current which runs in the opposite direction is negative.
Applications of Ampere’s law:

In certain cases, Ampere’s law can be used together with symmetry arguments to find an unknown magnetic field.

- Magnetic field by a long, straight wire
- Magnetic field by a long solenoid
Consider a long, straight Wire:

- \( \vec{B} \) must be cylindically symmetric here \( \Rightarrow \) Could be \( 1 \), \( 2 \), or \( 3 \)
- but also: magnetic field line must be closed loops \( \Rightarrow \) \( 3 \)

Which configuration of magnetic field along the integration path can be correct (use symmetry arguments)?

A. ①  B. ②  C. ③
D. None of the above.
Applications of Ampere’s Law:

**Magnetic Field outside of a Long, straight Wire**

- $\mathbf{B}$ point along integration path:
  - $|B_r| = |B_{\parallel}| = \text{const along path}$
  - $\int B \cdot d\mathbf{s} = B \, ds$

Use Ampere’s Law:

- $\oint B \cdot d\mathbf{s} = \mu_0 i_{enc}$

- $\oint B \, ds = B \oint ds = \mu_0 i$

- $B = \frac{\mu_0 i}{2\pi r}$

For long wire, for $r > R$ (outside of)
Consider two long straight current-carrying wires as shown below:

What is the value of

\[ \oint \mathbf{B} \cdot d\mathbf{s} \]

for the path shown?

\[ \oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 \ i_{\text{enc}, \text{net}} = \mu_0 (-i) \]

only current enclosed by path counts!

A. $2 \mu_0 i$
B. $\mu_0 i$
C. 0
D. $-\mu_0 i$
E. Can’t tell.
Consider two long straight current-carrying wires as shown below:

What is the value of

\[ \oint \vec{B} \cdot d\vec{s} \]

for the path shown?

\[ \oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc.,mt}} = \mu_0 (i - i) = 0 \]

A. \(2\mu_0 i\)  
B. \(\mu_0 i\)  
C. 0  
D. \(-\mu_0 i\)  
E. Can’t tell.
Wire, shown in cross section, carries a current \( i \) out of (\( \bigodot \)) the screen. Assume that the magnitude of the current density is constant across the wire.

Because of the cylindrical symmetry, the only coordinate that \( B \) can depend on is \( r \).

\[ B = B(r) = \text{const} \]

\( \mathbf{B} \) must point along the circular integration path along the path.

\[ \oint \mathbf{B} \cdot d\mathbf{s} = \oint \mathbf{B} ds = B \oint ds = B(2\pi r) = \mu_0 i \text{enc} \]
Magnetic Field inside of a Long, straight Wire

Wire, shown in cross section, carries a current $i$ out of ($\bigcirc$) the screen. Assume that the magnitude of the current density is constant across the wire.

What is the current enclosed by the integration path?

$$B = \frac{\mu_0 i \cdot \pi}{2 \pi R^2}, \quad \text{for } r < R \quad \text{(inside)}$$

A. $i$  B. $-i$  C. $ir^2/R^2$  D. $-ir^2/R^2$  E. $ir/R$
Magnetic field due to a circular current-carrying loop:
Applications of Ampere’s Law:

Magnetic Field inside a Solenoid

\( \mathbf{B}_{\text{inside}} \) is strong and uniform inside of solenoid.

\( \mathbf{B}_{\text{outside}} \times 0 \) outside.

helical coil of wire
**Magnetic Field inside a Solenoid**

\[ \mathbf{B}_i \text{ (out of)} \]

\[ \mathbf{B}_f \text{ (into)} \]

**Integration Path**

\[ \mathbf{B}_\text{inside} \]

\[ \mathbf{B}_\text{outside} \]

\[ \mathbf{A}_m \text{ per 'L'aw: } \oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 i_{enc} \]

\[ = B \cdot h \]

\[ \Rightarrow i_{enc} = \frac{\mu_0 B h}{l} i \]

\# of terms per length of solenoid

\[ \oint \mathbf{B} \cdot d\mathbf{s} = \oint \mathbf{B}_\text{inside} \cdot d\mathbf{s} + \oint \mathbf{B}_\text{outside} \cdot d\mathbf{s} \]

\[ = 0 \]

\[ = 0 \]

\[ = 0 \]
\[ = \text{for solenoid:} \]
\[ \oint \mathbf{B}' \cdot d\mathbf{s}' = B \int ds = Bh = \mu_0 i_{enc} = \mu_0 n i \]

\[ \text{since } B_{\text{inside}} \text{ is const., and } \mathbf{B}' \text{ points along } d\mathbf{s}' \]

\[ \Rightarrow B_{\text{inside of solenoid}} = \mu_0 n i \]

\[ n = \frac{\# \text{ of turns}}{\text{Unit length of solenoid}} \]