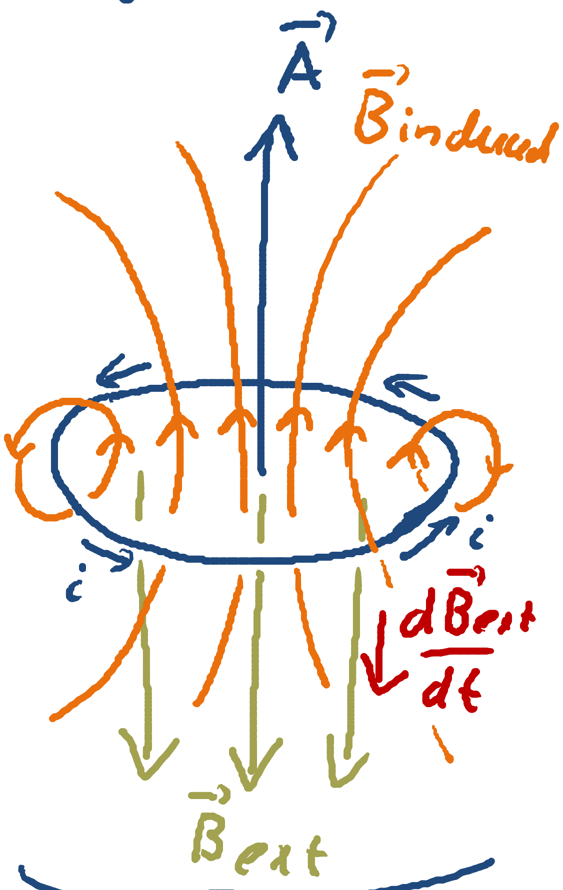
Recap

Lecture 21

- Magnetic Induction:



$\Phi_B = \vec{B}\cdot\vec{A} < 0$ here

$d\Phi_B/dt < 0$ here $\Rightarrow \mathcal{E} > 0$

$\Rightarrow$ current flows in "+ direction"

- Emf induced by changing flux through a conducting loop:

  Faraday's Law $\mathcal{E} = -\dfrac{d\Phi_{B,\text{through loop}}}{dt}$

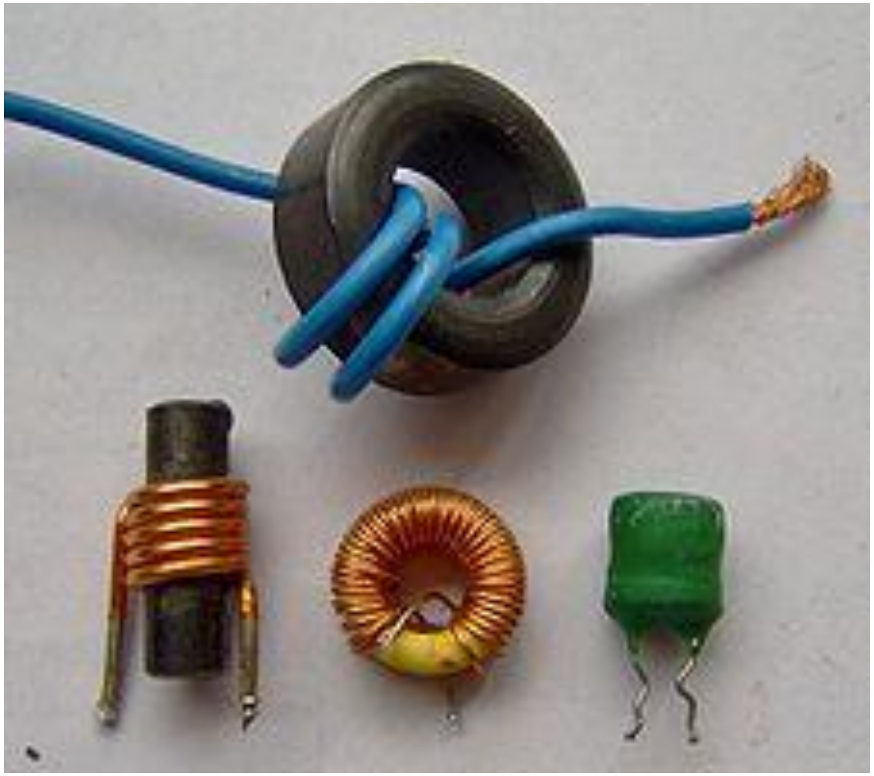
- resulting induced current:

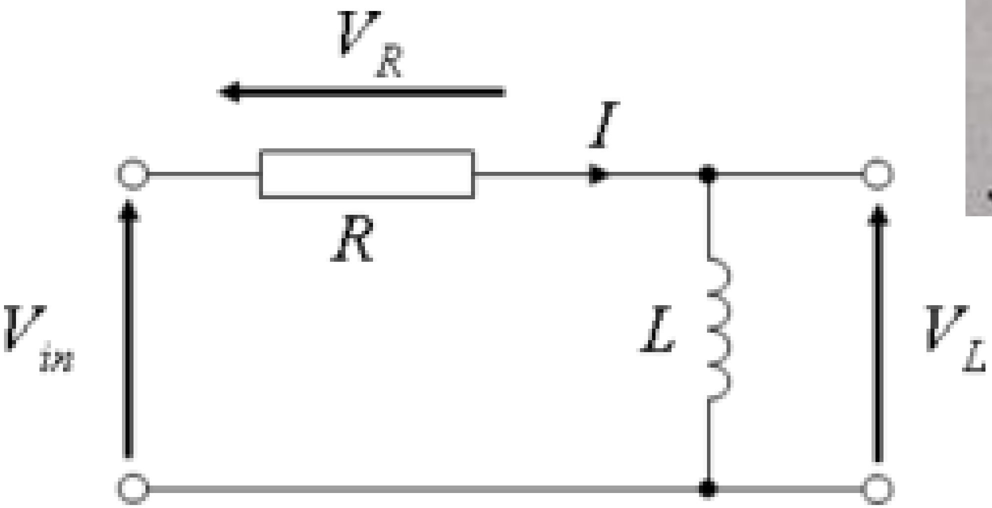
  $i_{ind} = \dfrac{\mathcal{E}}{R}$ ← Resistance of the loop

- Lenz's Law: An induced current has a direction such that the magnetic field due to the induced current opposes the change in the magnetic flux that induces the current!

Today:

- **Inductors and their inductance**
- **RL circuits**
- **Energy density of a magnetic field**





Inductors and Inductance L:

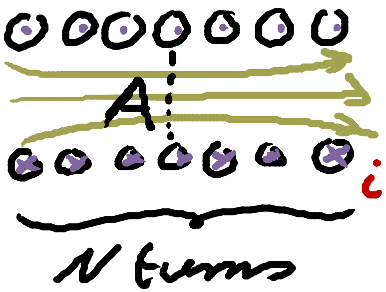
~ **Recall:** Capacitors and Capacitance

- Capacitor: produces electric field between two plates
  - Symbol: $-\mid\,\mid-$
  - Described by capacitance: $C = \dfrac{Q}{\Delta V_c}$ } depends on geometry of capacitor
  - Energy of electric field in capacitor: $\mathcal{U} = \frac{1}{2} C \Delta V_c^2$

~ **Now:**

- Inductor: produces a magnetic field around current carrying wire
  - Symbol: 
  - Described by inductance L

→ Consider a solenoid with N windings / turns:



- magnetic flux through central region of area A : Φ_B
- same flux goes through each winding of solenoid ⇒ are "linked" by shared flux
- define magnetic flux linkage $= N\Phi_B$

• Define Inductance L of solenoid / inductor:

$$\boxed{L \equiv \frac{N\Phi_B}{i}}$$

N: number of turns of wire

i: current going through wire

Units:

$$\underline{\underline{[L]}} = \frac{[\Phi_B]}{[i]} = \frac{Tm^2}{A} = 1\,\text{henry} = \underline{\underline{1\,H}}$$

$\Rightarrow$ for a long solenoid of length ℓ with N turns

$$B_{inside} = \mu_0 \underbrace{\frac{N}{\ell}}_{n} i \quad \Rightarrow \quad \Phi_B = \mu_0 \frac{N}{\ell} i \underbrace{A}_{\text{cross-sectional area of solenoid}}$$

$\Rightarrow$ Inductance of a solenoid:

$$\underline{\underline{L}} = \frac{N \Phi_B}{i} = \frac{N \mu_0 \frac{N}{\ell} i A}{i} = \underline{\underline{\mu_0 \frac{N^2 A}{\ell}}}$$

\# of turns

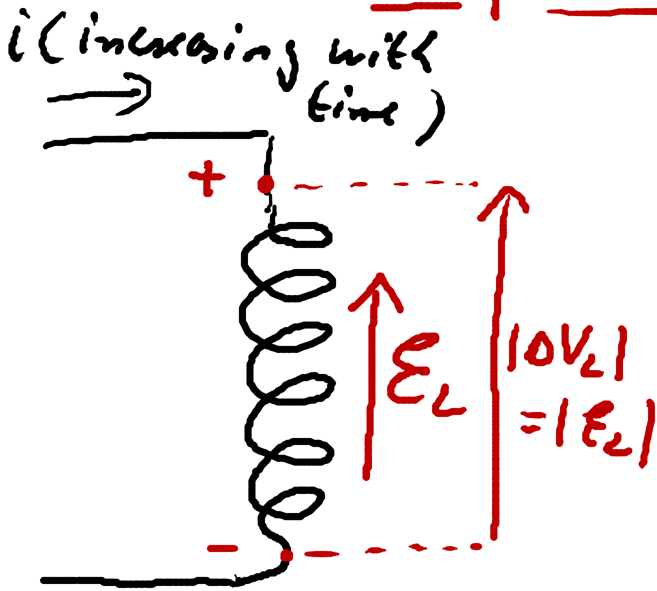
cross-sect. area

$$\Rightarrow \boxed{L_{of\ solenoid} = \mu_0 \frac{N^2 A}{\ell}}$$

length

} depends on geometry of solenoid only

Self-induction in an Inductor:



suppose that we let current i in the solenoid change with time

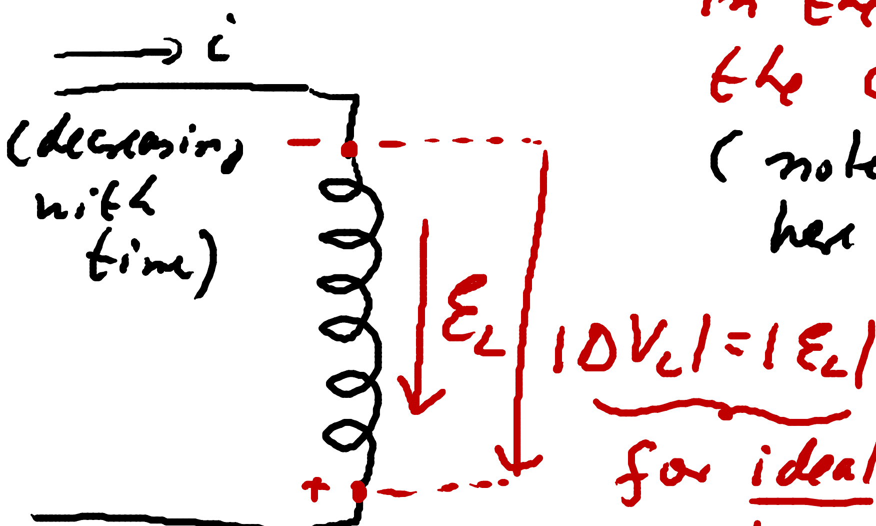
$\Downarrow$

magnetic flux Φ_B changes with time

$\Downarrow$

according to Faraday's law: an emf will be 'self-induced' in the solenoid, that opposes the change in current/flux

(note: only one solenoid involved here!)



for *ideal* inductor without resistance R in wire

⇒ from Faraday's Law:

$$\underline{\mathcal{E}_L} = -N\frac{d\Phi_B}{dt}$$ for solenoid with N turns

$$= -\frac{d(N\Phi_B)}{dt} = -\frac{d(Li)}{dt}$$ since $L = \frac{N\Phi_B}{i}$

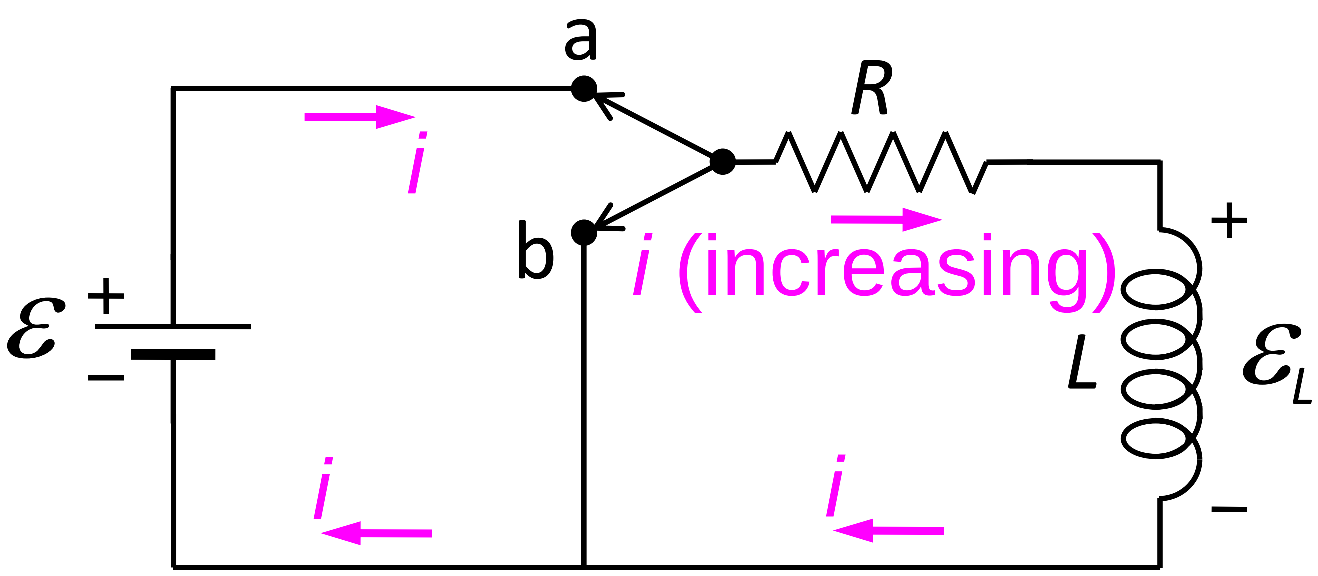
$$\underline{= -L\frac{di}{dt}}$$

⇒ if current is changing in an inductor ($\frac{di}{dt} \neq 0$) then there is a self-induced emf in the inductor:

$$\boxed{\mathcal{E}_{self\text{-}induced} = -L\frac{di}{dt}}$$

rate of change of current i

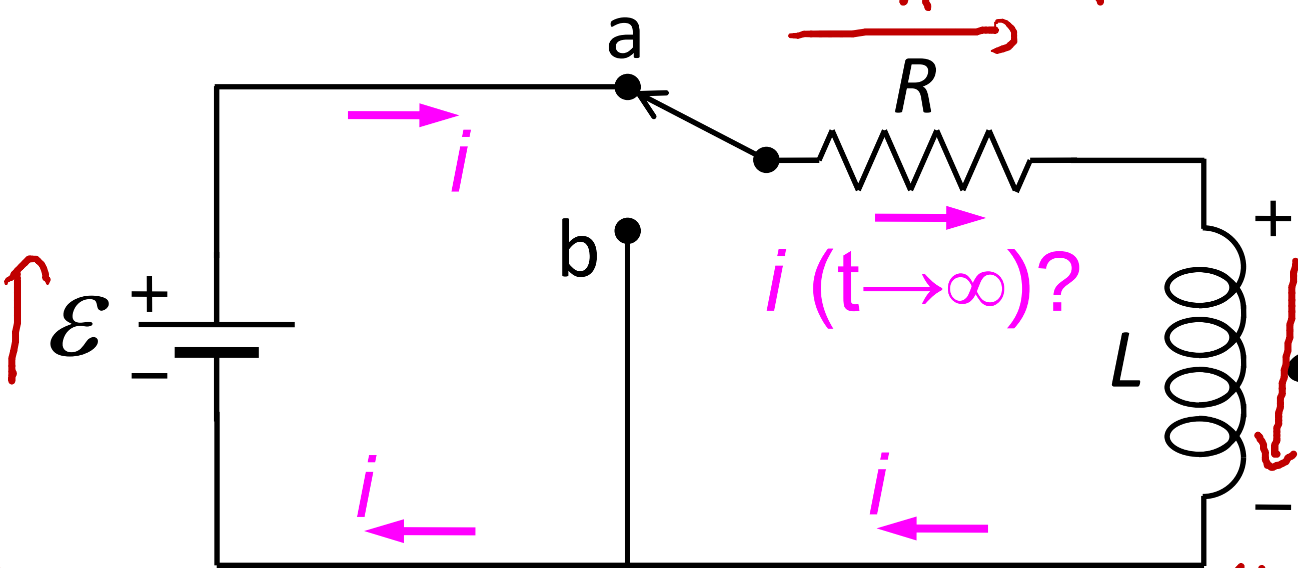
RL circuit: Rise of current



- At time $t = 0$ move the switch to position a.
- Current i begins to flow but the self-induced emf $\mathcal{E}_L$ in the inductor L opposes the rise in current.

-> Current starts out at 0 at t=0 and then increases until it approaches a steady state value asymptotically.

$\Delta V_R = -iR$



self induced emf

$\Delta V_L = -L\frac{di}{dt}$

$\Rightarrow \Delta V_L \to 0$ for $t \to \infty$

loop rule: $\varepsilon + \Delta V_R + \Delta V_L = 0 = \varepsilon - iR - \underbrace{L\, di/dt}_{=0} \Rightarrow \underline{\underline{i = \varepsilon/R}}$ for $t \to \infty$

At time $t = 0$ move the switch to position a.

After a very long time what will be **the magnitude of the steady state current** in the circuit?

← i = const ⇒ $di/dt = 0$

A. 0

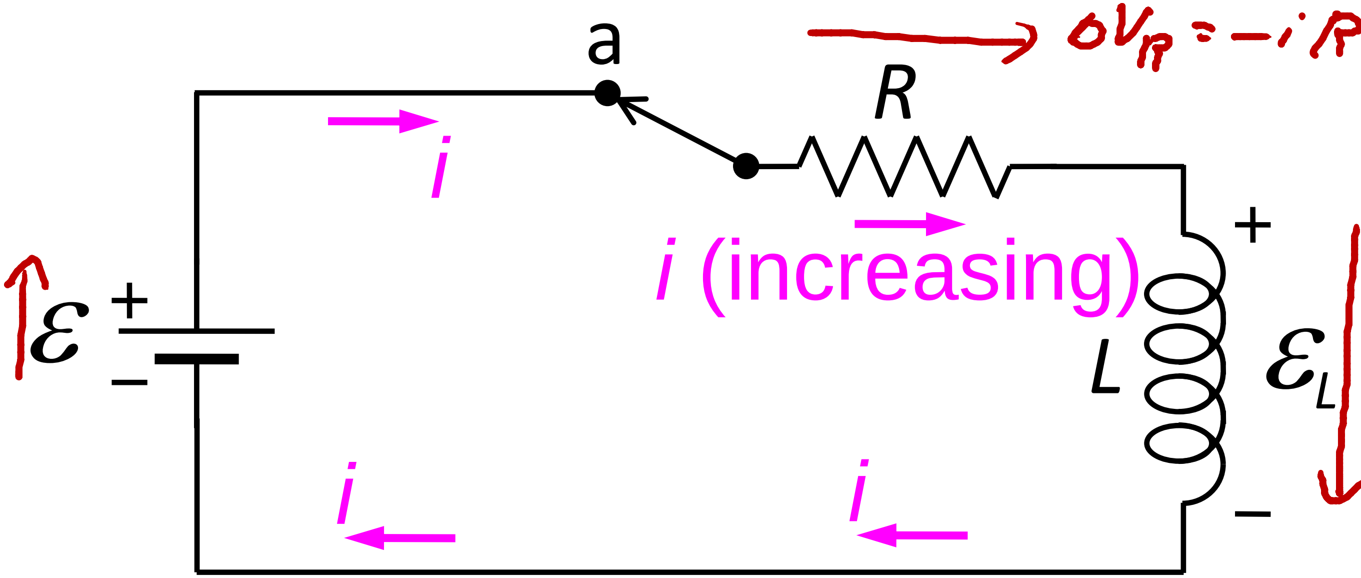
B. $|\varepsilon_L|/R$

C. ε/R

D. $(\varepsilon - |\cancel{\varepsilon_L}|)/R$ → 0

E. Both answers C & D above.

RL circuit: Rise of current



At time $t = 0$ move the switch to position a.

use loop rule: $\mathcal{E} + \Delta V_R + \Delta V_L = 0$

$\Rightarrow \quad \mathcal{E} - iR - L\frac{di}{dt} = 0$ } differential equation for current rise

at $t=0$: $i(t=0) = 0$ } initial condition

$\Rightarrow$ Solution: $i(t) = \frac{\mathcal{E}}{R}\left[1 - e^{-tR/L}\right]$

check: $i(t=0) = 0$ ✓ $\quad \frac{di}{dt} = +\frac{\mathcal{E}}{L}\, e^{-tR/L}$

RL circuit: Rise of current

current in inductor

$$\boxed{i(t) = \frac{\mathcal{E}}{R}\left[1 - e^{-t/\tau}\right]}$$

with time constant of current increase $\boxed{\tau = L/R}$

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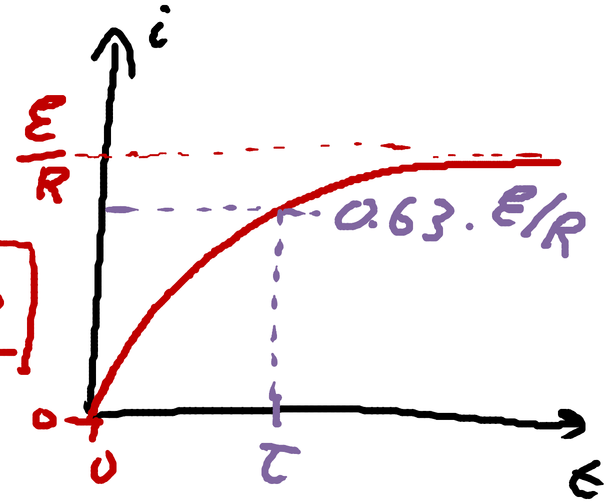
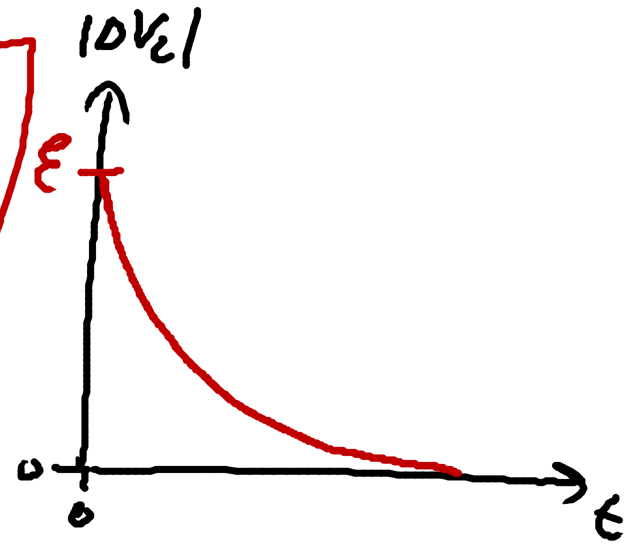


change in electric potential across the inductor:

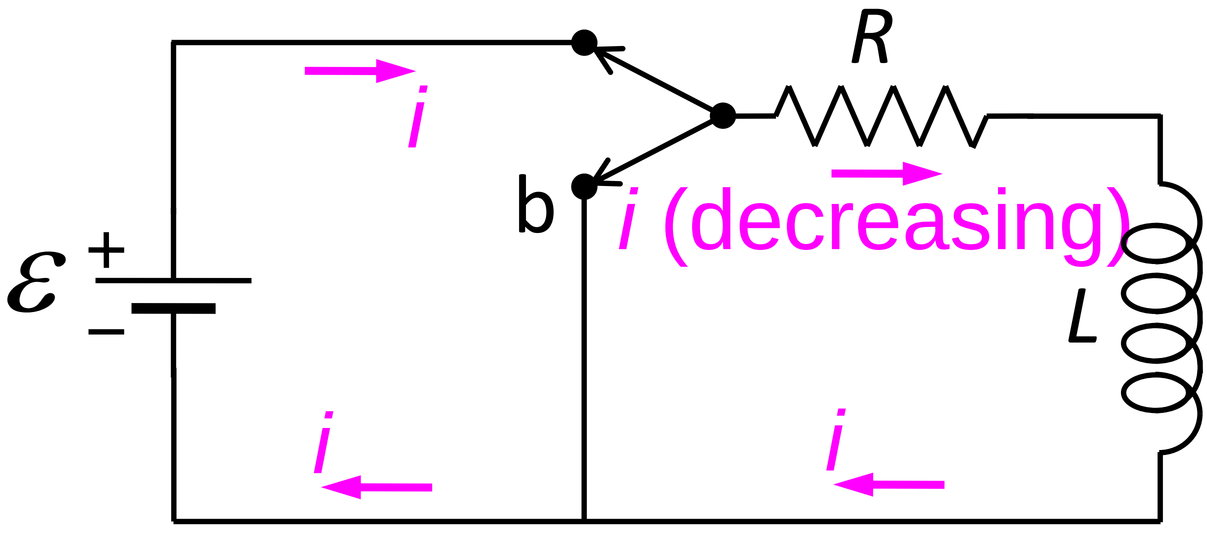
$$\boxed{|\Delta V_L| = L\frac{di}{dt} = \mathcal{E}e^{-t/\tau}}$$



Note:
- at $t=0$, $i=0$ and $|\Delta V_L| = \mathcal{E}$
- at $t \to \infty$, $i = \frac{\mathcal{E}}{R}$ and $|\Delta V_L| = 0$

  i.e. inductor eventually behaves like an ordinary connecting wire

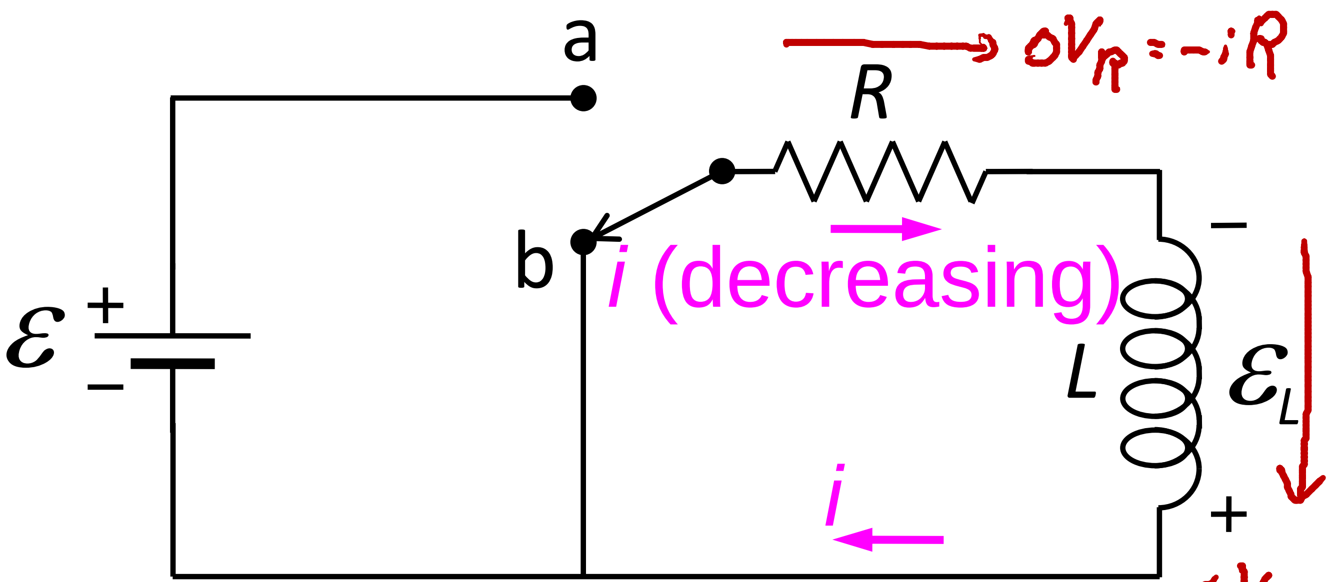
RL circuit: Decay of current



- **The switch has been in position a for a very long time.**
- **At time $t = 0$ move the switch to position b.**
- **Current *i* begins to decrease, but the self-induced emf $\mathcal{E}_L$ in the inductor L slows down the decease in current.**

-> Current starts out at the equilibrium value, and then decays to zero over time.

RL circuit: Decay of current



At time $t = 0$, switch to position b using a make-before-break switch.

$\Delta V_L = -L \underbrace{\frac{di}{dt}}_{< 0 \text{ now!}}$

use loop rule: $\Delta V_R + \Delta V_L = 0$

$\Rightarrow \left. -iR - L\frac{di}{dt} = 0 \right\}$ differential equation for current decay

at $t=0$: $\left. i(t=0) = i_0 \right\}$ initial current

$\Rightarrow$ solution : $i(t) = i_0 e^{-tR/L}$

check: $i(t=0) = i_0$

$\frac{di}{dt} = -\frac{i_0 R}{L} e^{-tR/L}$

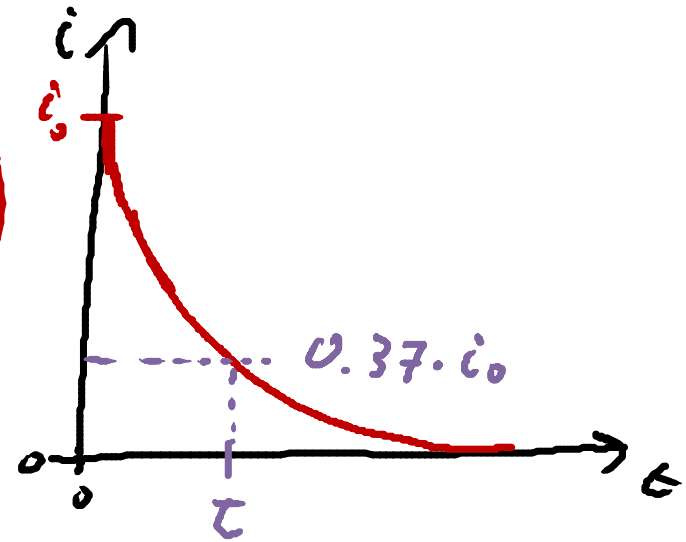
RL circuit: Decay of current

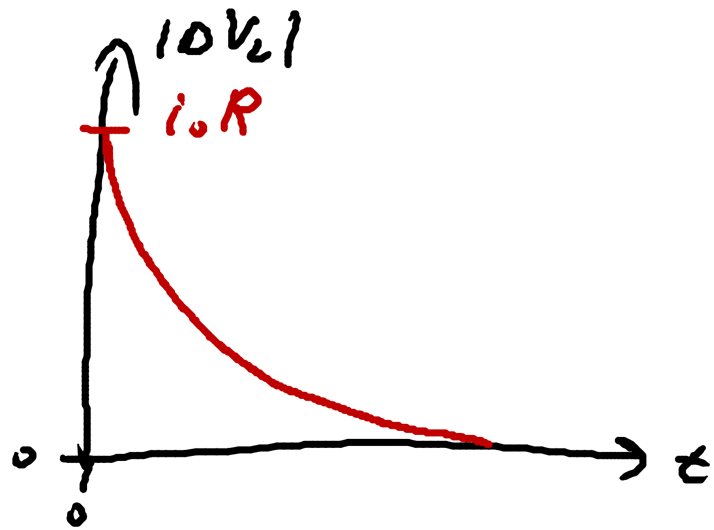
current in inductor: $\boxed{i(t) = i_0\, e^{-t/\tau}}$

with time constant $\boxed{\tau = L/R}$ for current decay

$\Downarrow$

change in electric potential across the inductor: $$\boxed{|\Delta V_L| = \left|L \frac{di}{dt}\right| = i_0 R\, e^{-t/\tau}}$$





<u>Note:</u>

if current has reached steady state value $i = \mathcal{E}/R$ before moving switch to position b:

$i_0 = \mathcal{E}/R \Rightarrow |\Delta V_L(t=0)| = \mathcal{E}$