**Inductor:**
- produce a well defined magnetic field in a specific region
- circuit symbol: \( \text{---} \)
- Inductance \( L \):
  \[ L = \frac{N \Phi B}{I} = \text{flux linkage over current} \]
  \[ \ell L J = \frac{T_m^2}{A} = H = \text{Henry} \]
- for a solenoid:
  \[ L = \mu_0 N^2 A \]
  \[ \ell < \text{length of solenoid} \]

- **Self-induced emf:**
  if current in a coil changes, an emf is induced in the coil:
  \[ \mathcal{E}_c = \mathcal{E}_{\text{self-induced}} = -L \frac{di}{dt} \]
  Potential change over inductor: \( 10 \mathcal{V}_L = \mathcal{E}_L \)
Recap II

**RL Circuits:**

- **Current Rise:**
  \[ i(t) = \frac{E}{R} \left[ 1 - e^{-t/\tau} \right] \]
  
  with inductive time constant \( \tau = L/R \)

- **Current Decay:**
  \[ i(t) = i_0 e^{-t/\tau} \]
  
  with inductive time constant \( \tau = L/R \)
Today:

- Energy density of a magnetic field
- Alternating current and power
  - Transmission lines and transformers
- Ideal LC circuit
**RL circuit:**

Power supplied and dissipated in the circuit

Loop rule gives: \[ \mathcal{E} = iR + L \frac{di}{dt} \quad (\mathcal{E} - iR - L \frac{di}{dt} = 0) \]

Multiply both sides by current \( i \) to get:

\[ \mathcal{E}i = i^2R + iL \frac{di}{dt} \]

This term must be the power delivered to the inductor.
\[\text{(power delivered to inductor)} = \left(\text{rate at which the magnetic potential energy } U_B \text{ is stored in the magnetic field of the inductor}\right) = \frac{dU_B}{dt}\]

\[\Rightarrow \frac{dU_B}{dt} = iL \frac{di}{dt} \Rightarrow dU_B = Li \, di\]

Integrate:

\[U_B = \int dU_B = \int_0^i L \, di = L \int_0^i idi = \frac{1}{2} L i^2\]

\[\Rightarrow \text{Energy stored in the magnetic field of an inductor:}\]

\[U_B = \frac{1}{2} L i^2\]

\{ Magnetic fields have stored energy.\}
Energy density of a magnetic field:

Energy density $U_B = \frac{U_B}{\text{volume}} = \frac{1}{2} L i^2 \frac{\mu_0 N^2}{A L}$

for solenoid

Cross-sectional area of solenoid

Length of solenoid

Inductance $L$ of solenoid:

\[ L = \mu_0 \frac{N^2}{A} \]  

\# of turns

\[ U_B = \frac{1}{2} \mu_0 \frac{N^2}{A} i^2 \]  

now use: \[ B = \mu_0 \frac{N}{L} i \]

\[ (\text{energy density of a magnetic field}) = \frac{U_B}{\text{volume}} = \frac{1}{2} \frac{B^2}{\mu_0} \propto B^2 \]

Recall:

\[ (\text{energy density of an electric field}) = \frac{U_E}{\text{volume}} = \frac{1}{2} \varepsilon_0 E^2 \propto E^2 \]
At time $t = 0$ the switch is moved to position a.

After $t = 0$, how does the power delivered to the inductor’s magnetic field vary with time?

A. It starts low & steadily increases.
B. It starts high & steadily decreases.
C. It starts low, then increases until it reaches a peak, & then decreases.
D. It’s constant.
E. It oscillates.

\[ P_{\text{to inductor}} = i L \frac{di}{dt} \]

\[ \begin{align*}
  &\text{at } t=0: \ i=0 \Rightarrow P=0 \\
  &\text{at } t\to\infty: \ i=\text{const} \Rightarrow P=0
\end{align*} \]
Why is power transmitted at very high voltages in power transmission lines (several 100,000 volts)?

A. Because it reduce the energy lost in long-distance transmission
B. Because it maximized the power that can be transmitted
C. Because it is easier to generate high voltages
Alternating current (ac):

- Direct current (dc): flow of electric charge carriers is only in one direction, i.e. non-oscillating.

- Alternating current (ac): movement of electric charge carriers periodically reverses direction -> oscillating emf and current.

Example: Household voltage in North America:

\[ E(t) = E_{\text{max}} \sin (2\pi f t) = E_{\text{max}} \sin (\omega t) \]

- Varies sinusoidally?
- with \( E_{\text{max}} = 120 \, \text{V} \)
- and \( f = 60 \, \text{Hz} \)
- \( T = \frac{1}{f} = \frac{1}{60} \, \text{s} \)
Consider a circuit describing a device with resistance $R$ plugged into a power outlet.

![Circuit Diagram]

- **Loop rule:** $\mathcal{E} - \Delta V_R = 0$
- $\Delta V_R = \mathcal{E} = \mathcal{E}_{\text{max}} \sin(\omega t)$

\[ \text{circuit } R = \frac{\Delta V_R}{i_R} \]

- $i(t) = \frac{\Delta V_R}{R} = \frac{\mathcal{E}_{\text{max}}}{R} \sin(\omega t) = i_{\text{max}} \sin(\omega t)$

**with:**

\[ i_{\text{max}} = \frac{\mathcal{E}_{\text{max}}}{R} = \frac{\Delta V_R}{R} \]

**for a resistor in an AC-circuit**

\[ i(t) = \frac{\Delta V_R}{R} = \frac{\mathcal{E}_{\text{max}}}{R} \sin(\omega t) \]

Note: the potential drop $\Delta V_R$ and current $i_R$ oscillate in phase for a resistor!
now: calculate power dissipated in the resistor $R$

from before: $P = i^2 R$ \(\Rightarrow\) for ac: $P = i_{\text{max}}^2 \sin^2(\omega t) R$

take time average: 

\[
\begin{align*}
P_{\text{avg}} &= \langle i^2 \rangle_{\text{avg}} R \\
&= \frac{1}{2} i_{\text{max}}^2 R
\end{align*}
\]

average of $\sin^2(\omega t) = \frac{1}{2}$

\[
\text{Average power (dissipated in resistor in AC circuit)} = P_{\text{avg}} = \frac{1}{2} i_{\text{max}}^2 R = \left(\frac{i_{\text{max}}}{\sqrt{2}}\right)^2 R = i_{\text{RMS}}^2 R
\]

with "root-mean-square" (RMS) current: $i_{\text{RMS}} \equiv \frac{i_{\text{max}}}{\sqrt{2}}$

\[
\begin{align*}
\Delta V_{\text{RMS}} &= \Delta V_{\text{max}} \sqrt{\frac{1}{2}} \\
\varepsilon_{\text{RMS}} &= \varepsilon_{\text{max}} \sqrt{\frac{1}{2}}
\end{align*}
\]
Note:

- in U.S.: $E_{\text{rms}} = \frac{E_{\text{max}}}{\sqrt{2}} = \frac{170V}{\sqrt{2}} = 120V$

- Voltmeters, ammeters, etc... read RMS values of ac current

- Power transmission line:

  Power transmission lines have resistance $R$

  $\Rightarrow$ power lost in transmission

  $P_{\text{aux, lost}} = i^2 R$

  $\Rightarrow$ need to keep current low in lines!

  Power delivered by power plant/ transmission line:

  $P_{\text{aux, deliv.}} = E_{\text{rms}} i_{\text{rms}} \Rightarrow P_{\text{aux, lost}} = \left(\frac{P_{\text{aux, deliv.}}}{E_{\text{rms}}}\right)^2 R$

  $\Rightarrow$ high voltage $E_{\text{rms}} \Rightarrow$ low current $i_{\text{rms}}$

  $\Rightarrow$ reduce power lost in transmission line!
Electrical transmission system:

Power is transmitted at high voltage and low current! 

Transmission lines: 765, 500, 345, 230, and 138 kV

Generating Station

Generating Step Up Transformer

Transmission Customer 138kV or 230kV

Substation Step Down Transformer

Subtransmission Customer 26kV and 69kV

Primary Customer 13kV and 4kV

Secondary Customer 120V and 240V

easy to do for ac, more difficult for dc

Transformer: transform ac input voltage to different output voltage, while keeping the product current x voltage = constant