Recap

- **Transformer:**
  - Transformation of voltage: \( V_s = \frac{N_s}{N_p} V_p \)
  - Transformation of current: \( i_s = \frac{N_p}{N_s} i_p \)

- **Ideal LC-Circuit:**

  Energy oscillates between electric and magnetic fields, but the total sum of energy remains constant:
  \[ U_{\text{total}} = U_E(t) + U_B(t) = \frac{q(t)^2}{2C} + \frac{\Phi(t)^2}{2} \]
  \[ = \frac{1}{2} \frac{Q_{\text{max}}^2}{C} = \text{const} \]

  Oscillation of charge: \( q(t) = Q_{\text{max}} \cos(\omega t + \phi) \)

  Oscillation of current: \( i(t) = \frac{dq}{dt} = -\omega Q_{\text{max}} \sin(\omega t + \phi) \)

  Angular frequency: \( \omega_0 = \frac{1}{\sqrt{LC}} = 2\pi f_0 \)
Today:

• RLC circuit: damping and driven

• Another look at Faraday’s law

• Next time: Maxwell’s equations

\[ \oint E \cdot dA = \frac{q_{\text{enc}}}{\varepsilon_0} \]
\[ \oint B \cdot dA = 0 \]
\[ \oint E \cdot ds = -\frac{d\Phi_B}{dt} \]
\[ \oint B \cdot ds = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{\text{enc}} \]
Ideal $LC$ circuit (no resistance):

The capacitor starts with charge $q = Q > 0$ with the polarity shown. At time $t = 0$ the switch is closed and current $i \equiv dq/dt$ flows in the circuit.

A graph of $i$ versus $t$ is shown below.

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$$\Delta V_C = L \frac{di}{dt} \implies \Delta V_C = 0 \text{ when } \frac{di}{dt} = \text{slope} = 0$$

Which of the labeled points correspond(s) to no voltage across the inductor?

A. A  B. B  C. C  D. D  E. Both A & C
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Which of the labeled points correspond(s) to no voltage across the capacitor?

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Which of the labeled points correspond(s) to charge $+Q$ on the capacitor?

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The capacitor starts with charge $q = Q > 0$ with the polarity shown. At time $t = 0$ the switch is closed and current $i \equiv dq/dt$ flows in the circuit.

A graph of $i$ versus $t$ is shown below.

Which of the labeled points correspond(s) to counterclockwise current flow in the circuit?

A. A  B. B  C. C  D. D  E. Both A & C
**RLC circuit:**

- Resistance $R$
  - dissipation of power
  - damping of oscillation

- Loop rule now gives:

$$\frac{\Phi}{C} + L \frac{d^2 q}{dt^2} + \frac{d q}{dt} \frac{q}{L} R = 0$$

$\frac{\Phi}{C} \quad \frac{d^2 q}{dt^2} \quad \frac{d q}{dt} \quad \frac{q}{L} R = 0$

$\frac{d^2 q}{dt^2} + \frac{1}{L} \frac{d q}{dt} + \frac{q}{C R} = 0$

$\Rightarrow$ Solution:

$$q(t) = Q_0 e^{-t/2T} \cos(\omega'T + \phi)$$

with energy decay time constant $T = L/R$

and $\omega' = \sqrt{\omega_0^2 - (R/2L)^2}$

$\Rightarrow$ for small $R$: $\omega' \approx \omega_0 = \frac{1}{\sqrt{LC}}$
\[ q(t) = Q_0 e^{-\frac{t}{2C}} \cos(\omega t + \phi) \]

\[ U_{\text{total}} \text{ (total energy stored in circuit)} \]
also decreases exponentially:

\[ U_{\text{total}}(t) = \frac{1}{2C} Q_{\text{max}}(t)^2 = \frac{1}{2C} Q_0^2 e^{-\frac{t}{2C}} \]

\[ c = \frac{1}{\sqrt{LC}} \]
Driven RLC circuit:

Current will oscillate at the driving frequency: \( f_d = \frac{\omega_d}{2\pi} \)

- Maximum current amplitude when driving frequency matches natural frequency of circuit:

\[
\Delta \omega = \frac{1}{Q} \quad \text{(energy decay time)}
\]

- Define quality factor:

\[
Q \equiv \frac{\omega_0}{\Delta \omega} = \frac{\omega_0}{2} = \frac{2\pi}{T_0}
\]
Another look at Faraday’s Law:

A changing magnetic field induces an electric field

- Induced electric field drives the current in the conducting loop (\( i = \epsilon / R \))
- Work done by the electric field on a charge \( q_0 \)

\[
dW_{\text{ind}} q_0, B' = \vec{F} \cdot d\vec{s} = q_0 \vec{E} \cdot d\vec{s}
\]

\( \Rightarrow \) Total work done on charge \( q_0 \) while it moves around the loop:

\[
W_{\text{ind}} q_0, B' = \oint_{\text{loop}} \vec{F} \cdot d\vec{s} = q_0 \oint_{\text{loop}} \vec{E} \cdot d\vec{s}
\]
But: induced emf in loop \( \epsilon \equiv \frac{W}{q_0} = \frac{\text{work done on charge}}{\text{charge}} \)

So:
\[
\epsilon = \frac{W}{q_0} = \oint \mathbf{E'} \cdot d\mathbf{s}
\]

closed path

\(\Rightarrow\) Can write Faraday's law \( \epsilon_{\text{ind}} = -\frac{d\Phi_B}{dt} \) in the following way:

\[
\oint \mathbf{E'} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}
\]
closed path

This is true whether or not the conducting loop is present!
The magnetic field is confined to the cylindrical region shown and is spatially uniform but its magnitude is increasing with time.

When Faraday’s law,

\[ \oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}, \]

is applied to the circular integration path, which best describes \( E_s \), the component of the electric field along the direction of \( d\vec{s} \)?

- A. \( E_s > 0 \)
- B. \( E_s < 0 \)
- C. \( E_s = 0 \)
- D. \( E_s \) depends on where \( d\vec{s} \) is along the integration path.
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- A. $E_s > 0$
- B. $E_s < 0$
- C. $E_s = 0$
- D. $E_s$ depends on where $d\vec{s}$ is along the integration path.
Ampère-Maxwell Law:

Symmetry is powerful in physics!

Changing magnetic field produces an electric field

\[ \downarrow \text{symmetry} \]

Changing electric field produces a magnetic field

Faraday's Law for magnetic induction:

\[ \oint E \cdot ds = -\frac{d\Phi_B}{dt} \]

Closed path

Maxwell's Law for electric induction:

\[ \oint B \cdot ds = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \]

Closed path

Rate of change of electric flux through interior of closed path
Also have Ampère's Law: \( \oint B \cdot d\mathbf{s} = \mu_0 i_{\text{enclosed}} \) for \( B \) produced by a current-closed path.

Combine: **Ampère- Maxwell Law:**

\[
\oint B \cdot d\mathbf{s} = \mu_0 i_{\text{enclosed}} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}
\]

**Closed Path**

"display next current"

**Example:**

\[ i \quad \rightarrow \quad B \quad \rightarrow \quad +q \quad E \quad -q \quad \left( \text{increasing} \right) \]

\[ \oint B \cdot d\mathbf{s} = \mu_0 i_{\text{enclosed}} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \]

**Note:** Magnetic field lines must surround either currents or changing electric fields (or both).