# Recap I



- Maxwell's Equations:

I) Gauss' Law for electric fields:

$$\oint_{\text{closed surface}} \vec{E}\cdot d\vec{A} = \frac{Q_{net,\,inside}}{\varepsilon_0}$$

} electric charges produce electric fields

II) Gauss' Law for magnetic fields:

$$\oint_{\text{closed surface}} \vec{B}\cdot d\vec{A} = 0$$

} magnetic charges do not exist

III) Faraday's Law:

$$\oint_{\text{closed path}} \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

} changing magnetic fields produce electric fields

IV Ampère - Maxwell Law:

$$\oint_{\text{closed path}} \vec{B}\cdot d\vec{s} = \mu_0 i_{encl} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

} currents and changing electric fields produce magnetic fields

# Recap II

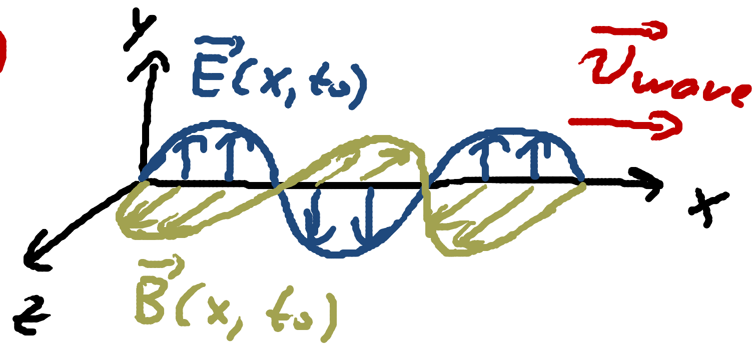
- *Electromagnetic Waves:*
  - Plane EM wave propagating along +x direction

$$E_y(x,t) = E_{max} \sin(kx - \omega t)$$

$\updownarrow$ in phase!

$$B_z(x,t) = B_{max} \sin(kx - \omega t)$$

$B_{max} = E_{max}/c$



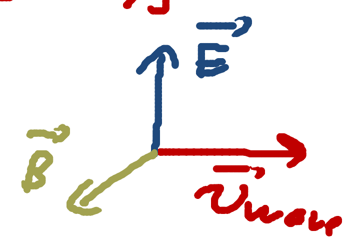

  - wave speed in vacuum:

$$v_{wave} = c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = \text{"speed of light"} = 3.0 \cdot 10^8 \text{ m/s}$$

  - Directions:

$\vec{E} \perp \vec{B}$ always

$\vec{E} \times \vec{B}$ points along $\vec{v}_{wave}$

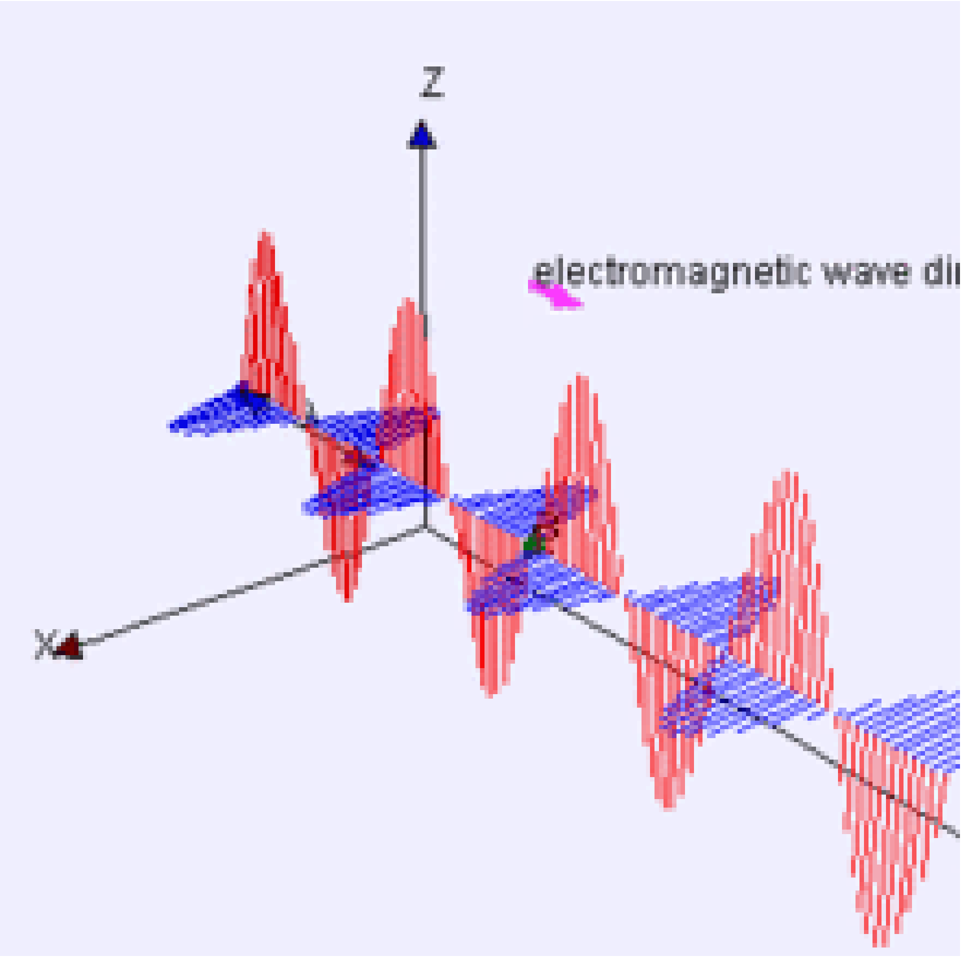



  - Polarization:

Plane polarized, if $\vec{E}$ oscillates along one specific direction $\Rightarrow$ direction of polarization = direction along which $\vec{E}$ points

# Today:

- More on electromagnetic waves
  - Spectrum
  - Energy transport
  - Polarization
    - Why is the sky blue, and why does it turn dark blue at 90 degrees from the sun?

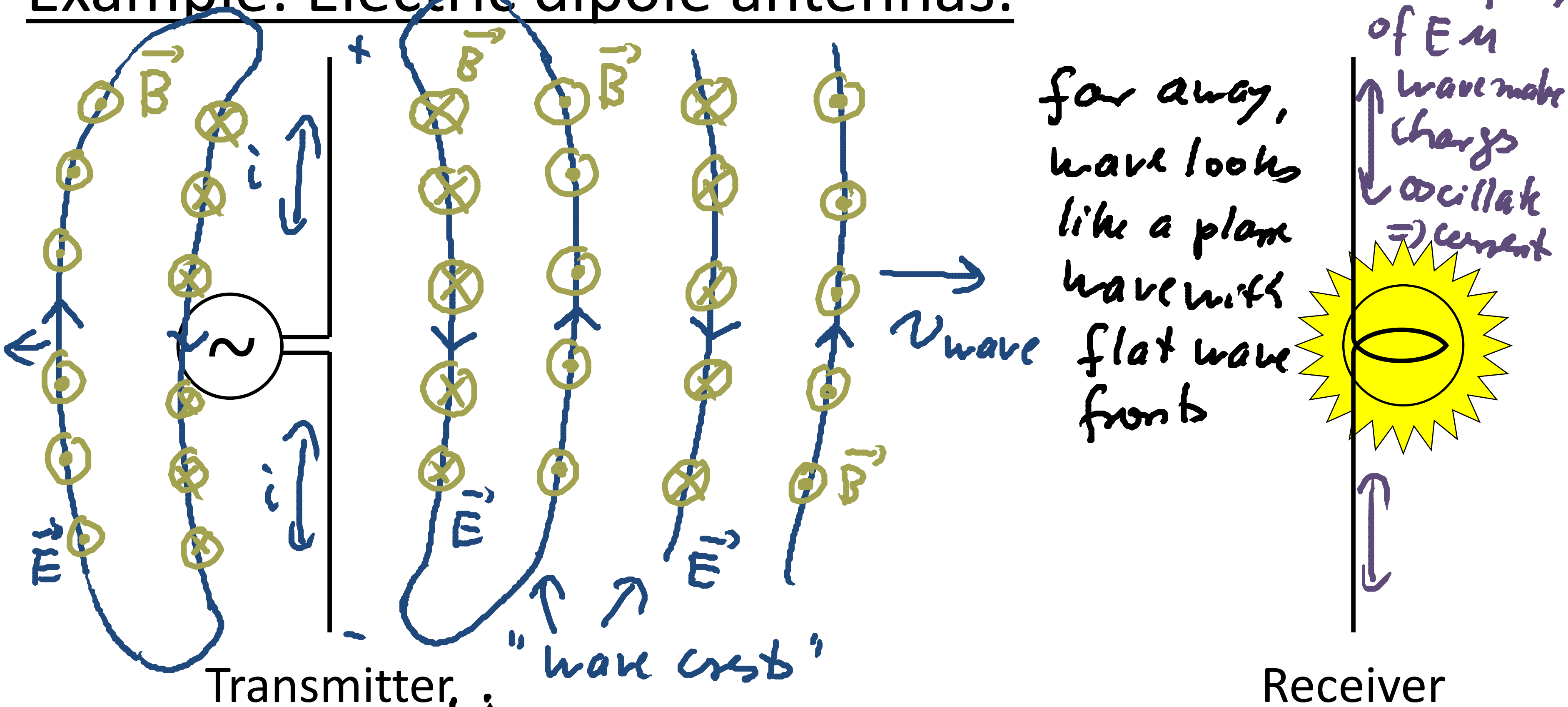


# V: Sources of EM waves:

**Accelerating charges (changing currents) radiate EM waves.**

↑ no radiation this way

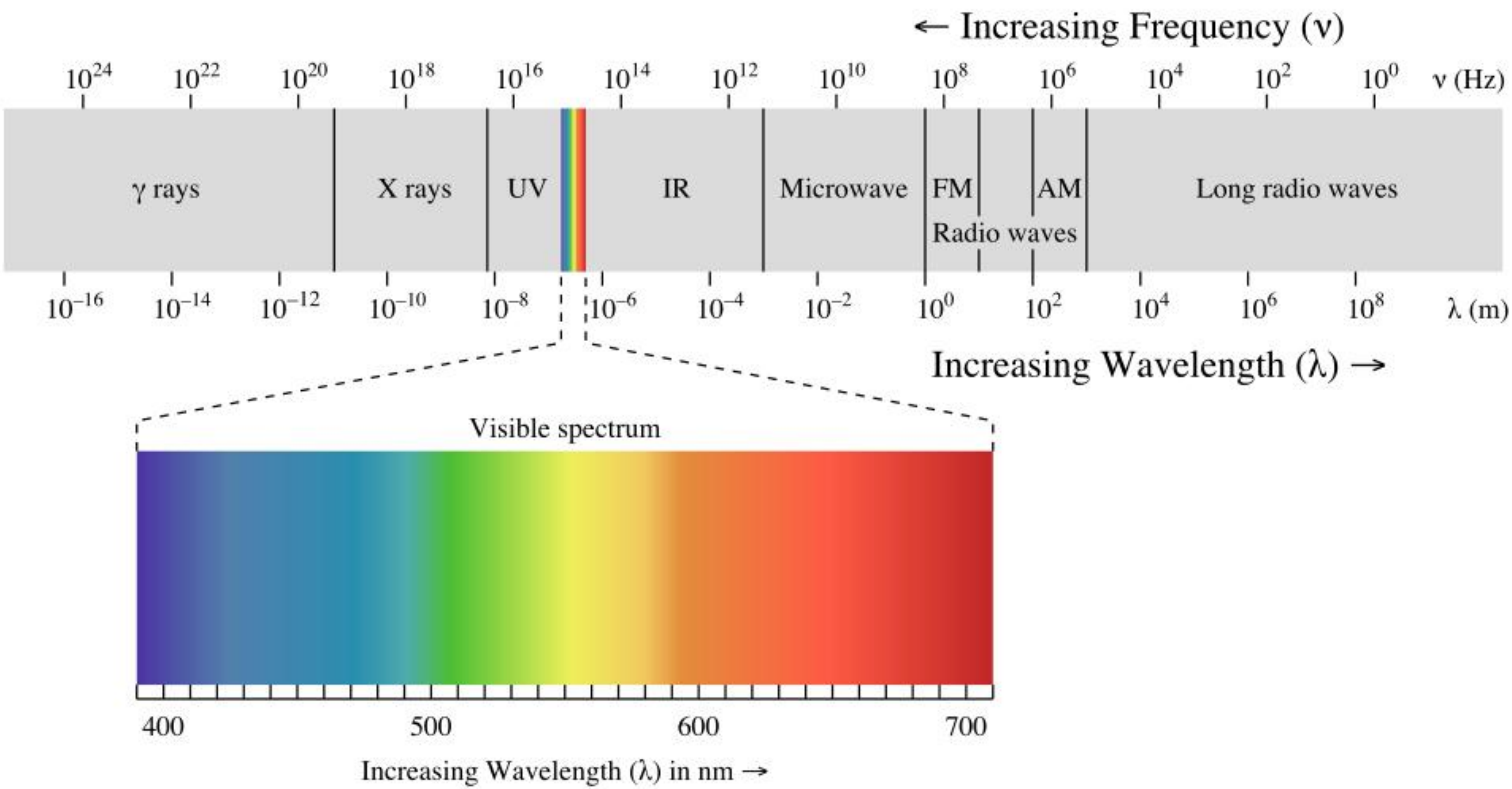
Example: Electric dipole antennas:




far away, wave looks like a plane wave with flat wave fronts

note: ↓ no radiation this way!

# VI: Spectrum of EM waves:




**Note: These are <u>all</u> electromagnetic waves! Only difference is frequency (wavelength)!**

# VII Energy Transport by EM waves

Electric and magnetic fields have energy:

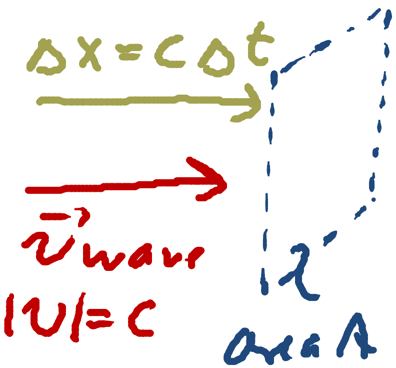
$$u_E = \frac{1}{2}\varepsilon_0 E^2$$

$$u_B = \frac{1}{2}\frac{B^2}{\mu_0}$$

$\Rightarrow$ traveling EM wave transport ("radiate") energy!

$\Rightarrow$ Average Power transported by wave per area:

$$\text{intensity } I = \left(\frac{\text{energy / time}}{\text{area} \perp \text{to wave direction}}\right)_{avg} = \left(\frac{\text{power}}{\text{area}}\right)_{avg}$$




$$= \underbrace{\left[\frac{1}{2}\varepsilon_0 (E^2)_{avg} + \frac{1}{2}\frac{(B^2)_{avg}}{\mu_0}\right]}_{\text{average energy density of EM wave}} \cdot \frac{\overbrace{A\, c\Delta t}^{A\cdot\Delta x = \text{volume}}}{\Delta t\, A}$$

$$(E^2)_{avg} = E_{max}^2 \{\sin^2(kx - \omega t)\}_{avg} = \frac{1}{2}E_{max}^2$$

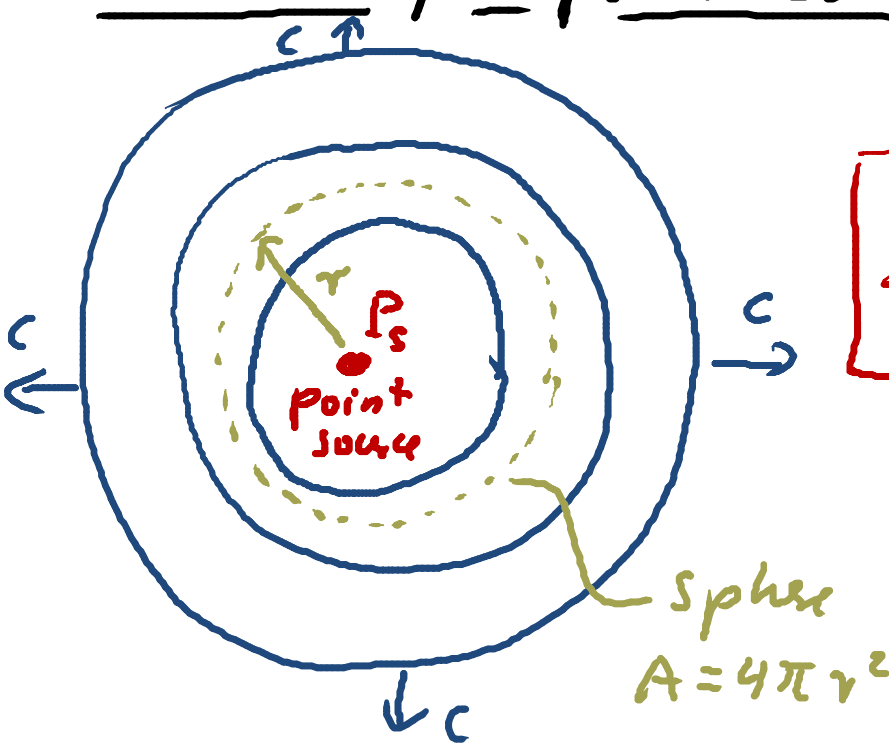
same for $(B^2)_{avg} = \frac{1}{2}B_{max}^2 = \frac{1}{2}E_{max}^2\,\mu_0\varepsilon_0$

$\Rightarrow$ Intensity $I$ of EM wave:

$$\boxed{I = \frac{\text{power}}{\perp\ \text{area}} = \frac{1}{2} c \varepsilon_0 E_{max}^2 = c\varepsilon_0 \left(\frac{E_{max}}{\sqrt{2}}\right)^2 = c \varepsilon_0 E_{rms}^2}$$

with root-mean-square value: $E_{rms} \equiv \dfrac{E_{max}}{\sqrt{2}}$

- For isotropic point source: (radiate with equal intensity in all direction)

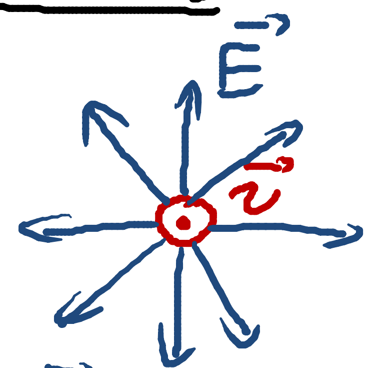



$$\boxed{I(r) = \frac{\text{power}}{\text{area}} = \frac{P_{\text{point source}}}{4\pi r^2}}$$

$r$ = distance from point source

# Polarization of EM waves:

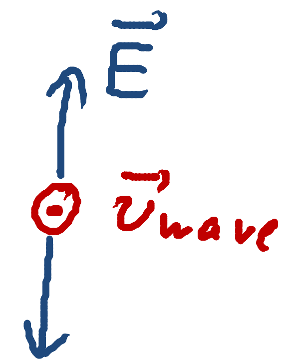
- Some sources produce randomly polarized ("unpolarized") EM waves, e.g. light bulb, sun...




note: $\vec{E} \perp \vec{v}_{wave}$ always!

$\vec{E}$ changes direction randomly.

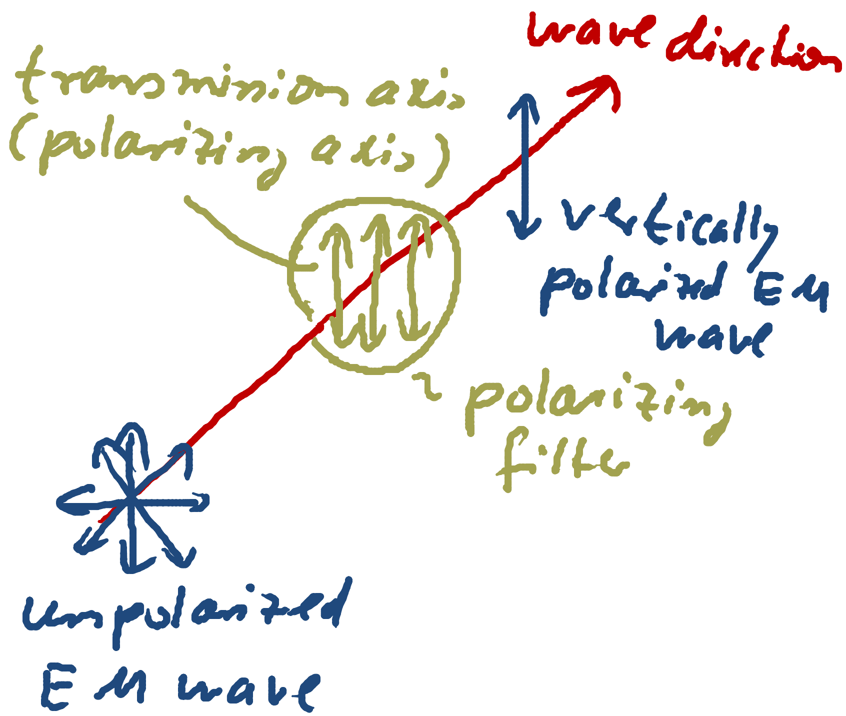
- Some sources produce plane polarized EM waves, e.g. radio/TV antennas




vertically polarized

$\vec{E}$ oscillates along a fixed direction

- Can use a polarizing filter (sheet) to produce a plane polarized wave from an unpolarized wave:



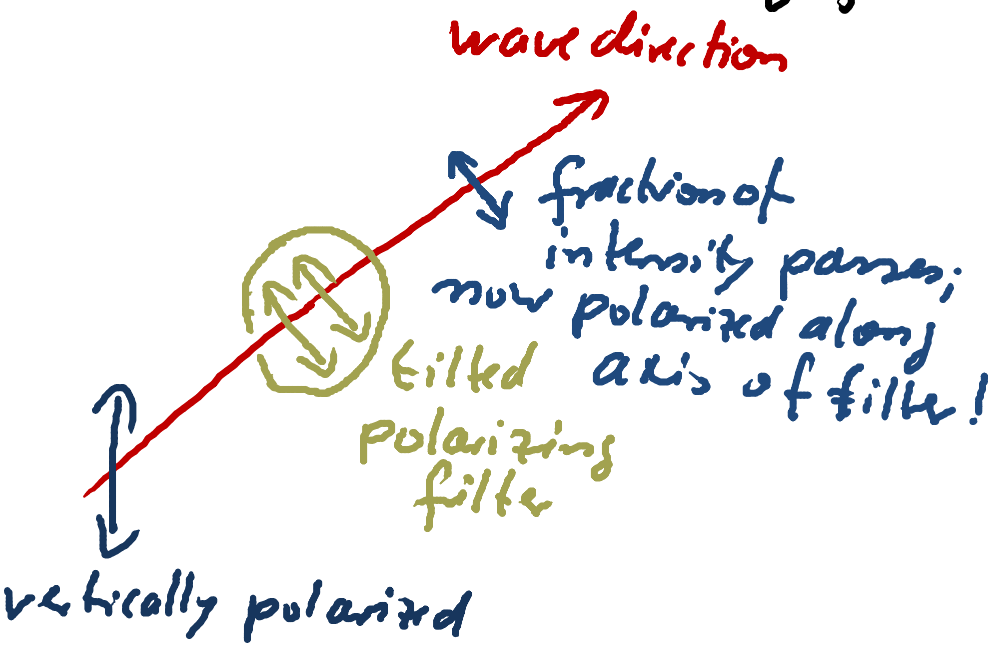

- An electric field component parallel to the filter's transmission axis is passed.
- A component perpendicular to it is absorbed.

⇒ for an incident randomly-polarized EM wave:

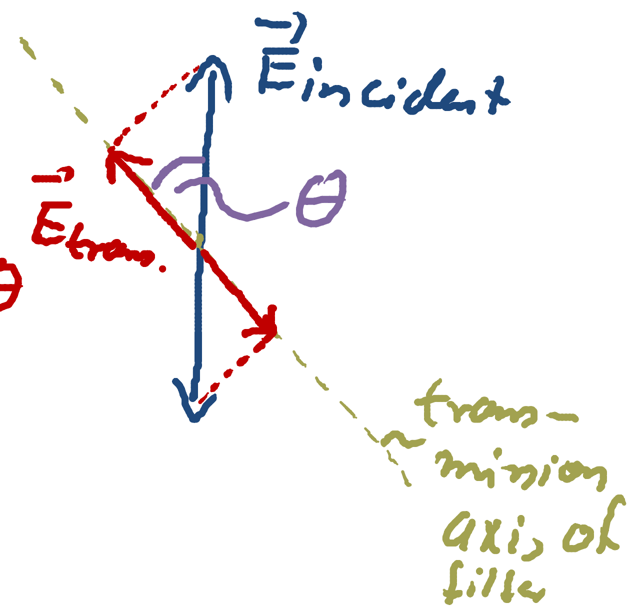
half of the incident intensity is transmitted, and half is absorbed:

$$I_{\text{after filter}} = \frac{1}{2} I_{\text{incident, randomly pol}}$$

⇒ Transmission of a plane polarized EM wave through a polarizing filter:




filter only transmits components of a plane-polarized wave's electric field that is parallel to the filter's transmission axis:




$$E_{max,transmitted} = E_{max,incident} \cdot \cos\theta$$

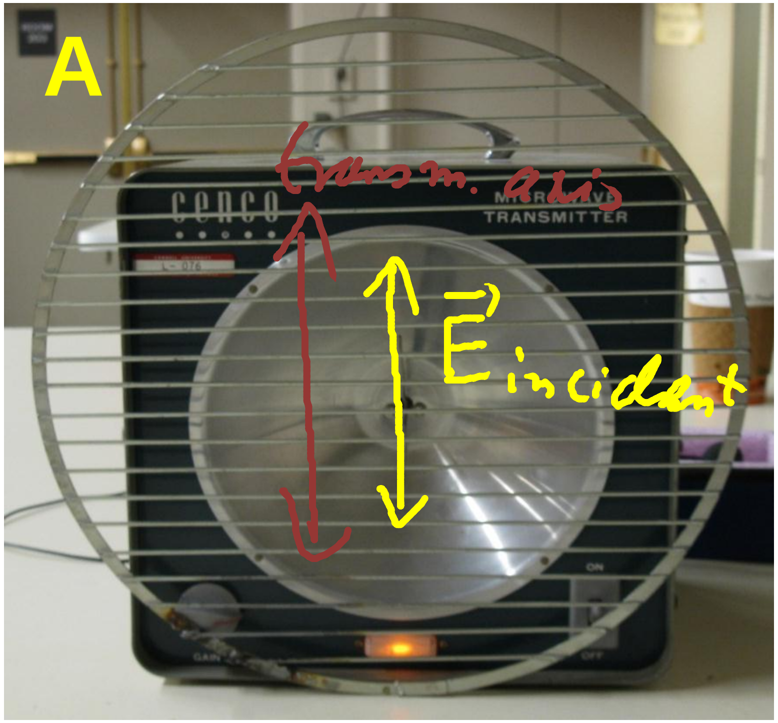
$\theta$ = angle between $\vec{E}_{incident}$ and polarizing direction of the filter

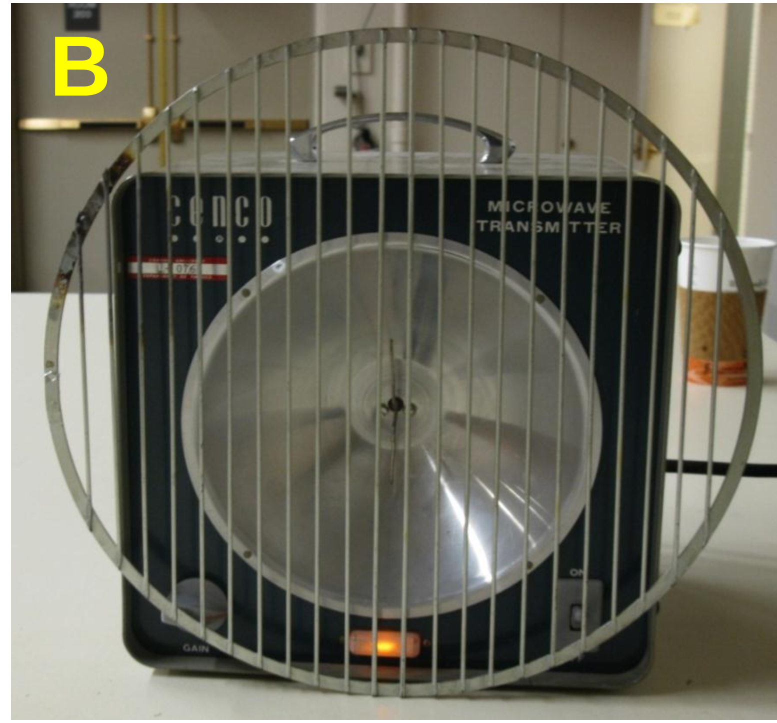
$\Rightarrow$ since $I \propto E_{max}^2$

$$\boxed{I_{transmitted} = I_{incident} \cdot \cos^2\theta}$$

for transmission of a plane-polarized EM wave through a polarizing filter

$\theta$ = angle between $\vec{E}_{incident}$ and transmission axis of filter



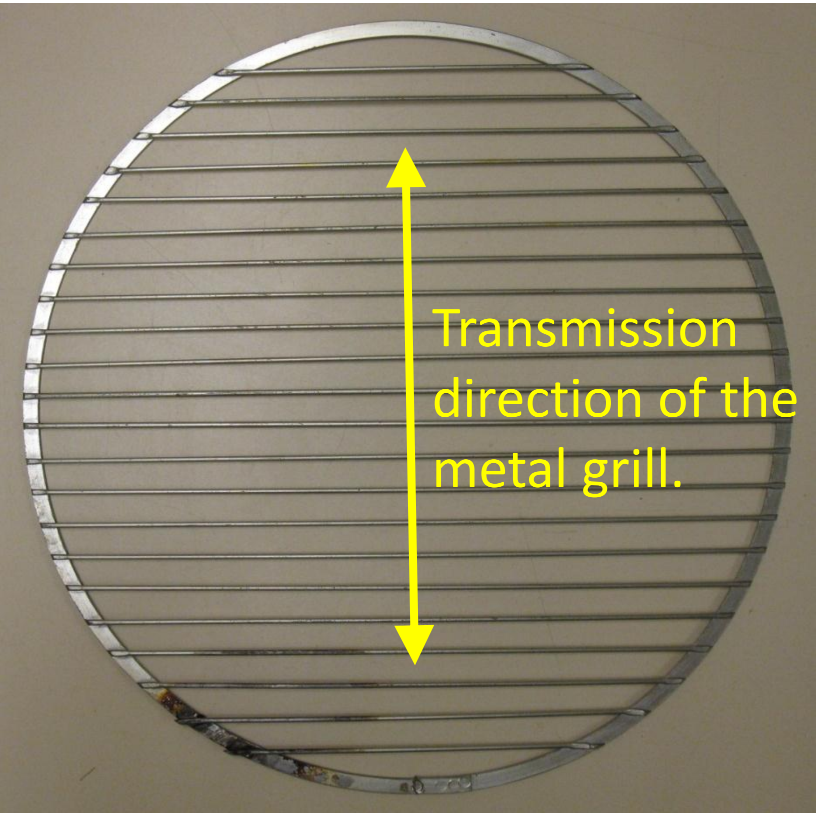




The electric dipole antenna of the microwave transmitter is vertical. Which orientation of the metal grill will allow the highest transmission of microwaves?

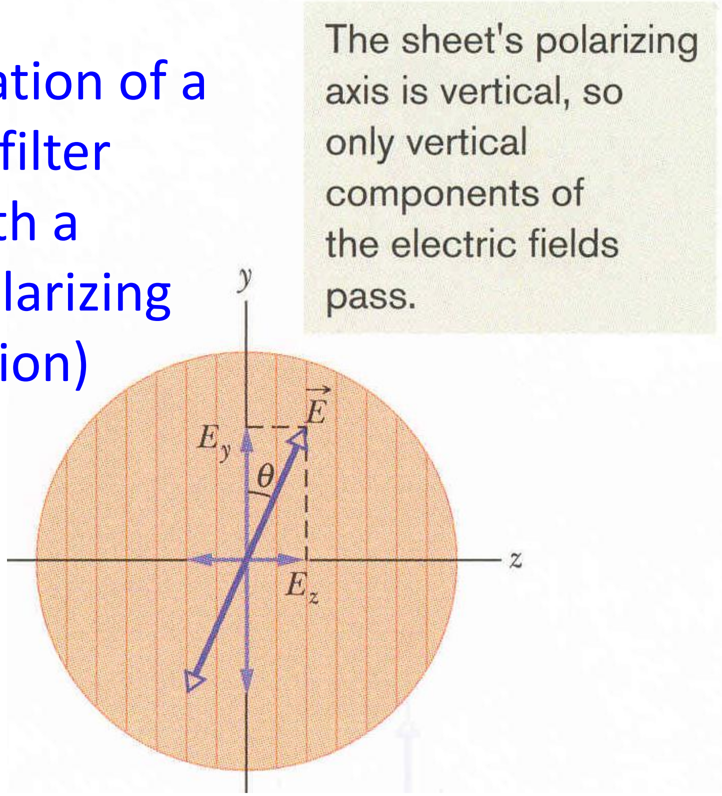
A. **A** B. **B**

C. Both will have about the same transmission.

The metal grill acts as a polarizing filter for microwaves.




Textbook representation of a polarizing filter (sheet) with a vertical polarizing (transmission) direction.




Be careful to distinguish the polarizing direction of a filter from its actual physical shape.

**It is desired to rotate the plane of polarization of a plane-polarized EM wave by 90° using ideal polarizing filters.**

$n$ filters $\Rightarrow$ $\theta = 90°/n$ angular spacing from filter to next

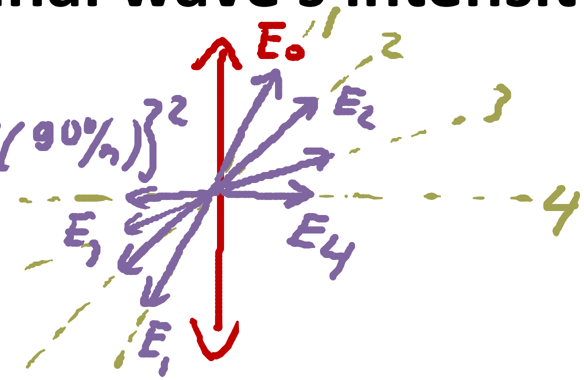
**What minimum number of such ideal polarizing filters, with equal angular spacing between successive filters, would be needed to do this if the intensity of the final transmitted wave is to be 50% or more of the original wave's intensity?**

after 1st filter: $I_{out,1} = I_{in,1} \cos^2(90°/n)$

after 2nd filter: $I_{out,2} = I_{in,2} \cos^2(90°/n) = I_{in,1}\{\cos^2(90°/n)\}^2$

after 4th filter: $I_{out,4} = I_{in,1} \cdot \{\cos^2(90°/n)\}^4$

$= I_{in,1} \cdot \underline{0.53}$




A. 2 B. 3 C. 4 D. > 5

E. It can't be done this way.