Recap I

Sources of phase shift between two waves: $\Delta \Phi_{A,B} = \Phi_B - \Phi_A$

1. Waves start out with different phases.
2. Path length difference: $\Delta \Phi = \frac{2\pi}{\lambda} (\Delta \text{path length})$
3. Waves travel through mediums with different index of refraction: $\lambda = \frac{\lambda_{\text{vacuum}}}{n}$
4. Phase shift upon reflection

<table>
<thead>
<tr>
<th>Reflection Type</th>
<th>Phase Shift</th>
</tr>
</thead>
<tbody>
<tr>
<td>slow $\rightarrow$ fast</td>
<td>$0$</td>
</tr>
<tr>
<td>$n_{\text{incident}} &gt; n_{\text{transm.}}$</td>
<td>$\pi$</td>
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Recap II

- Thin-Film Interference:

Total phase shift between reflected waves A and B results from:
- Path length difference = 2L
- Different wavelength in film: \( \lambda_f = \frac{\lambda_{\text{vacuum}}}{n_f} \)
- Phase shifts upon reflection

\[ 2L \text{ constructive, } m = (m + \frac{1}{2}) \frac{\lambda_{\text{vacuum}}}{n_{\text{film}}} \quad 2L \text{ destructive, } m = m \frac{\lambda_{\text{vacuum}}}{n_{\text{film}}} \]

\( m = 0, 1, 2, \ldots \)

Air \( n=1 \) → Coating \( n_f \) \( \xrightarrow{L} \) Glass \( n_g = 1.5 \)

Anti-reflective Coating on Glass:
- Choose \( L \) to get destructive interference between waves A and B for visible light:
  \[ 2L \text{ destructive, } m = (m + \frac{1}{2}) \frac{\lambda_{\text{vacuum}}}{n_{\text{film}}} \quad m = 0, 1, 2, \ldots \]
Today:

- Diffraction
  - Single slit
  - Circular aperture
  - Double slit (again)
What is the smallest object (finest detail) the human eye can resolve?

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A. ~1 mm
B. ~0.5 mm
C. ~0.05 mm
D. ~0.005 mm

~0.01° angular resolution

=) ~0.05 mm at near point distance of 25 cm
Diffraction:

Wavefronts are ‘bent’ near edges & apertures.

Example: Diffraction of wave passing through a narrow slit:

Recall: Huygens' Principle

All points on a wavefront act as point sources

⇒ Light emerges in all directions rather than just passing straight through the narrow slit.

⇒ Important for aperture/objec/slit of size ≤ λ.
Example: Single-slit diffraction:

Mask with slit (top view).

Key idea: waves from each part of the slit can interfere with others.

→ Interference pattern by single slit observed on screen.
Single slit diffraction:
Single-slit diffraction pattern
Interference Pattern from a Single Slit:

- Break slit into $N$ zones, each of width $a/N$
- Rays from different zones interfere on distant screen

Path length difference between two adjacent rays:

$$\Delta \text{path} = \frac{a}{N} \cdot \sin \theta$$

Phase difference between waves of adjacent rays:

$$\Delta \phi_{\text{adjacent rays}} = \frac{2\pi}{\lambda} \Delta \text{path} = \frac{2\pi}{\lambda} \frac{a}{N} \sin \theta$$
Side Note:

**Phasors**

- **To add two waves:**
  
  
  \[ E(t) = E_{\text{max}} \cos(\kappa x_0 - \omega t + \phi) \]

- \[ E_{\text{sum}} = 0 = E_1(t) + E_2(t) \]
2 turns:
\[ \Delta \phi_{\text{total}} = \sum \Delta \phi_{\text{adjacent}} \]
\[ E_{\text{total}} = 2 \cdot (2\pi) \]

\[ E_{\text{total}} = \sum \Delta \phi_{\text{adjacent}} \]
\[ E_{\text{total}} = 2\pi \]

E from individual ray

E_{\text{total}} (\theta = 0)

E_{\text{total}} (\theta)

Full circle!
Constructive Interference at highest intensity at $\theta = 0$

Dark Fringes (Intensity minima) each time the electric field vectors of the individual waves add up to zero:

$$\Rightarrow \text{ for } \phi_{\text{total}} = \sum_{i=1}^{N} \phi_{i, \text{adjacent ray}} = m (2\pi)$$

$$m = \pm 1, \pm 2, \pm 3, \ldots$$

but not $m = 0$

$$\Rightarrow \phi_{\text{total}} = N \cdot \phi_{\text{adjacent ray}}$$

$$= N \cdot \frac{2\pi}{\lambda} \frac{a}{N} \sin \theta = m (2\pi)$$

Minima for single slit diffraction:

$$a \sin \theta = m \lambda$$

$m = \pm 1, \pm 2, \pm 3, \ldots$ (but not $m = 0$)

width of slit
Also can find the single-slit diffraction intensity pattern: \( I(\theta) \propto (E_{\text{cutoff}})^2 \)

\[
I(\theta) = I_{\text{max}} \left( \frac{\sin \alpha}{\alpha} \right)^2
\]

where \( \alpha = \frac{\pi a}{\lambda} \sin \theta \)

Note: Wider slit (larger \( a \)) \( \rightarrow \) more, sharper intensity maxima and minima

Minima: \( \sin \theta_{\text{min}} = m \frac{\lambda}{a} \leq 1 \)
Single-slit diffraction pattern for different slit widths:

\[ \frac{\lambda}{a} = 1/1 \]

\[ \frac{\lambda}{a} = 1/2 \]

\[ \frac{\lambda}{a} = 1/4 \]

\[ \frac{\lambda}{a} = 1/8 \]
Diffraction of red laser beam on a Hole (Circular Aperture)
Diffraction by a circular Aperture

Interference of waves diffracted by the hole results in circular intensity pattern on the screen.

$$\sin \theta_1 = 1.22 \frac{\lambda}{a}$$

$$a = \text{diameter of hole}$$