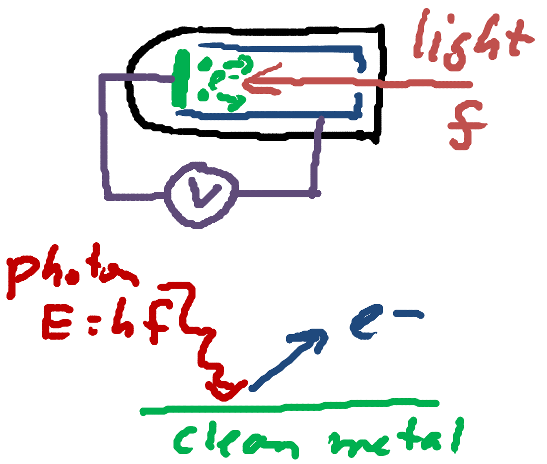
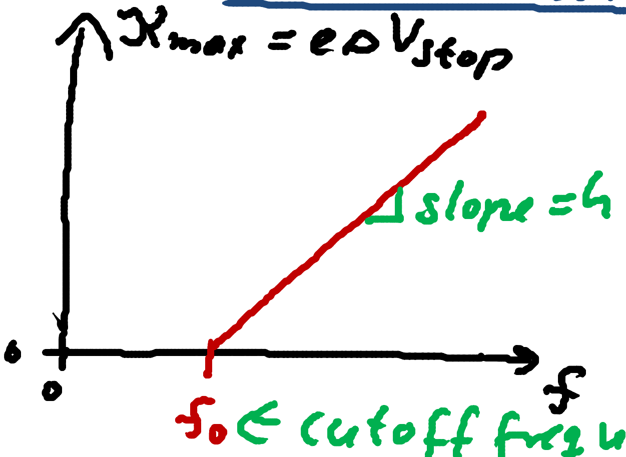
Recap I

Lecture 37

- Evidence of Photons: The Photoelectric Effect





f_0 ← cutoff frequency (depends on target)

data are not explained by classical EM wave model

⇒ Photon model: $$\boxed{E_{ph} = hf = K_{max} + \Phi}$$

max. kinetic energy of photoelectrons

work function of target material = minimum energy to remove an electron from the target material ≈ few eV

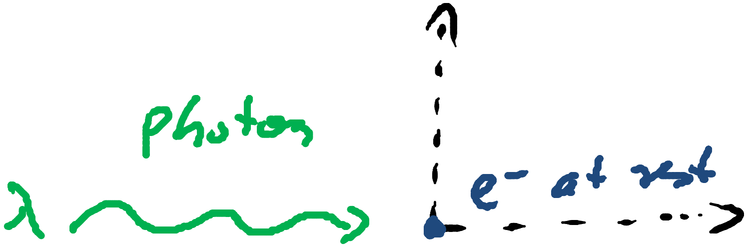
- Photon energies:
  - visible light: $700\,nm \lesssim \lambda \lesssim 400\,nm$
    $1.8\,eV \lesssim E_{ph} \lesssim 3.1\,eV$
  - ultraviolet: $400\,nm \lesssim \lambda \lesssim 10\,nm$
    $3.1\,eV \lesssim E_{ph} \lesssim 100\,eV$
  - x-rays: $\lambda \gtrsim 10\,nm$ ⇒ $E_{ph} \gtrsim 100\,eV$

Recap II

- Evidence of Photons: The Compton Effect

Collision of a photon with a free electron

Before collision:



$E_{ph} = \frac{hc}{\lambda}$

$p_{ph} = h/\lambda$

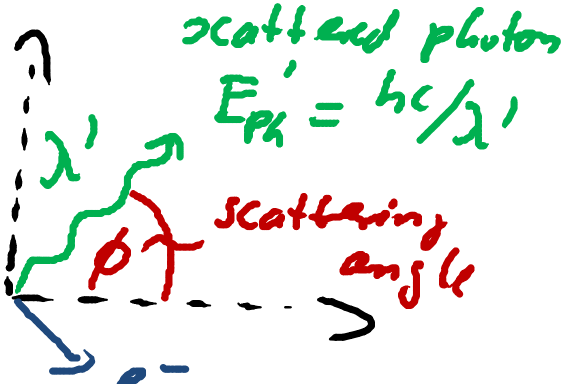
$x_{e^-,i} = 0$

$p_{e^-,i} = 0$

⇒ elastic collision (photon gives part of its energy to the free electron)

$\boxed{\lambda' > \lambda}$

After collision:



$E'_{ph} = hc/\lambda'$

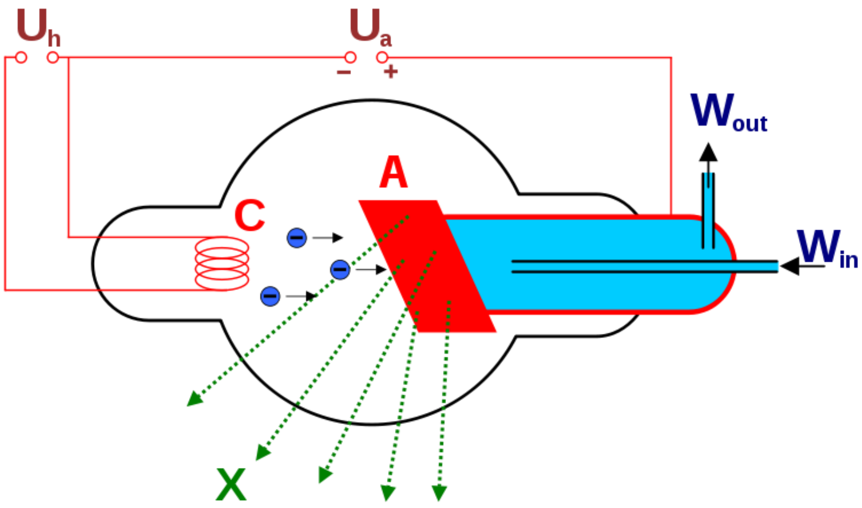
$x_{e^-,f} > 0$

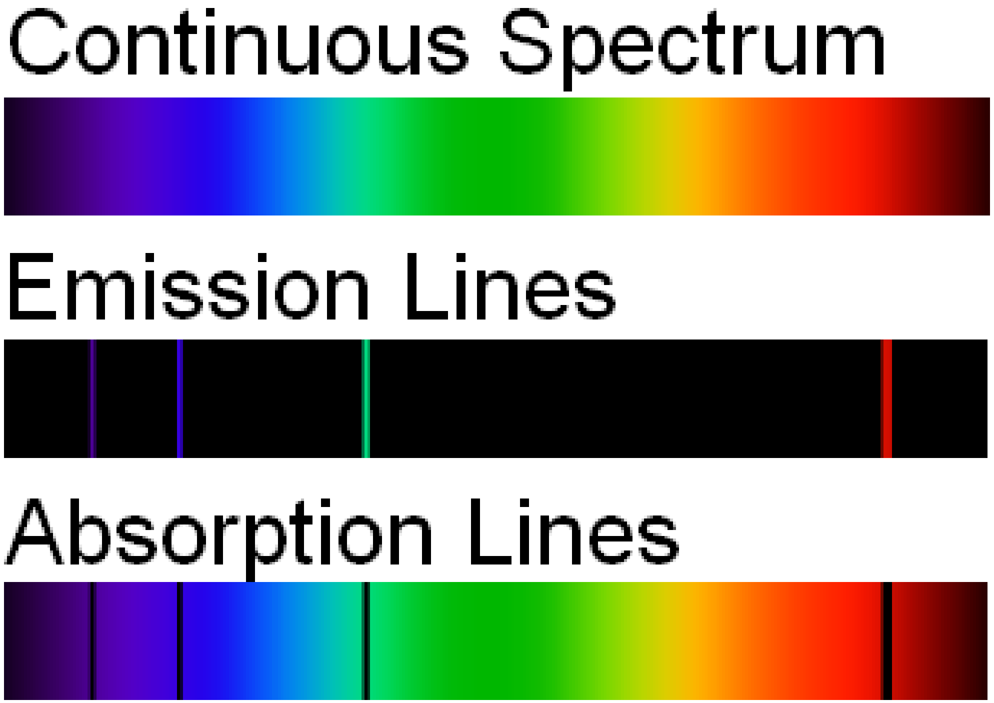
$|p_{e^-,f}| > 0$

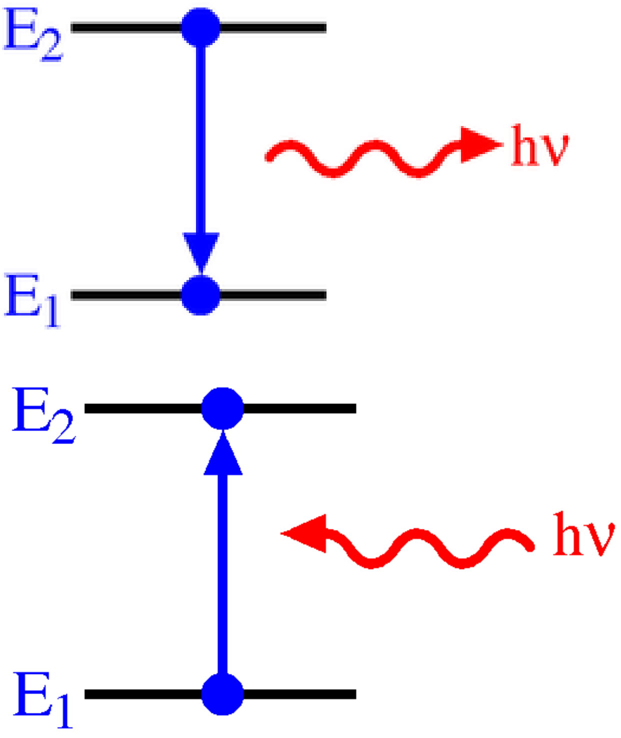
Key Idea: Photons carry energy and momentum, and obey energy and momentum conservation laws.

Today:

- **Photons:**
    - **X-ray production**
    - **Spectroscopy and spectra**







Compton Effect:

Conservation of energy and momentum in collision of a photon with a free electron gives correct value for wavelength λ' of scattered photon at angle ϕ:

scattering angle

$$\boxed{\lambda' - \lambda = \Delta\lambda = \frac{h}{m_e c}(1-\cos\phi)} \geq 0$$

shifted wavelength after scattering

initial wavelength

mass of electron

$$\Rightarrow \Delta\lambda(\phi = 0) = 0$$

$$\Delta\lambda(\phi = 180^\circ) = \Delta\lambda_{max} = \frac{2h}{m_e c} \approx \underline{0.0049\,\mathrm{nm}}$$

$\Rightarrow$ need x-rays to observe this small shift!

If one doubles the wavelength of the incoming light, the Compton shift Δλ …

A. … doubles

[✓ B.] … stays the same

C. … decreases by a factor ½

D. Something else

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0 c}(1-\cos\phi)$$

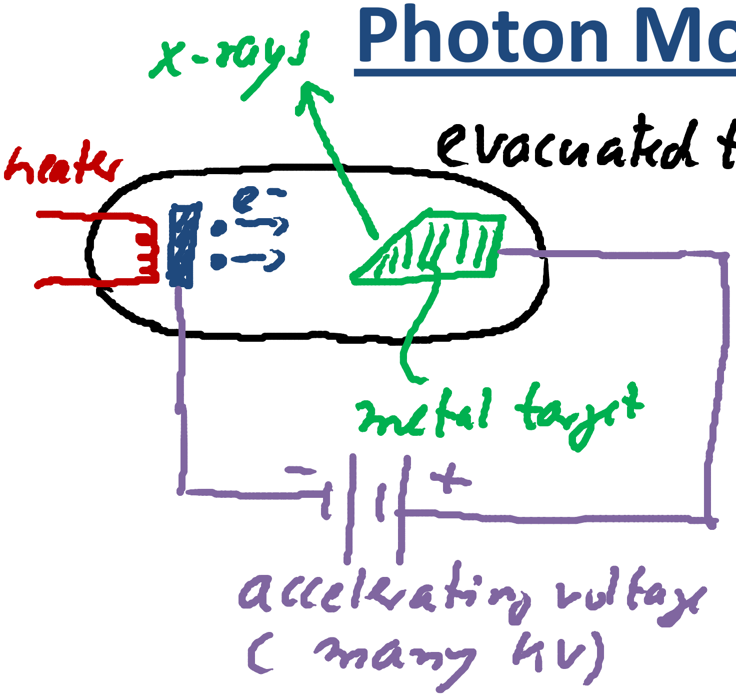
Why, in Compton scattering, would you expect $\Delta\lambda$ to be independent of the scatter material?

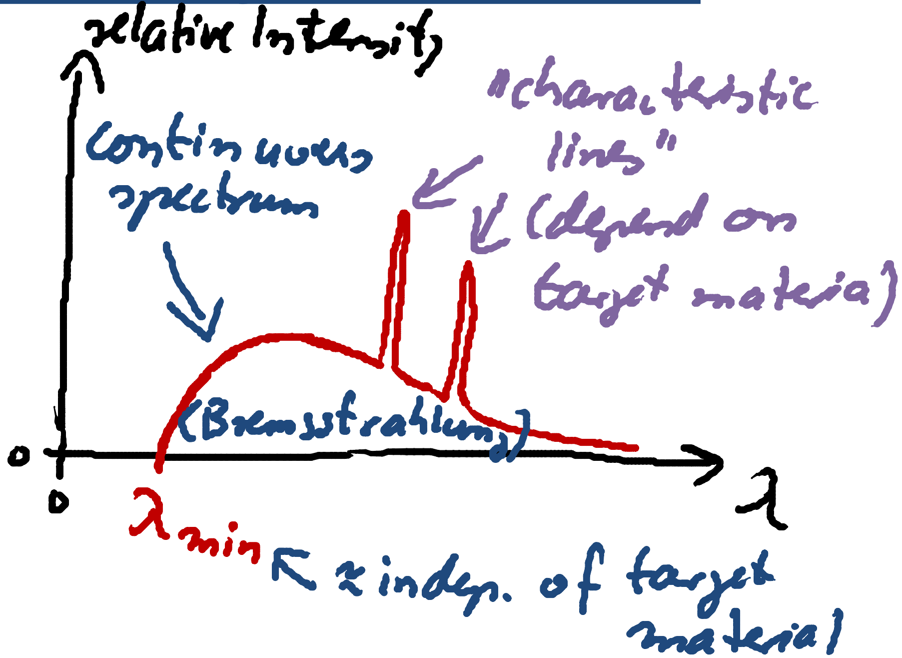
A. Because the equation says so

[✓ B.] Because the photon scatters of a free electron in the Compton effect

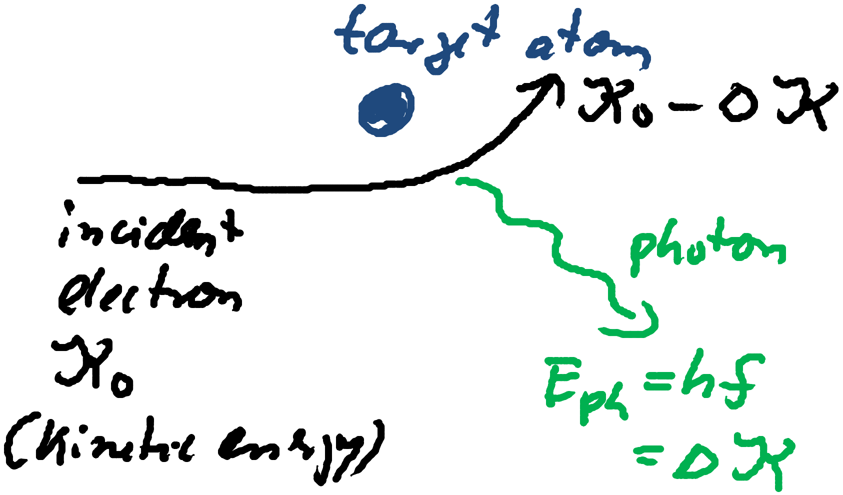
C. It is not independent of the scatter material

Photon Model for X-Ray Production:





⇒ Electron with keV kinetic energy collides (interact) with target atoms and may lose part (or all) of its energy by generating an x-ray photon.



⇒ for energy conservation
(neglect eV-scale energy from work function of target material here)

$$E_{photon} = hf = \frac{hc}{\lambda} = \Delta\mathcal{K} \le \mathcal{K}_0$$

$$\Rightarrow \boxed{\lambda \ge \frac{hc}{\mathcal{K}_0}}$$

} get continuous spectrum above λ_{min}

$$\boxed{\lambda_{min} = \frac{hc}{\mathcal{K}_0}}$$

minimum wavelength of continuous spectrum

Quantum Mechanics:

So far:

light → wave properties

light → particle like properties

⇒ Photons : $E_{ph} = hf$

⇒ energy of radiation is

quantized: $E_{rad} = \text{integer} \cdot E_{photon}$

now:

Lesson II:

Small particles → wave properties

at small scales → energy quantization

if particle is confined

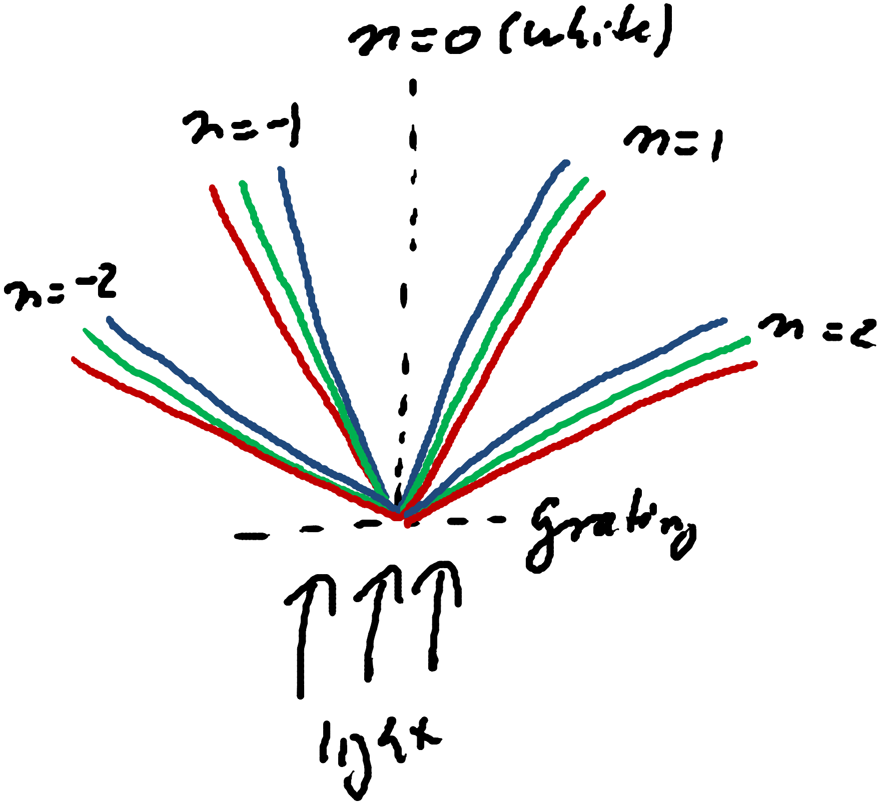
to a small volume

Example: The quantized atom

Spectroscopy:

What can be learned about a substance (atom) from the light it emits or absorbs?

⇒ Use diffraction grating to separate light into its component wavelengths:



Primary maxima:

$$\boxed{d \sin \theta_n = n \lambda}$$

$n = 0, \pm 1, \pm 2 \ldots$

where n = order number

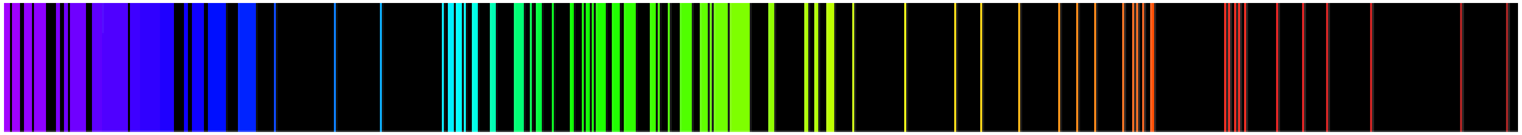
d = center-to-center spacing of slits

Continuous vs. Line Spectra

- **A glowing solid produces a continuous spectrum from the material as a whole.**
  - **Example:** filament in a light bulb

- **A glowing gas at low pressure produces a characteristic line spectrum of specific wavelengths doe to the emission form individual atoms or molecules**
  - **Example:** Emission spectrum of hydrogen

  - **Example:** Emission spectrum of iron



⇒ Radiation emitted by independent atoms shows sharp spectral lines

apply conservation of energy to the atom

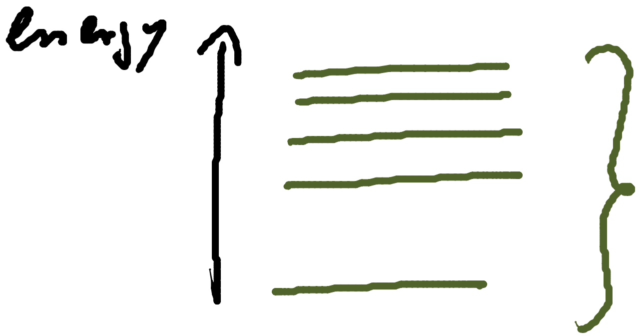
$$E_i = E_f + E_{photon}$$

$$\Rightarrow E_i - E_f = E_{photon} = hf$$

⇒ Key Idea: – Atoms exist in state of discrete quantized internal energy

– associate energy of emitted photons as difference between two quantized energy states of the atom!

- **Energy level diagram for a given atom:**

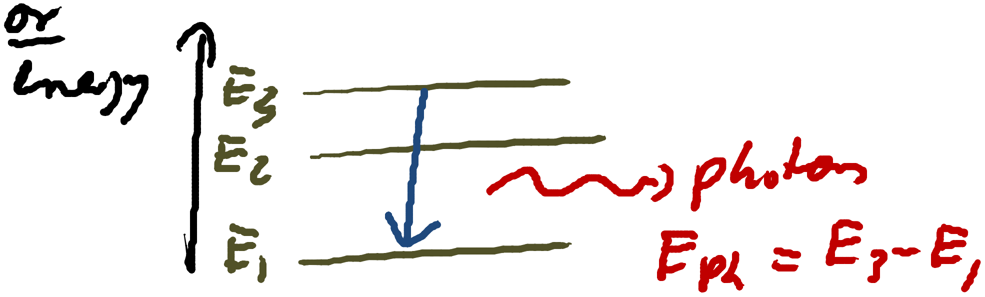


quantized energy levels (states) of a given atom ⇒ only these energy levels are possible

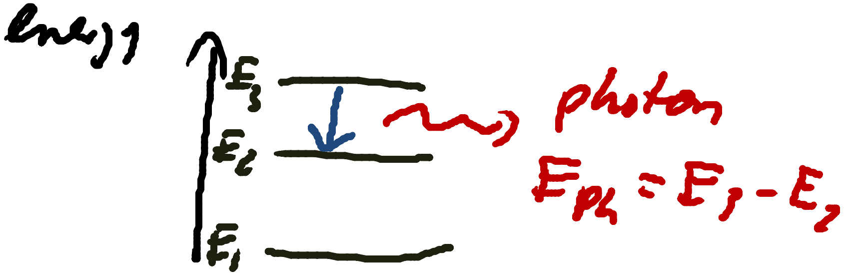
- **Photon emission:**

$E_{photon} = hf = E_i - E_f$

or

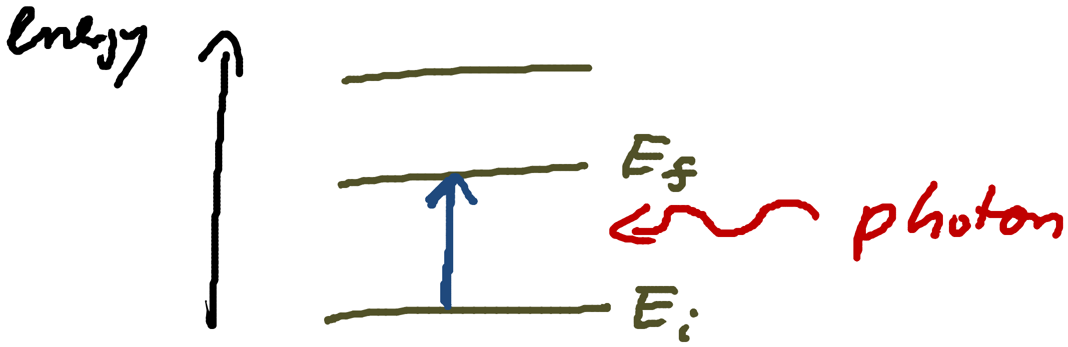


$E_{ph} = E_3 - E_1$



$E_{ph} = E_3 - E_2$

- **Photon absorption:**



$$E_{photon} = hf = E_f - E_i$$

Note: Photon is only absorbed, if

$$E_{photon} = E_{\substack{\text{some final state}\\\text{of atom}}} - E_{\substack{\text{initial state}\\\text{of atom}}}$$

Emission Spectrum for Hydrogen

Visible Balmer Series:

Discharge Lamp

J.J. Balmer (1885):

$$\frac{1}{\lambda_n} = -109700\text{cm}^{-1}\left(\frac{1}{n^2} - \frac{1}{2^2}\right)$$

$$n = 3,4,5\ldots$$

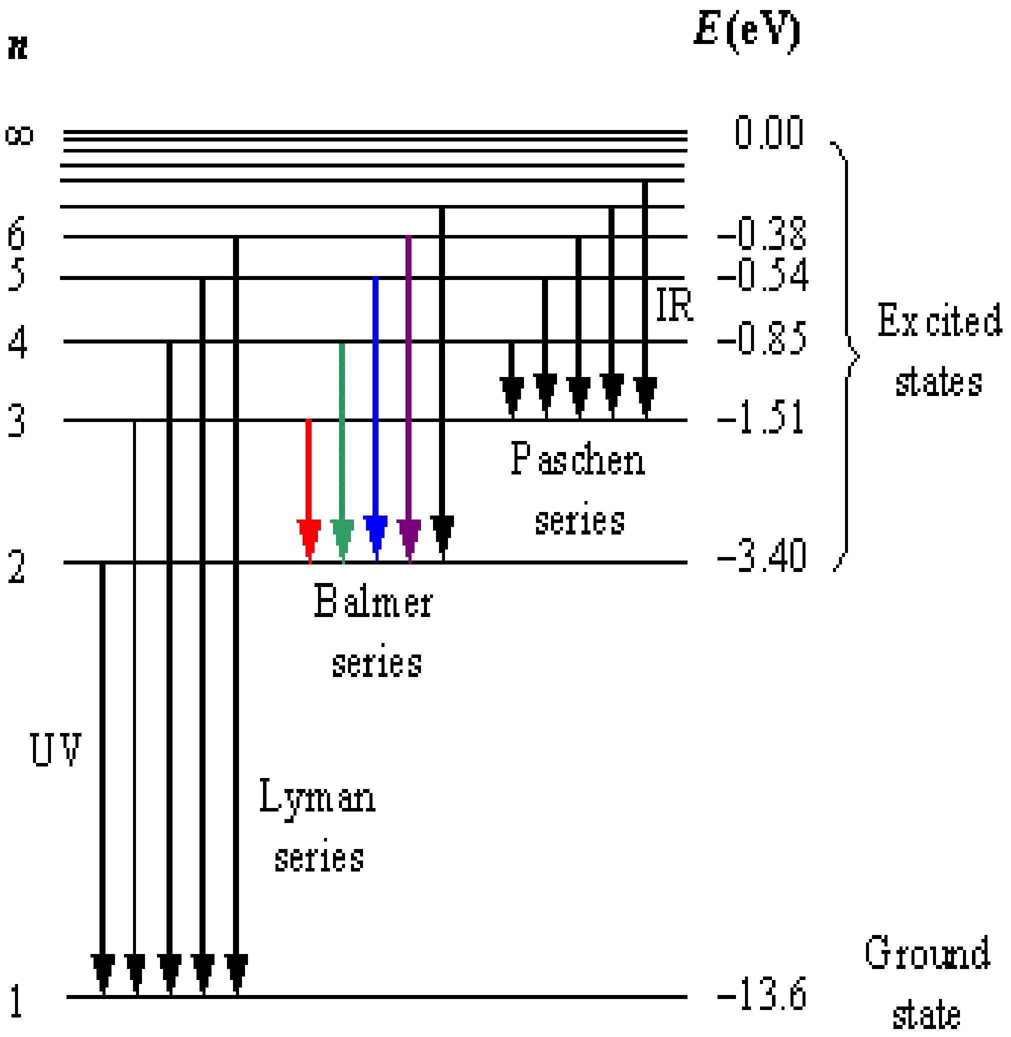
General empirical result:

$$E_{photon} = hf = \frac{hc}{\lambda} = E_i - E_f = -13.6\,\text{eV}\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)$$

$$n_i = 2,3,4\ldots$$

$$n_f = 1,2,3,\ldots < n_i$$

Energy Levels of the Hydrogen Atom



$$E_{\text{hydrogen, n}} = -13.6\,\text{eV}\,\frac{1}{n^2}$$

$$n = 1,2,3\ldots$$

$\Rightarrow$ **Energy levels of atoms are quantized!**

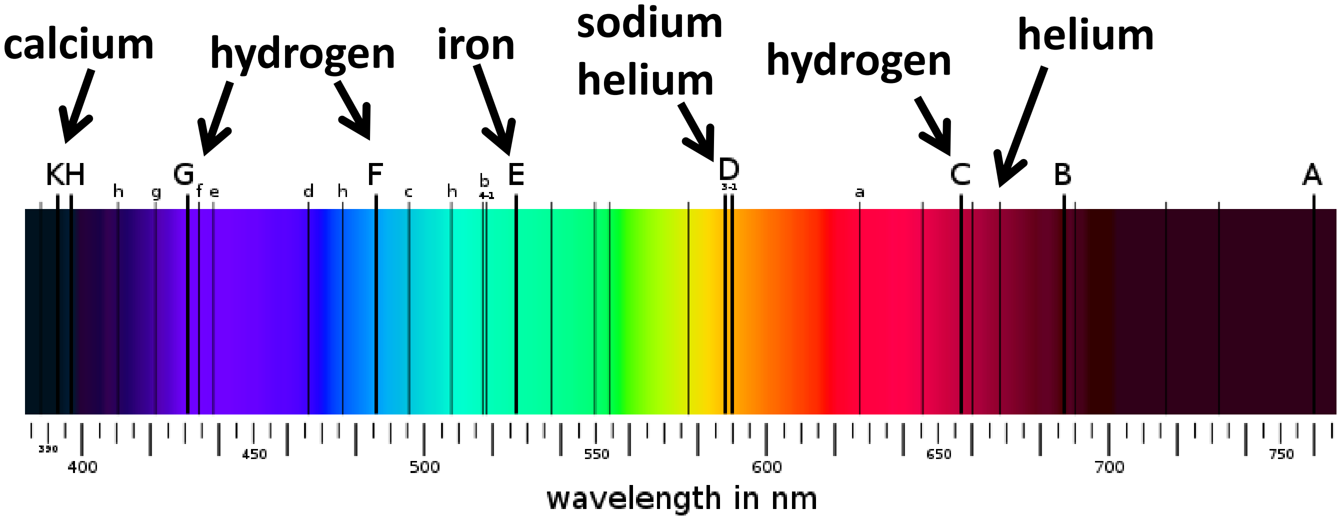
- $\Rightarrow$ Particles confined to small volume have quantized energies!

⇒ All atoms and molecules have characteristic (unique) energy level structures.

⇒ Atoms have **characteristic (unique) line spectra** in the gas phase.

⇒ Emission or absorption spectra can be used to check for the presence of certain atoms in a gas

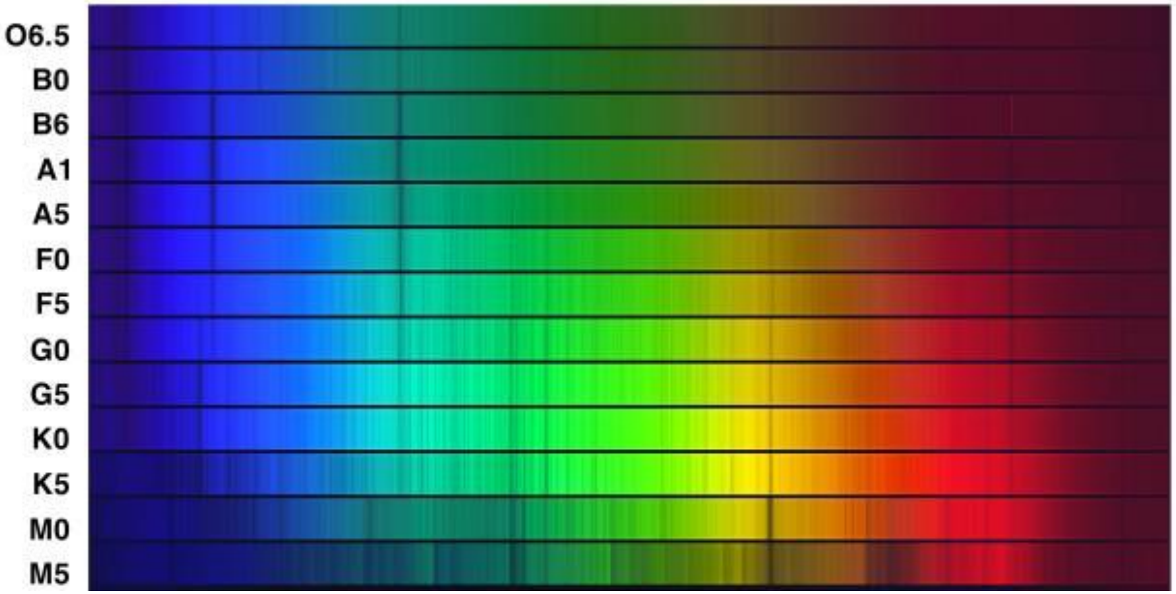
⇒ Example: **Absorption lines in the sun's continuous spectrum**

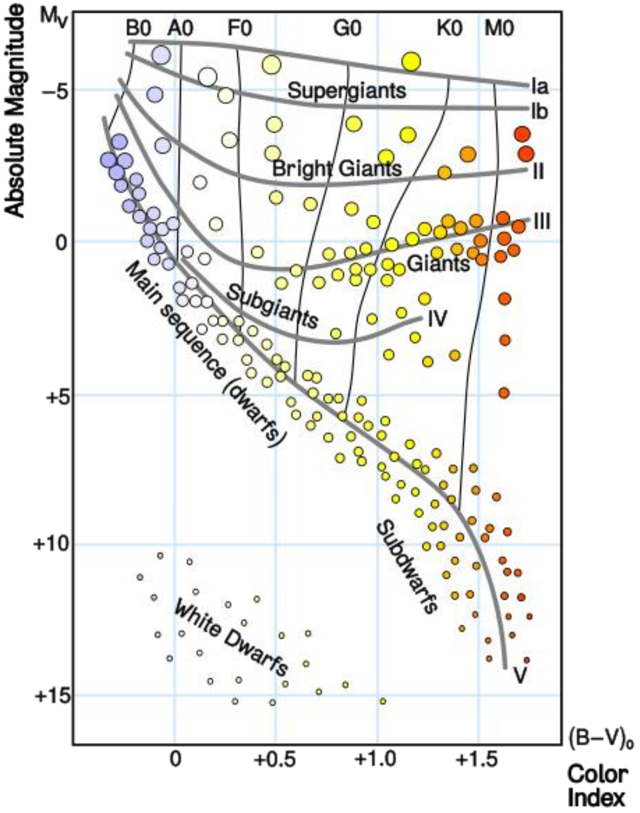


Dark lines in the solar spectrum are caused by absorption by chemical elements in the Solar atmosphere.

Stellar Classification

In astronomy, stellar classification is a classification of stars based on their spectral characteristics.





Class	Surface temperature[8] (kelvins)	Conventional color	Apparent color[9][10][11]	Mass[8] (solar masses)	Radius[8] (solar radii)	Luminosity[8] (bolometric)	Hydrogen lines	Fraction of all main-sequence stars[12]
O	≥ 33,000 K	blue	blue	≥ 16 $M_\odot$	≥ 6.6 $R_\odot$	≥ 30,000 $L_\odot$	Weak	~0.00003%
B	10,000–33,000 K	blue to blue white	blue white	2.1–16 $M_\odot$	1.8–6.6 $R_\odot$	25–30,000 $L_\odot$	Medium	0.13%
A	7,500–10,000 K	white	white to blue white	1.4–2.1 $M_\odot$	1.4–1.8 $R_\odot$	5–25 $L_\odot$	Strong	0.6%
F	6,000–7,500 K	yellowish white	white	1.04–1.4 $M_\odot$	1.15–1.4 $R_\odot$	1.5–5 $L_\odot$	Medium	3%
G	5,200–6,000 K	yellow	yellowish white	0.8–1.04 $M_\odot$	0.96–1.15 $R_\odot$	0.6–1.5 $L_\odot$	Weak	7.6%
K	3,700–5,200 K	orange	yellow orange	0.45–0.8 $M_\odot$	0.7–0.96 $R_\odot$	0.08–0.6 $L_\odot$	Very weak	12.1%
M	≤ 3,700 K	red	orange red	≤ 0.45 $M_\odot$	≤ 0.7 $R_\odot$	≤ 0.08 $L_\odot$	Very weak	76.45%