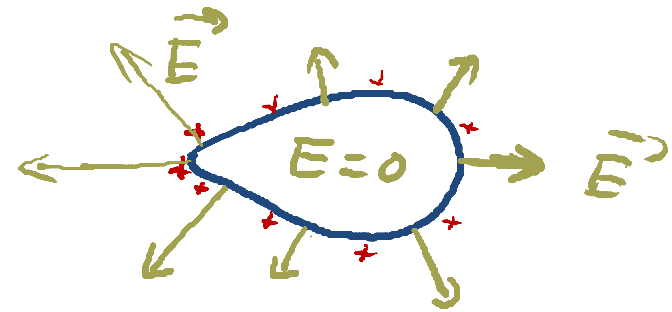
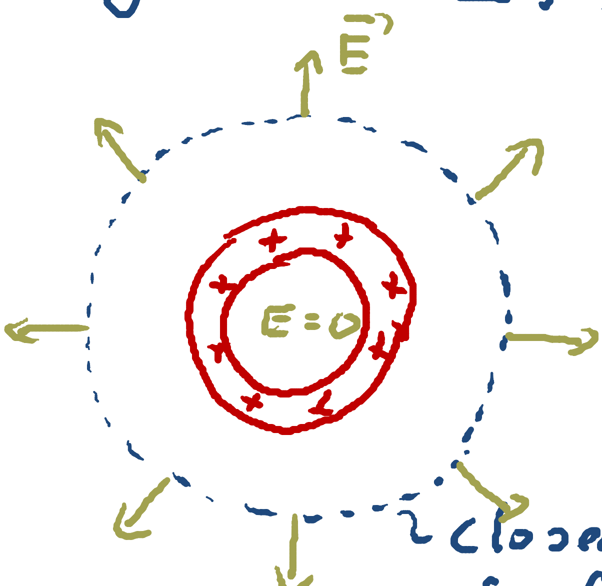
Recap

Lecture 6

- Conductors in Electrostatic Equilibrium:
  - $\vec{E} = 0$ inside
  - Excess charge on surface only, more concentrated at regions of great curvature
  - $\vec{E}$ at surface $\perp$ to surface



- Gauss' Law for Electric Fields:



closed, gaussian surface (3D!)

$$Q_{\text{net, inside closed gaussian surface}} = \varepsilon_0 \, \Phi_{\text{net, through closed surface}}$$

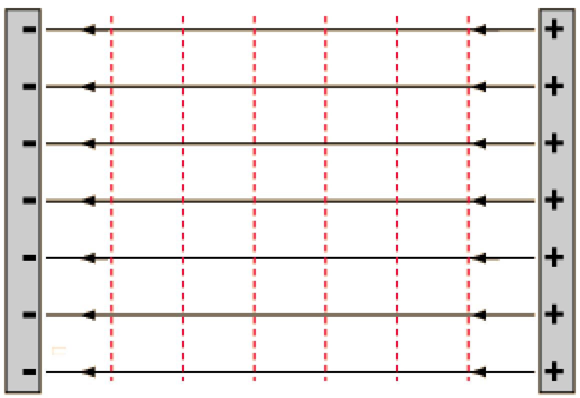
$$\Phi_{net} = \sum_i E_i \, \Delta A_i \cos\theta_i = \sum_i E_{\perp,i} \, \Delta A_i = \oint \vec{E} \cdot d\vec{A}$$

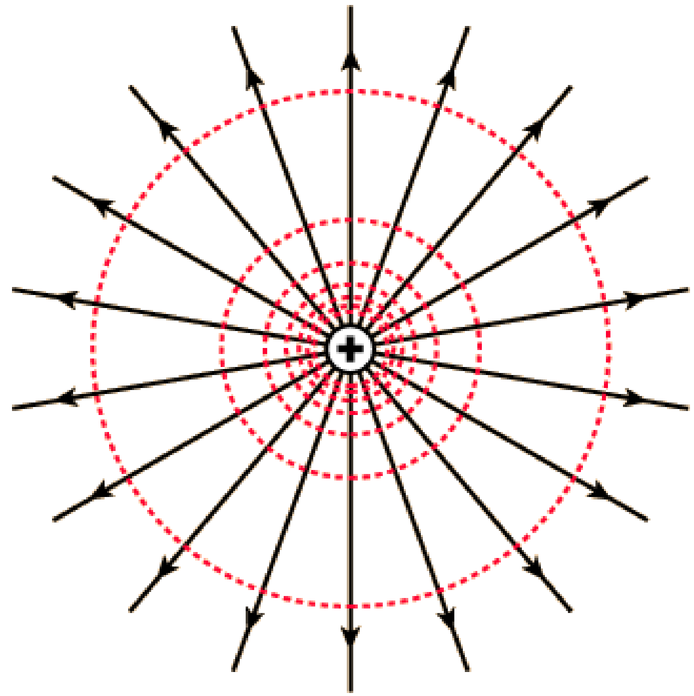
component of $\vec{E} \perp$ to surface

angle between vector $\vec{A}$ normal ($\perp$) to surface and $\vec{E}$

Today:

- Electric potential energy and potential





Gauss: Q enclosed $\propto$ # of "field lines" crossing

Consider a rectangular Gaussian surface surrounding a dipole that has 16 field lines emanating from its positively charged end.

If you move the Gaussian rectangle around (anywhere in the plane), the field line flux through the rectangle:

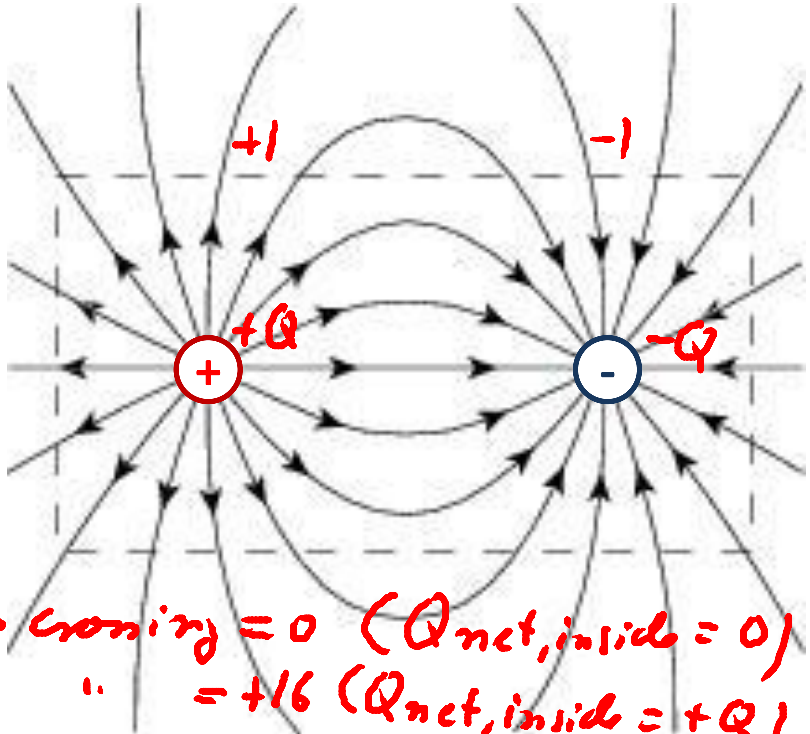
A. Always remains zero

B. Varies between -32 and +32.3

C. Varies between -16 and +16

D. Is -16, zero, or 16

E. Other



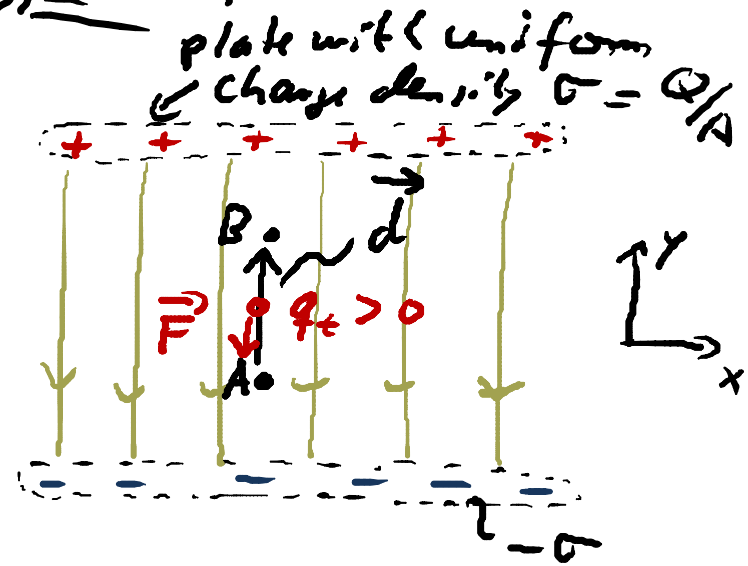
if both charges inside ⇒ # of field lines crossing = 0 ($Q_{net,inside} = 0$)

if only + charge inside ⇒ " " " " " = +16 ($Q_{net,inside} = +Q$)

if only − charge inside ⇒ " " " " " = −16 ($Q_{net,inside} = -Q$)

Electric Potential Energy: $\mathcal{U}_{el}$

A) Consider small positive test charge ($q_t > 0$) in a uniform electric field, which moves from point A to point B



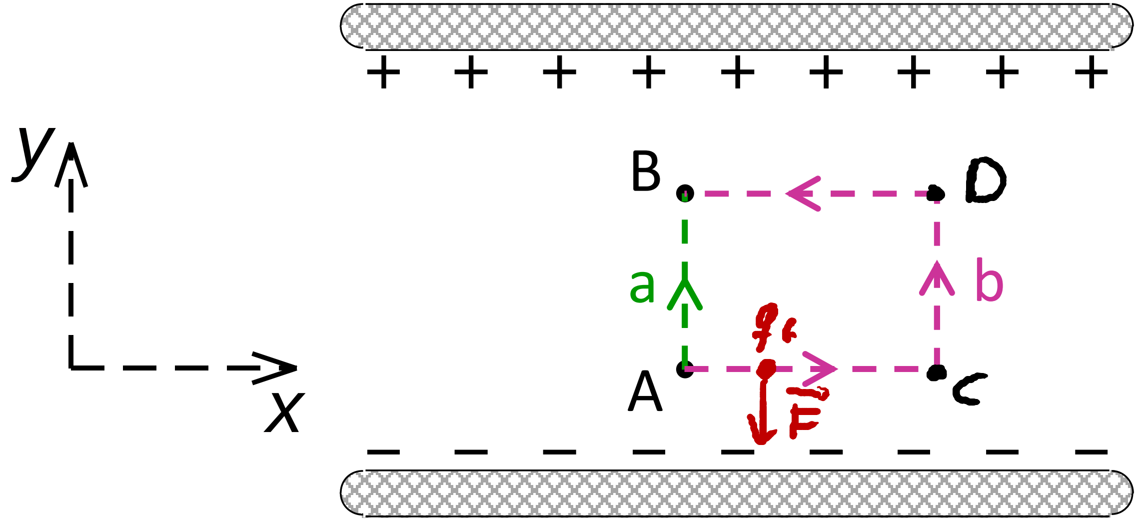
$\Rightarrow$ Work done by electrostatic force on test charge:

$$W_{on\, q_t} = \vec{F}_{el} \cdot \vec{d} = F_{el}\, d \cos 180^\circ = -q_t E d$$

$$= -q_t E (y_B - y_A)$$

displacement vector from point A to point B

If the charge is moved along path b how does the work done by the electric force compare with that done when the charge is moved along path a?



A. $W_b < W_a$

B. $W_b = W_a$

C. $W_b > W_a$

$$W_b = W_{A\to C} + W_{C\to D} + W_{D\to B}$$

$$= \underbrace{0}_{\cos 90^\circ = 0} + q_t E d \cos 180^\circ + 0$$

$$= -q_t E d = W_a$$

$\Rightarrow$ Work by electrostatic force is path independent!

⇒ Electrostatic force is a conservative force!

⇒ can define electric potential energy:

for example above:

$\Delta U_{el,A\to B}$

$= -W_{el,A\to B}$

$= +q_t E(y_B - y_A)$

> 0

$$\boxed{\Delta U_{el,\, of \text{ some charge } q_t} = U_{el,f} - U_{el,i} = -W_{el \text{ on charge } q_t \text{ by electrostatic force, } i\to f}}$$

Δ ↑ change!

(change in electric potential energy of charge q_t) = − (work done by electrostatic force acting on charge q_t while it moves from initial to final point)

⇒ recall from P2207:

$$\Delta E_{mech} = \Delta K + \Delta U = W_{non.\,cons.\,force}$$

⇒ if we choose $U_{el,i} = 0$ at infinity

$$\Rightarrow \boxed{U_f = -W_{el \text{ on } q_t,\, \infty\to f}}$$

Electric potential: V ("Potential", "Voltage")

potential change defined as:

$$\boxed{\Delta V = V_f - V_i = \frac{\Delta U_{el,i\to f}}{q_t} = -\frac{W_{el\,on\,q_t,i\to f}}{q_t}}$$

$\uparrow$ change!

for example above:

$$\Delta V_{A\to B} = \frac{\Delta U_{el,A\to B}}{q_t}$$

$$= +E(y_B - y_A)$$

(electric potential difference between two points) = (change in electric potential energy per unit charge between the two points)

Units? $[V] = \frac{[U]}{[q]} = \frac{J}{C} = \underline{\underline{\text{Volt}}} = V$

$\Rightarrow [E] = \frac{N}{C} = \frac{J}{m}\frac{1}{C} = \underline{\underline{\frac{V}{m}}}$

<u>Note:</u> ΔV is independent of test charge q_t!

$\Rightarrow$ <u>Property of an electrostatic field!</u>

i.e. potential difference ΔV exists between any two points in an electrostatic field

$\Rightarrow$ if we choose $V_i = 0$ at infinity

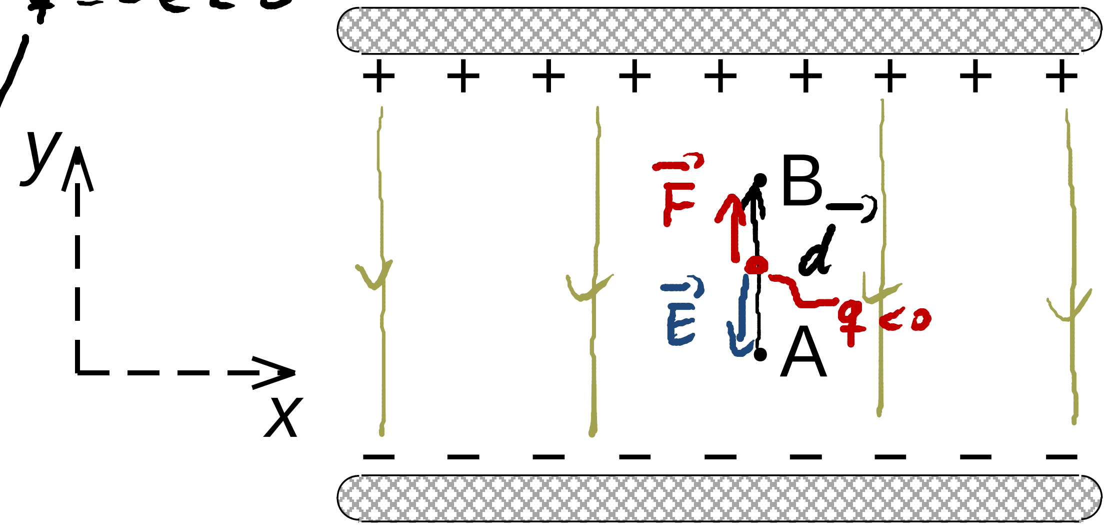
$$\boxed{V_f = V = \frac{U_{el,f}}{q_t} = -\frac{W_{el\ on\ q_t,\ \infty \to f}}{q_t}}$$

Electric potential

$\Rightarrow$ Potential is a <u>scalar</u>, not a vector, and can be 0, positive or negative!

$q = -e < 0$

There is a uniform electric field between the plates. An electron is moved from point A to point B.



Which of the following is true?

$W_{el,A\to B} = \vec{F}\cdot\vec{d}$
$= |F| d \cos 0° = eEd > 0$
$\Rightarrow \Delta U_{el,A\to B} = -W_{el,A\to B}$
$= -eEd < 0$
$\Rightarrow \Delta V_{A\to B} = V_B - V_A = \frac{\Delta U_{el}}{q_e}$
$= \frac{-eEd}{(-e)} = Ed > 0$

A. $U_{e,B} - U_{e,A} > 0$ and $V_B - V_A > 0$

B. $U_{e,B} - U_{e,A} < 0$ and $V_B - V_A < 0$

C. $U_{e,B} - U_{e,A} > 0$ and $V_B - V_A < 0$

D. $U_{e,B} - U_{e,A} < 0$ and $V_B - V_A > 0$

E. None of the above.

Directions:

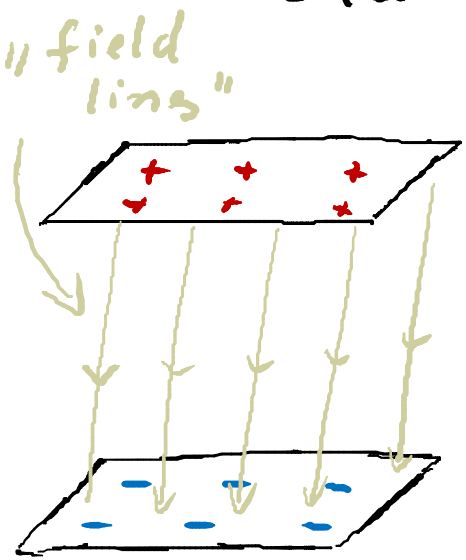
- $\vec{E}$ points away from positive charge
- Electric potential V increases with decreasing distance to positive charge

  $V_B > V_A \Rightarrow \Delta V_{A\to B} > 0$
- $\vec{E}$ always points in direction of decreasing potential!

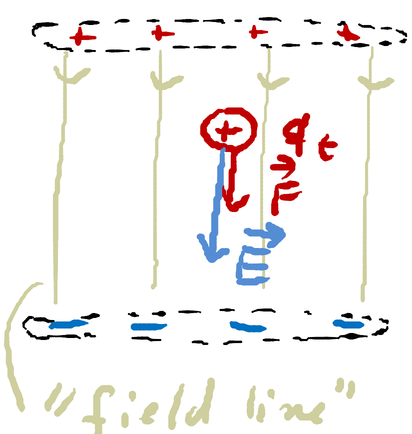
but: $\Delta U_{A\to B}$ = change in potential energy of test charge can be positive or negative

- for $q_t > 0$: $\Delta U_{A\to B} = q_t \cdot \Delta V_{A\to B} > 0$
- for $q_t < 0$: $\Delta U_{A\to B} = q_t \cdot \Delta V_{A\to B} < 0$

Example: Uniform Electric Field



side view:



Properties of test charge q_t		Properties of point in electric field
Electric force $\vec{F}_{el} = q_t \vec{E}$	per unit charge $\longrightarrow$	Electric field $\vec{E} = \dfrac{\vec{F}_{el\ on\ q_t}}{q_t}$