Recap

Lecture 6

- **Conductors in Electrostatic Equilibrium**:
  - \( \vec{E} = 0 \) inside
  - Excess charge on surface only, more concentrated at regions of great curvature
  - \( \vec{E} \) at surface \( \perp \) to surface

- **Gauss' Law for Electric Fields**:

  \[ Q_{\text{net, inside closed Gaussian surface}} = \vec{E} \cdot \vec{\Phi}_{\text{net, through closed surface}} \]

  \[ \Phi_{\text{net}} = \sum E_i \cdot \vec{O}A_i \cos \Theta_i = \sum E_i \cdot \vec{O}A_i = \oint E \cdot d\vec{A} \]

  Angle between vector \( \vec{A} \) normal (\( \perp \)) to surface and \( \vec{E} \).
Today:

- Electric potential energy and potential
Consider a rectangular Gaussian surface surrounding a dipole that has 16 field lines emanating from its positively charged end.

If you move the Gaussian rectangle around (anywhere in the plane), the field line flux through the rectangle:

A. Always remains zero
B. Varies between -32 and +32.3
C. Varies between -16 and +16
D. Is -16, zero, or 16
E. Other

if both charges inside =) # of field line crossing = 0 (Q_{net,inside} = 0)
if only + charge inside =) " " " " " = +16 (Q_{net,inside} = +Q)
if only - charge inside =) " " " " " = -16 (Q_{net,inside} = -Q)
Electric Potential Energy: $U_{el}$

Consider small positive test charge ($q_t > 0$) in a uniform electric field, which moves from point A to point B.

$\Rightarrow$ Work done by electrostatic force on test charge:

$$W_{onq_t} = \overrightarrow{F_{el}} \cdot \overrightarrow{d} = F_{el} d \cos 180^\circ = -q_t E d$$

displacement vector from point A to point B

$$= -q_t E (y_B - y_A)$$
If the charge is moved along path b how does the work done by the electric force compare with that done when the charge is moved along path a?

A. $W_b < W_a$

B. $W_b = W_a$

C. $W_b > W_a$

\[
W_b = W_{A\rightarrow C} + W_{C\rightarrow D} + W_{D\rightarrow B}
\]
\[
= 0 + qE \cdot d \cos 180^\circ + 0
\]
\[
= -qE \cdot d = W_a
\]

\]
Electrostatic force is a conservative force!

can define electric potential energy:

\[ \Delta U_{el, of} = U_{el, f} - U_{el, i} = -W_{el} \text{ on charge } q_6 \]

by electrostatic force, \( i \rightarrow f \)

\[ \begin{array}{c}
\text{(change in electric potential energy of charge } q_6) \\
\text{work done by electrostatic force acting on charge } q_6 \text{ while it moves from initial to final point}
\end{array} \]

\[ \Rightarrow \text{ recall from P2207:} \]

\[ \Delta E_{\text{mech}} = \Delta K + \Delta U = W_{\text{non-cons. forces}} \]

\[ \Rightarrow \text{ if we choose } U_{el, i} = 0 \text{ at infinity} \]

\[ U_f = -W_{el} \text{ on } q_6, \infty \rightarrow f \]
Electric potential: $V$ ("Potential", "Voltage")

Potential change defined as:

$$
\Delta V = V_f - V_i = \Delta \frac{U_{el, i \rightarrow f}}{q} = \frac{-W_{el, i \rightarrow f}}{q}
$$

For example:

$$
\Delta V_{A \rightarrow B} = \Delta \frac{U_{el, A \rightarrow B}}{q} = z + E(\gamma_0 - \gamma_A)
$$

Electric potential difference between two points:

$$
(\text{electric potential difference between two points}) = (\text{change in electric potential energy per unit charge between the two points})
$$

Unit: $\frac{\text{VJ}}{\text{C}} = \frac{\text{VU}}{\text{C}} = \frac{2}{C} = \text{Volt} = V
$$

$$
\Rightarrow \left[ \text{EJ} \right] = \frac{N}{C} = \frac{\text{V}}{\text{C}} = \frac{\text{V}}{\text{m}}
$$

Note: $\Delta V$ is independent of test charge $q$?

$\Rightarrow$ Property of an electrostatic field

i.e. potential difference $\Delta V$ exist between any two points in an electrostatic field
if we choose $V_i = 0$ at infinity

$$V_f = V = \frac{U_{el, f}}{q} = -\frac{W_{el on q, \omega \to f}}{q}$$

Electric potential

Potential is a scalar, not a vector, and can be 0, positive or negative?
There is a uniform electric field between the plates. An electron is moved from point A to point B.

Which of the following is true?

\[ W_{e, A \rightarrow B} = \mathbf{F} \cdot d = |\mathbf{F}|d \cos 0^\circ = eE d > 0 \]

\[ \Rightarrow D U_{e, A \rightarrow B} = -W_{e, A \rightarrow B} = -eE d < 0 \]

\[ \Rightarrow \Delta V_{A \rightarrow B} = V_B - V_A = \frac{D U_{e}}{q} = -e \frac{E d}{-e} = E d > 0 \]

A. \( U_{e,B} - U_{e,A} > 0 \) and \( V_B - V_A > 0 \)

B. \( U_{e,B} - U_{e,A} < 0 \) and \( V_B - V_A < 0 \)

C. \( U_{e,B} - U_{e,A} > 0 \) and \( V_B - V_A < 0 \)

D. \( U_{e,B} - U_{e,A} < 0 \) and \( V_B - V_A > 0 \)

E. None of the above.
Direction:

- $\vec{E}$ points away from positive charge
- Electric potential $V$ increases with decreasing distance to positive charge
  \[ V_B > V_A \Rightarrow \Delta V_{A\rightarrow B} > 0 \]
- $\vec{E}$ always points in direction of decreasing potential!

but: $\Delta V_{A\rightarrow B} =$ change in potential energy of test charge can be positive or negative

- For $q_\epsilon > 0$: $\Delta V_{A\rightarrow B} = q_\epsilon \cdot \Delta V_{A\rightarrow B} > 0$
- For $q_\epsilon < 0$: $\Delta V_{A\rightarrow B} = q_\epsilon \cdot \Delta V_{A\rightarrow B} < 0$
Example: Uniform Electric Field

Properties of test charge $q_t$

- Electric force $\mathbf{F}_{el} = q_t \mathbf{E}$

Properties of point in electric field per unit charge

- Electric field $\mathbf{E} = \frac{\mathbf{F}_{el}}{q_t}$