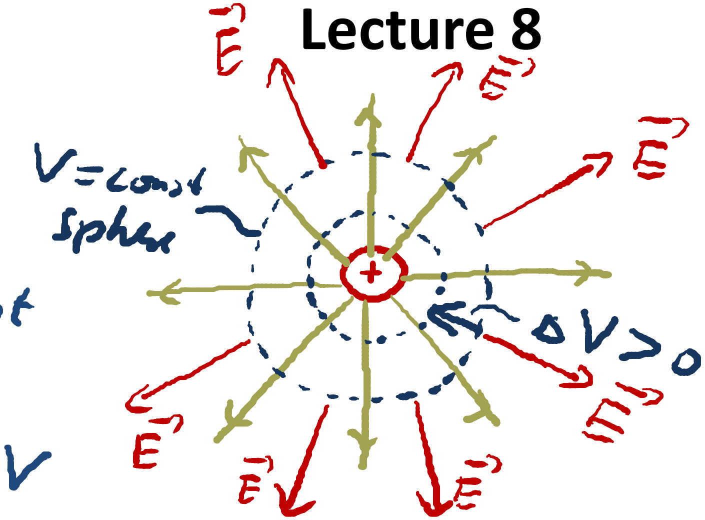
Recap

Lecture 8



- Equipotential Surface
  - V = const on surface
  - $\vec{E}$ is ⊥ to surface at each point
  - $\vec{E}$ always points in direction of maximum decrease in potential V
  - V = const inside conductor in electrostatic equilibrium
- Electric field $\vec{E}$ ⟺ electric potential V

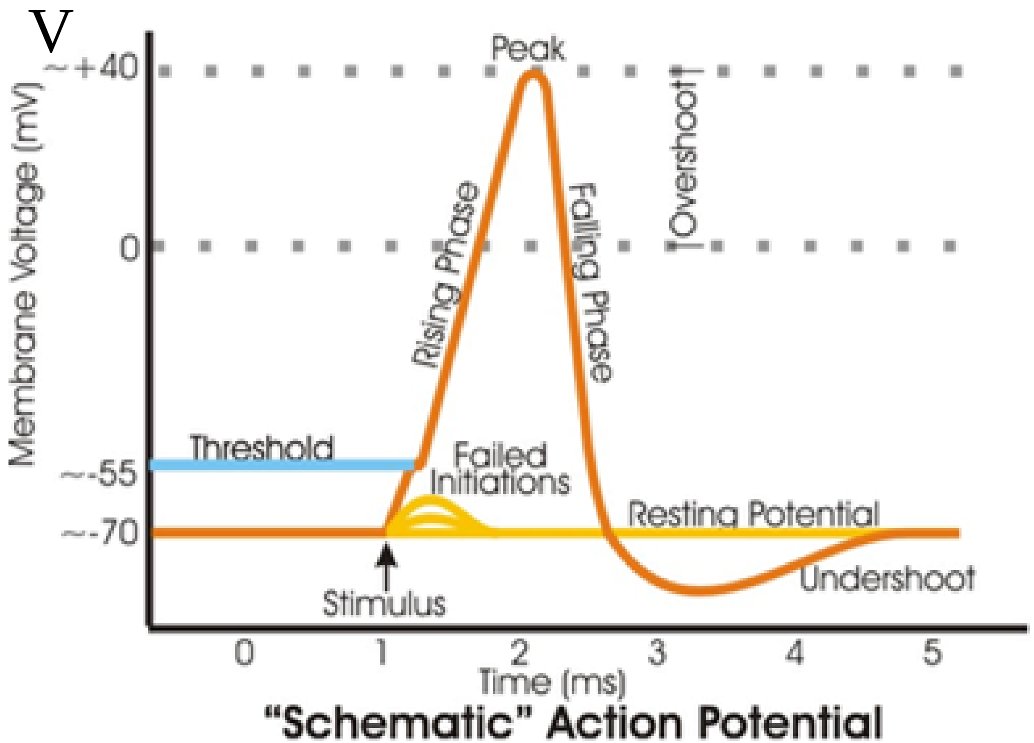
$$\Delta V = V_f - V_i = -\int_i^f \vec{E}\cdot d\vec{s} = \underbrace{-\vec{E}\cdot\Delta\vec{s} = -E\,\Delta s\cos\theta}_{\text{for uniform field}}$$

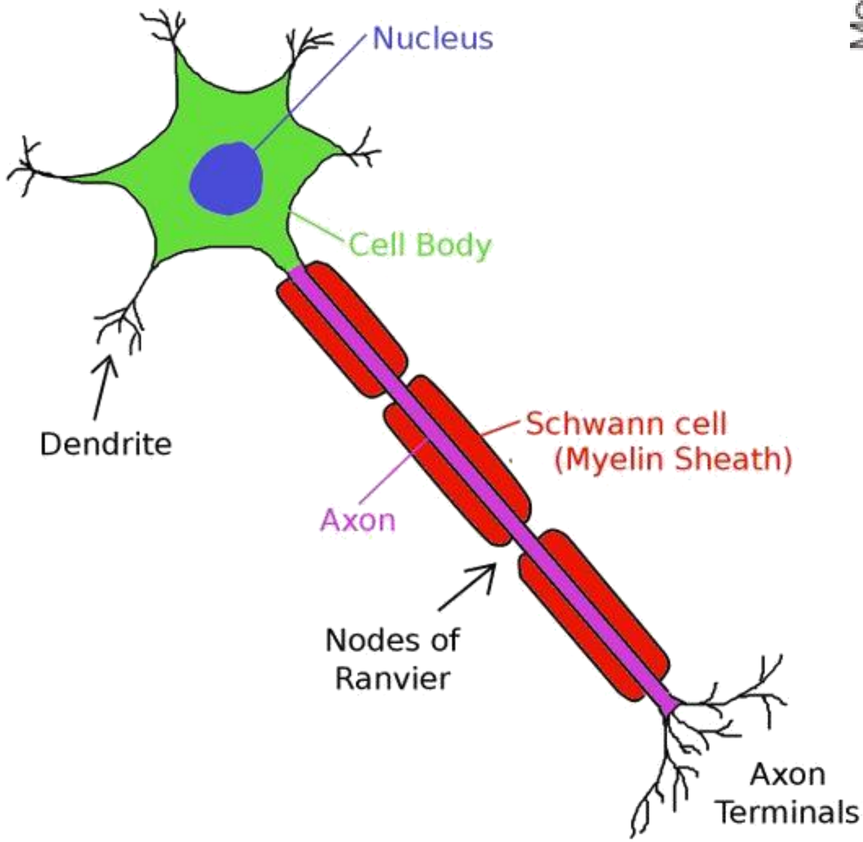
$$E_x = E_{\substack{\text{component}\\ \text{along } x}} = -\left.\frac{dV}{dx}\right|_{y,z=\text{const}} = -\left(\text{rate at which the electric potential changes with distance along } x\right)$$

$$E_y = -\left.\frac{dV}{dy}\right|_{x,z=\text{const}} \qquad E_z = -\left.\frac{dV}{dz}\right|_{x,y=\text{const}}$$

Transmission of Nerve Impulses

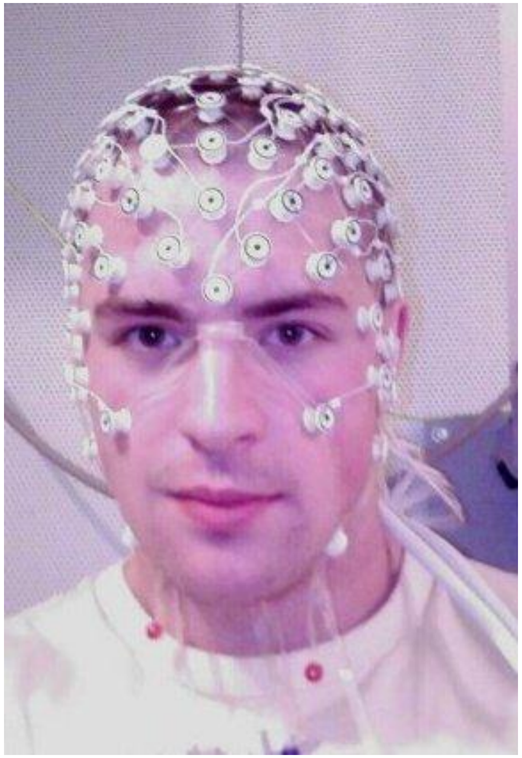
- **Axon: transmits nerve impulses**
- **In resting state: -70 mV potential of fluid inside relative to fluid outside (negative ions on inner surface of membrane and positive ions on outside)**



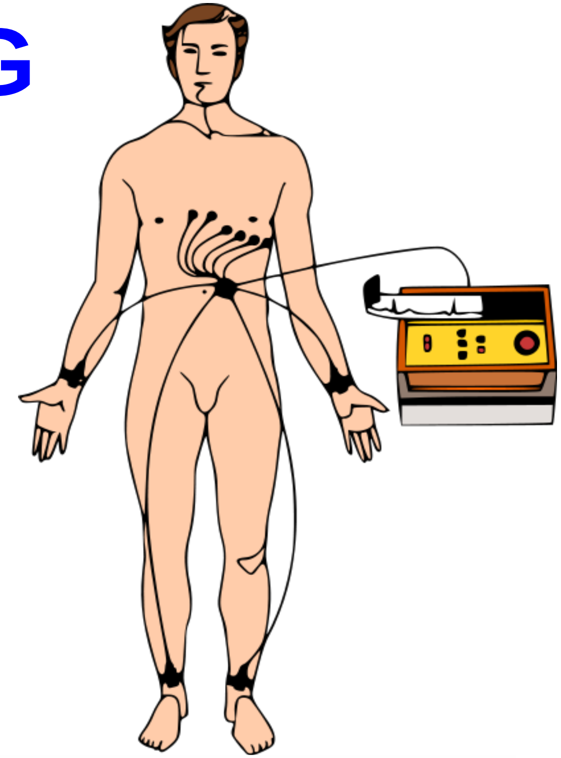


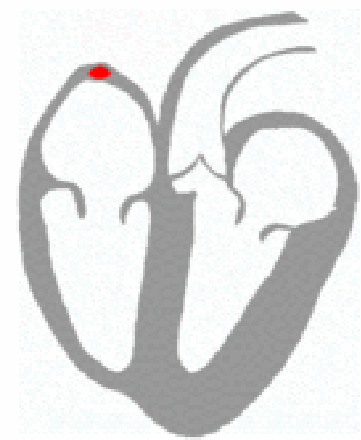
- **Nerve impulse changes the potential difference across the membrane (by sodium ion flow though membrane) to ~+40 mV**
- **Action potential propagates with 30 m/s down the axon**
- **~20% of resting energy of human body goes into active pumping of sodium ions!**

EEG and ECG



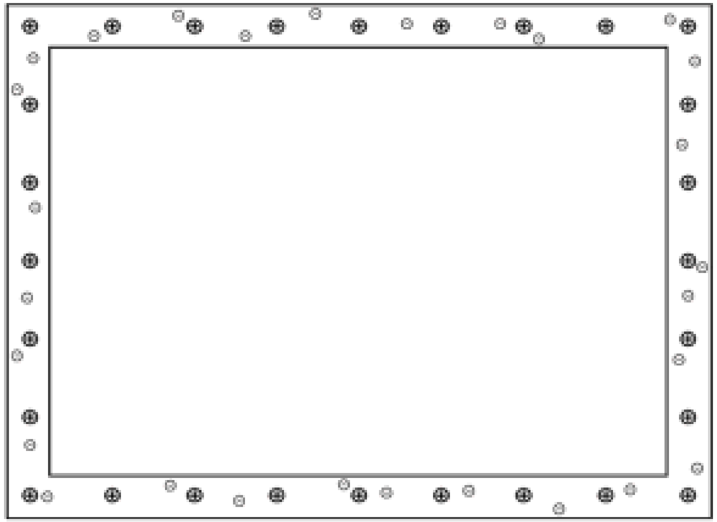
- **Electrocardiography** (ECG or EKG) is a transthoracic (across the thorax or chest) interpretation of the electrical activity of the heart
  - Detected by electrodes attached to the skin
  - **Measures potential difference** (voltage) do to changes on the skin that are caused when the heart muscle depolarizes during each heartbeat
- **Electroencephalography** (EEG) is the recording of electrical activity along the scalp.
  - Measures voltage fluctuations resulting from ionic current flows within the neurons of the brain.





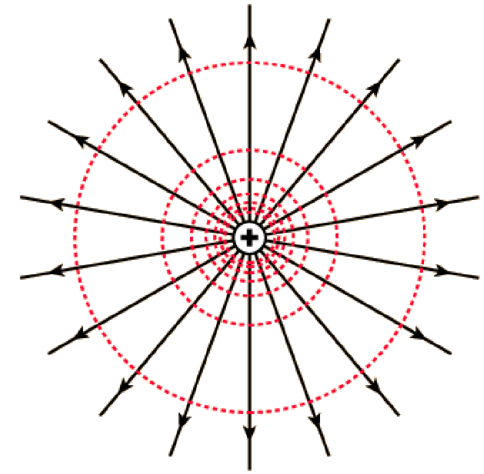
Faraday's Cage

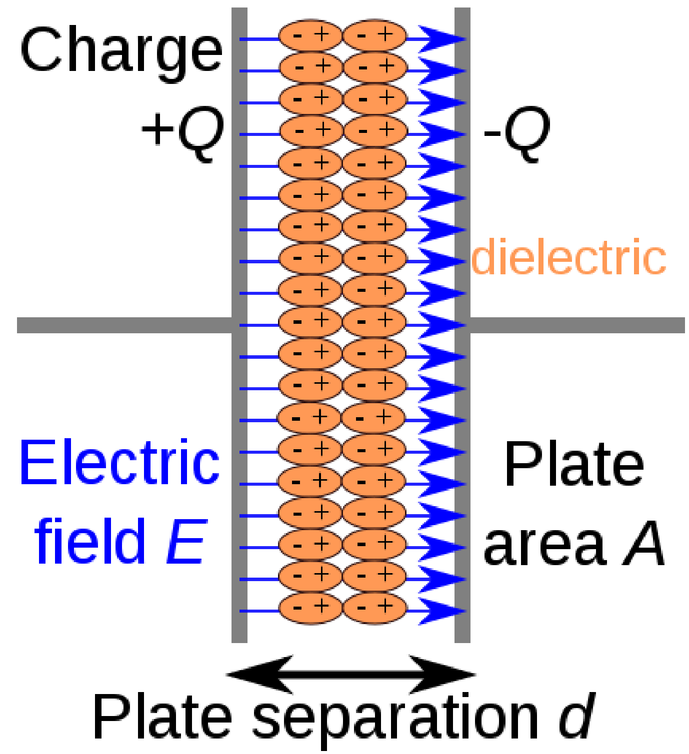
- Enclosure formed by conducting material or by a mesh of such material.
- Blocks out external static electric fields
- Recall: E=0 inside a hollow conductor !
- Shielding effect first observed by Benjamin Franklin in 1755



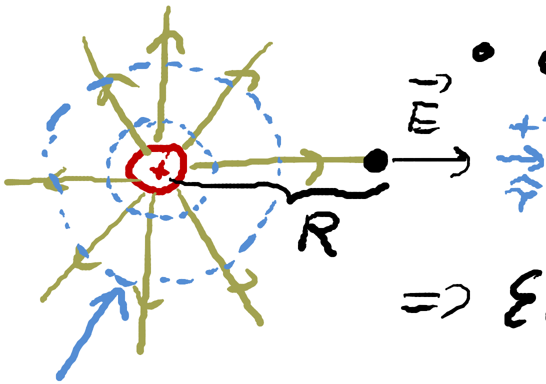
Today:

- Potential of a point charge
- Capacitors
- Energy density of the electric field





Example: Point charge



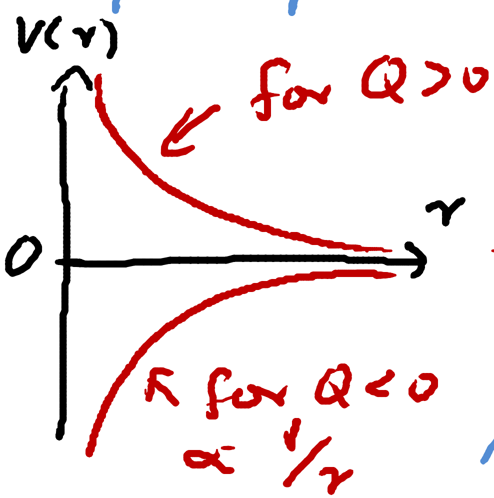
- Electric field:

$$|E| = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} \quad \text{points along } \hat{r}$$

⇒ Electric potential: take $V_i = 0$ at $r \to \infty$

$$V(R) = -\int_\infty^R \vec{E}\cdot d\vec{r} = -\int_\infty^R \frac{Q}{4\pi\varepsilon_0}\frac{1}{r^2}\,dr$$

$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r}\right]_\infty^R = \frac{1}{4\pi\varepsilon_0}\frac{1}{R} \propto \frac{1}{R}$$



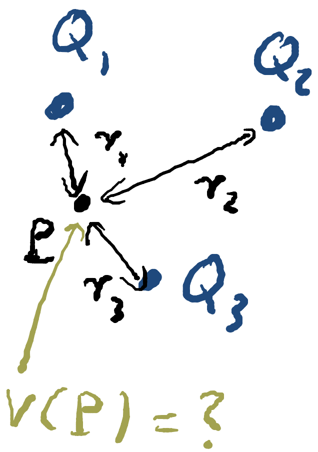
$$\Rightarrow \boxed{V(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}}$$

for point charge (only!)

check: $E_{\text{along } r} = -\frac{dV}{dr}$ ✓

Note:
- $V > 0$ for positive point charge ($Q>0$)
- $V < 0$ " negative " " ($Q<0$)
- only depends on distance r to charge

Potential due to a group of Point Charges



Recall: Principle of superposition for electric field of group of N charges:

$$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 + \dots + \vec{E}_N$$

⇒ Potential at point P:

$$V(P) = -\int_i^f \vec{E}_{net} \cdot d\vec{s} = -\underbrace{\int_i^f \vec{E}_1 \cdot d\vec{s}}_{} - \int_i^f \vec{E}_2 \cdot d\vec{s} - \dots - \int_i^f \vec{E}_N \cdot d\vec{s}$$

$V_1(P)$ = potential at point P just due to charge #1

$$= V_1(P) + V_2(P) + \dots + V_N(P)$$

$$\Rightarrow \boxed{V = \sum_{i=1}^{N} V_i = \frac{1}{4\pi\varepsilon_0} \sum \frac{Q_i}{r_i}}$$

⇒ principle of superposition applies to electric force, electric field $\vec{E}$, <u>and</u> electric potential V!

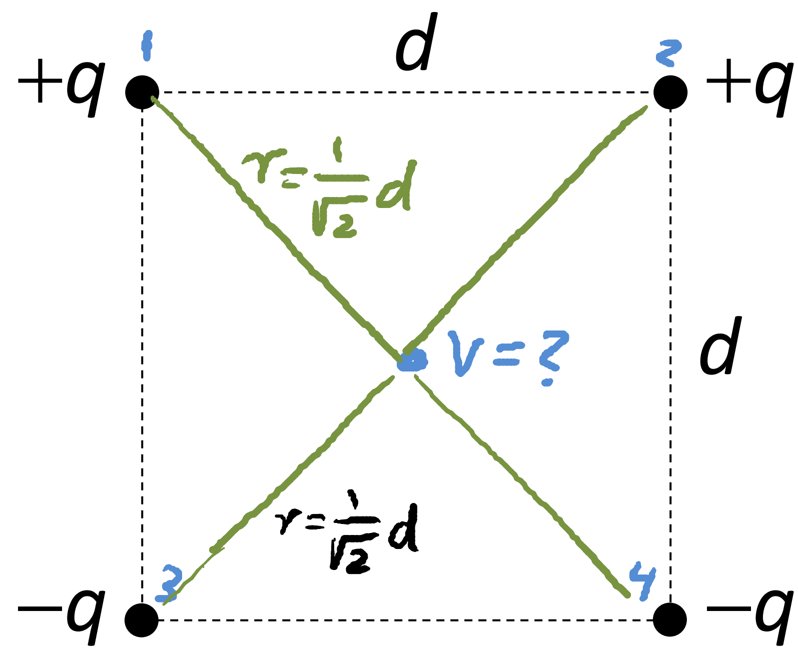
What is the potential at the center of the square? Take $V = 0$ at infinity.

A. $V_{\text{center}} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{8q}{d^2}$

B. $V_{\text{center}} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{8q}{\sqrt{2}d}$

C. $V_{\text{center}} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{4q}{d}$

D. $V_{\text{center}} = 0$



$$V_{net} = V_1 + V_2 + V_3 + V_4$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{+q}{\frac{1}{\sqrt{2}}d} + \frac{+q}{\frac{1}{\sqrt{2}}d} + \frac{(-q)}{\frac{1}{\sqrt{2}}d} + \frac{(-q)}{\frac{1}{\sqrt{2}}d}\right)$$

$$= \underline{0}$$

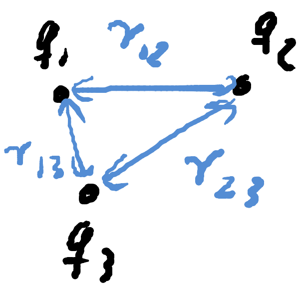
Total Potential Energy of a Group of Charges

Key idea: U_{total} of a system of fixed point charges is equal to the work that must be done by an external agent to assemble the system, bringing each charge in from an infinite distance

~ start with charge #1: $U_{total} = 0$

~ add charge #2: $U_{total} = 0 + U_2 = 0 + q_2 V_{\text{by charge \#1}}$ at point we put charge #2

$$= \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_{12}}$$



~ add charge #3: $U_{total} = 0 + U_2 + U_3$

$$= 0 + q_2 V_{\text{by \#1 at \#2}}$$

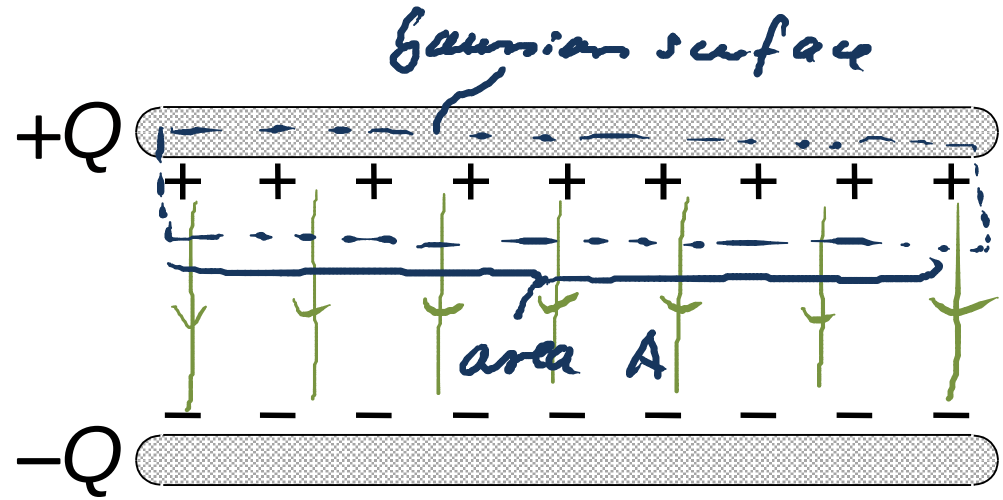
$$+ q_3 V_{\text{at point of charge \#3 by charge \#1 and \#2}} \leftarrow = V_{\text{by 1}} + V_{\text{by \#2}}$$

$$\Rightarrow U_{total} = q_2 V_{\text{by \#1 at \#2}} + q_3 V_{\text{by \#1 at \#3}} + q_3 V_{\text{by \#2 at \#3}}$$

$$= \frac{1}{4\pi\varepsilon_0} \left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right)$$

... and so on ...

If the charge on both metal plates ($\pm Q$) were doubled, what would happen to the magnitude E of the uniform electric field between the plates?



A. Decrease by a factor of 1/4.
B. Decrease by a factor of 1/2.
C. Stay the same.
D. Increase by a factor of 2.
E. Increase by a factor of 4.

$Q \to 2Q \Rightarrow \sigma \to 2\sigma \Rightarrow E \to 2E$

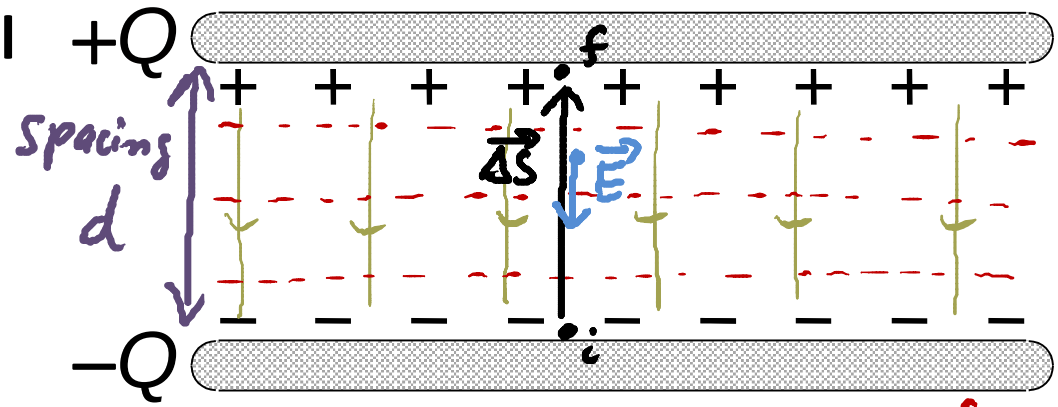
- Define surface charge density $\sigma = Q/A$
- Use Gauss Law:

$Q_{inside} = \sigma A = \varepsilon_0 \Phi = \varepsilon_0 A \cdot E$

$\Rightarrow E = \sigma / \varepsilon_0$

field just outside of a conductor

If the charge on both metal plates (±*Q*) were doubled, what would happen to the potential difference (voltage) between the plates?



A. Decrease by a factor of 1/4.
B. Decrease by a factor of 1/2.
C. Stay the same.
D. Increase by a factor of 2.
E. Increase by a factor of 4.

$\Delta V = V_f - V_i = -\int_i^f \vec{E}\cdot d\vec{s}$

$= -\vec{E}\cdot\vec{\Delta s} = -E\,\Delta s\cos 180°$

$= E\,\Delta s$

$\Rightarrow \boxed{\Delta V = Ed} \propto E$

$Q \to 2Q$

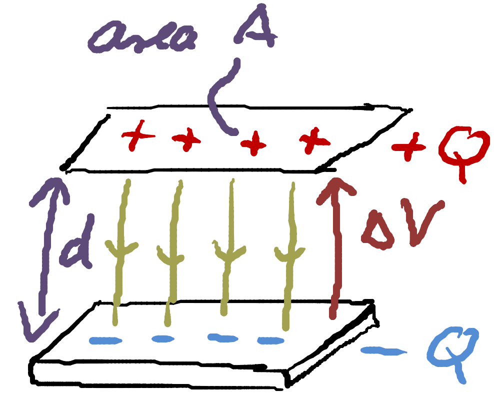
$\Downarrow$

$E \to 2E$

$\Downarrow$

$\Delta V \to 2V$

Capacitance:



- for two parallel metal plates separated by distance d, one with charge $+Q$ and one with charge $-Q$:

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\,\varepsilon_0}$$

$$\Delta V_{\text{between plates}} = E d$$

$$Q_{\text{per plate}} = \varepsilon_0 A E = \varepsilon_0 \frac{A}{d} \Delta V_{\text{between plates}} = C \, \Delta V_{\text{plates}}$$

with capacitance C of "parallel plate capacitor"

$$C = \varepsilon_0 \frac{A}{d}$$ for ∥ plates

Units: $[C] = \frac{\text{coulomb}}{\text{volt}} \equiv \text{farad} = 1\,F$

typically: $C \sim \underbrace{10^{-12}\,F}_{1\,pF}$ to $\underbrace{10^{-6}\,F}_{1\,\mu F}$

Notes:

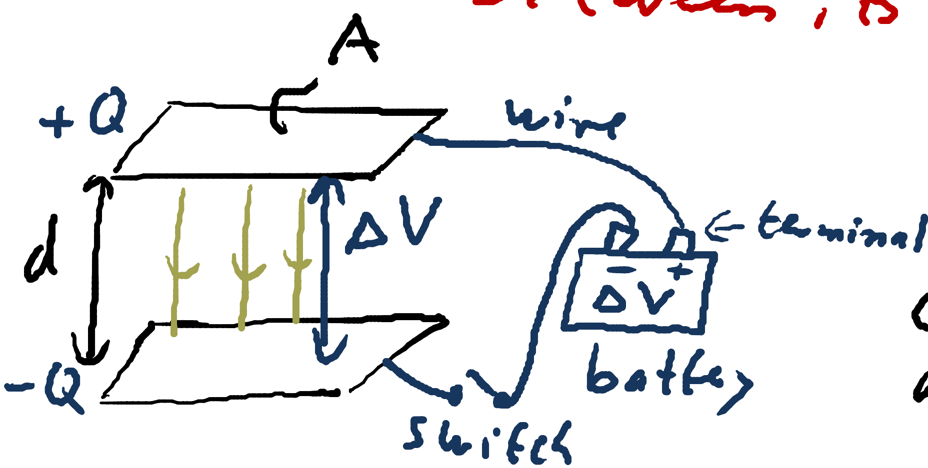
1. In general for two spaced conductors of charge $+Q$ and $-Q$: $\Delta V = V_+ - V_- \propto Q$ $\Rightarrow$ Capacitance $C = \frac{Q_{\text{per cond.}}}{\Delta V}$

2. C only depends on the geometry of the two conductors, and on the material between them.

3. Charging a capacitor:

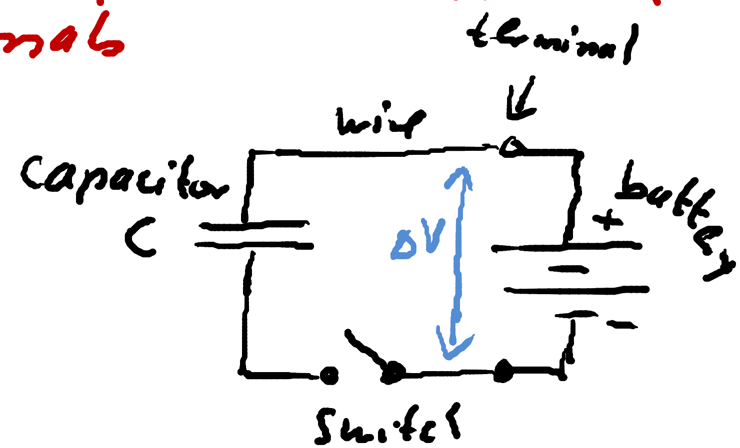
→ place capacitor in an electric circuit with a battery

Battery: maintains a certain potential difference between its terminals



⟹

Circuit diagram



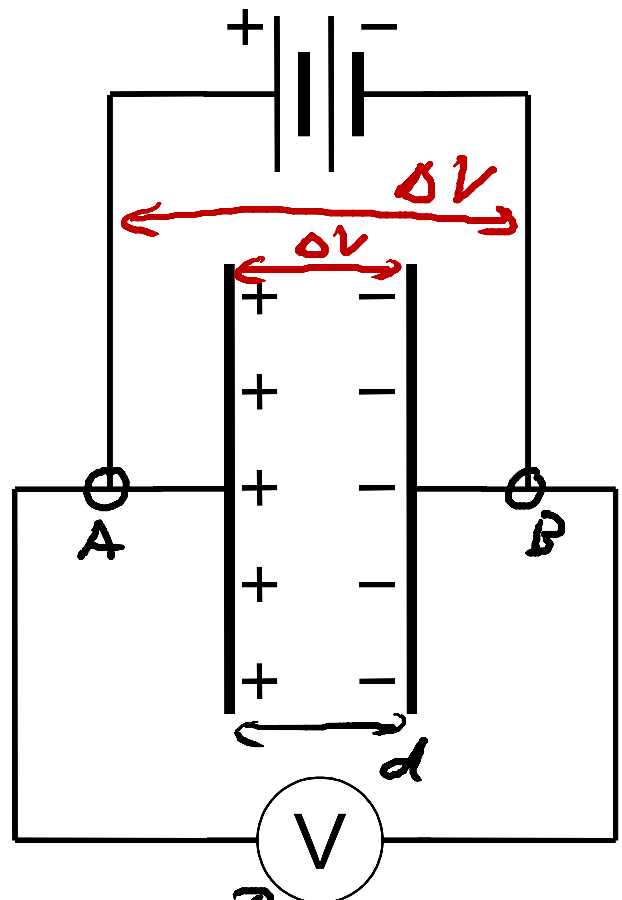
When the plates are moved apart, what happens to the voltmeter reading?

A. Goes up.

B. Goes down.

C. Stays the same.

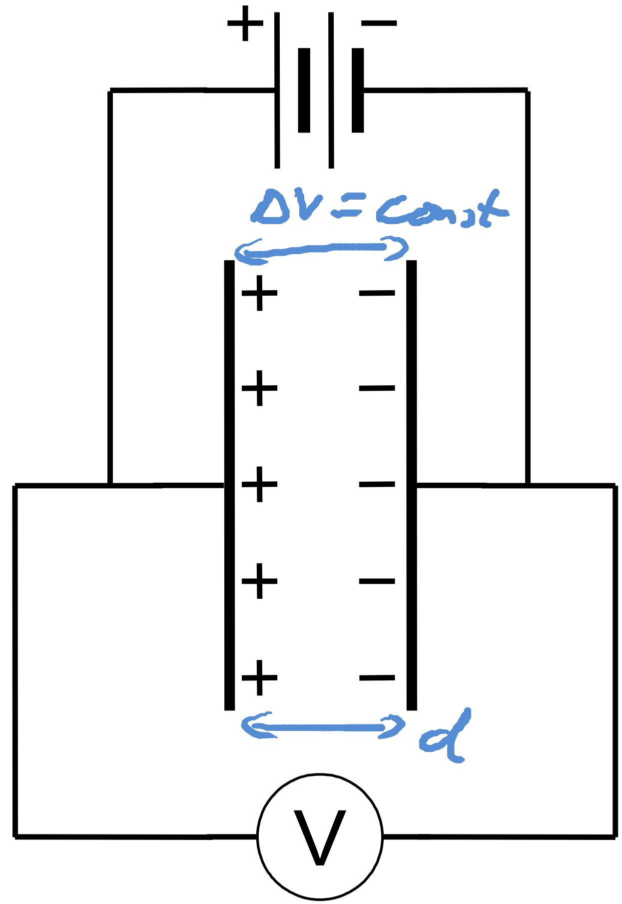
Battery maintains a constant potential difference ΔV



measures potential difference between A and B

When the plates are moved apart, what happens to the magnitude of the charge on each plate?

A. Goes up.
B. Goes down.
C. Stays the same.



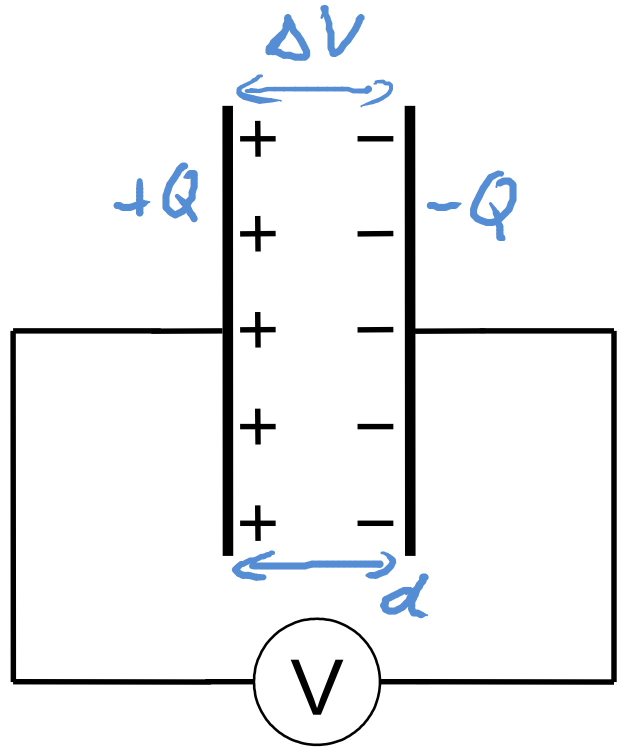
$\Delta V = \text{const here!}$

$\Delta V = E d \Rightarrow E \downarrow \text{ as } d \uparrow$

$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A \varepsilon_0} \Rightarrow Q \downarrow \text{ as } E \downarrow$

When the plates are moved apart, what happens to the voltmeter reading?

A. Goes up.
B. Goes down.
C. Stays the same.



no battery ⇒ ΔV can change!
charge Q is constant here!

$Q = \text{const} \Rightarrow E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0} = \text{const}$

$\Rightarrow \Delta V = Ed$ increases with d!