Recap

- Electric point charge:

\[ E(r) = \frac{1}{4\pi \varepsilon_0} \frac{Q}{r^2} \propto \frac{1}{r^2} \quad \iff \quad V(r) = \frac{1}{4\pi \varepsilon_0} \frac{Q}{r} \propto \frac{1}{r} \]

- Potential due to a group of point charges:

Principle of superposition: \( V(P) = \sum_{i=1}^{N} V_i(P) \)

- Potential energy of a point charge:

\[ U = q \cdot V \text{ at place of point charge by other charge} \]

- Capacitance:

- Capacitance = \( C = \frac{Q_\text{per-plate}}{\Delta V \text{ between plates}} \)

\[ E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A \varepsilon_0} \quad \Delta V = E d = \varepsilon_0 \frac{A}{d} \]
Touch Screens

- Technologies:
  - Infrared of optical Touch
  - Capacitive Touch
    • touching the screen surface results in a distortion of the screen's electrostatic field, measurable as a change in capacitance
  - Resistive Touch Technology
  - Surface Acoustic Wave
Today:

• Energy density of the electric field
• Dielectrics
• Electric current
• Electrical resistance
Energy stored in a Capacitor / Electric Field

- Potential difference between plates:
  \[ \Delta V = V_+ - V_- = \frac{q}{C} \]

- Move small positive charge \( dq \) from the - to the + plate, \( \Delta V \) = increase in potential energy stored in capacitor

\[ \Delta U = dq \cdot \Delta V = dq \cdot \frac{q}{C} = W \text{ we need to do to move charge} \]

\( \Rightarrow \) Total potential energy when charging capacitor from \( q_i = 0 \) to \( q_f = Q \):

\[ (\text{Energy stored in capacitor}) = U = \int_0^Q \Delta U = \int_0^Q dq \cdot \frac{q}{C} = \frac{1}{2C} Q^2 \]

= area under \( \Delta V(q) \) graph
\[ U_{\text{capacitor}} = \left( \text{energy stored in capacitor} \right) = \frac{Q^2}{2C} = \frac{1}{2} QDV = \frac{1}{2} CV^2 \]

\[ C = \frac{Q}{DV} \]

- Can think of this energy as being stored in the electric field \( E \) between the two plates:

\[ \begin{align*}
\text{Area} & \quad +Q \\
\text{d} & \quad V_+ \\
\text{Voltage} & \quad -Q \\
\text{d} & \quad V_- \\
E & \quad \text{Electric field}
\end{align*} \]

\[ \Rightarrow \left( \text{Energy of electric field of capacitor} \right) = \frac{1}{2} CV^2 \]

\[ \Rightarrow \text{Energy density } \mu_0 \text{ is associated with an electric field:} 
\]

\[ \mu_0 = \frac{\text{Energy in field}}{\text{Volume}} = \frac{\frac{1}{2} \varepsilon_0 \frac{A}{d} DV^2}{Ad} = \frac{1}{2} \varepsilon_0 \frac{(OV)^2}{d} = \frac{1}{2} \varepsilon_0 E^2 \]

true for any electric field.
Which capacitor stores more charge?

A. 1 μF  
B. 2 μF  
C. Both store the same charge

\[ Q = C \cdot \Delta V \propto C \]

Same for both.
What should be the value of $C_{\text{eff}}$ in terms of $C_1$ & $C_2$ so that the battery delivers the same charge in both circuits?

Same: $\Delta V_{\text{batt}} = \Delta V_1 = \Delta V_2$

Add: charge $Q = Q_1 + Q_2$

$Q = Q_1 + Q_2 = C_1 \Delta V_1 + C_2 \Delta V_2 = (C_1 + C_2) \Delta V_{\text{batt}}$

with $C_{\text{eff}} = \sum_{i=1}^{N} C_i$ for capacitors in parallel
Which capacitor stores more charge?

A. 1 $\mu$F
B. 2 $\mu$F
C. Both store the same charge
What should be the value of $C_{\text{eff}}$ in terms of $C_1$ & $C_2$ so that the battery delivers the same charge in both circuits?

**Same: charge $Q$:** $Q = Q_1 = Q_2$

**Add: $\Delta V_{\text{batt}} = \Delta V_1 + \Delta V_2$**

$$\Delta V_{\text{batt}} = \Delta V_1 + \Delta V_2 = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)Q = \frac{1}{C_{\text{eff}}}Q$$

**with** $rac{1}{C_{\text{eff}}} = \sum_{i=1}^{N} \frac{1}{C_i}$

**For capacitors in series**
Dielectrics and Electric Fields

**Dielectric:** Insulator that can be polarized by an applied electric field

**Two Types:**

- **Polar Dielectric**
  - Molecules have permanent electric dipole moments
  - No external field
  - With external field: $E' = 0_{Oy}$, $E = E_{ext}$, $E_{by\, dipole}$

- **Nonpolar Dielectric**
  - No permanent dipole moment, but external electric field induces dipole moments
  - No external field
  - With external field: $E' = 0_{Oy}$, $E = E_{ext}$, $E_{by\, dipole}$
In both cases:

\[ \vec{E}_{\text{inside}} = \vec{E}_{\text{ext}} + \vec{E}_{\text{by dipole}} \]

\[ |\vec{E}_{\text{inside}}| < |\vec{E}_{\text{ext}}| \]

\[ \vec{E}_{\text{inside}} \rightarrow \vec{E}_{\text{by dipole}} \rightarrow \vec{E}_{\text{ext}} \]

\[ \vec{E}_{\text{inside}} = \vec{E}_{\text{ext}} \]

The effect of the dielectric is to weaken the original external field:

\[ |\vec{E}_{\text{inside}}| = \frac{|\vec{E}_{\text{ext}}|}{\kappa} \]

with dielectric constant: \( \kappa \geq 1 \) (dimensionless)

- \( \kappa \) is a property of the dielectric material
- Example: \( \kappa_{\text{vacuum}} = 1 \); \( \kappa_{\text{paper}} = 3.5 \); \( \kappa_{\text{mica}} = 80 \)

Effect of dielectric:

All electrostatic equations containing the permittivity constant \( \varepsilon_0 \) are to be modified by replacing \( \varepsilon_0 \) with \( \kappa \varepsilon_0 \).
Examples: since: \( E_{\text{with}} = \frac{E_{\text{without}}}{\kappa} \)  

- Point charge:
  \[ E_{\text{without}} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \quad \Rightarrow \quad E_{\text{with}} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \]

- Capacitor:
  \[ E_{\text{without}} = \frac{\sigma}{\varepsilon_0} \quad \Rightarrow \quad E_{\text{with}} = \frac{\sigma}{\varepsilon_0} \]

\[ +Q \quad \begin{array}{c} \text{dial}\end{array} \quad -Q \]

\[ = \quad E_{\text{without}} = \frac{Q}{\partial V} = \frac{Q}{-\int_{V_{\text{without}}} \mathbf{E} \cdot d\mathbf{s}} = \frac{Q}{\varepsilon_0 \frac{A}{d}} \quad \Rightarrow \quad C_{\text{with}} = \frac{Q}{\partial V_{\text{with}}} = \frac{Q}{-\int_{V_{\text{with}}} \mathbf{E} \cdot d\mathbf{s}} = \kappa C_{\text{without}} = \frac{Q}{\varepsilon_0 \frac{A}{d}} \]

- Gauss' Law:

\[ Q_{\text{inside}} = \varepsilon_0 \oint \mathbf{E}_{\text{without}} \cdot d\mathbf{A} \quad \Rightarrow \quad Q_{\text{inside}} = \varepsilon_0 \kappa \oint \mathbf{E}_{\text{with}} \cdot d\mathbf{A} = \varepsilon_0 \frac{A}{d} \]
Moving Charges: Electric Current

**Electric Current**: net flow of moving charges through an area per time.

\[ i = \frac{dq}{dt} = \text{(charge passing through area per time)} \]

For constant / steady current:

\[ i = \frac{\Delta q}{\Delta t} \]

Units: \( [i] = \frac{C}{s} = \text{ampere} = 1 \text{A} \)