SUPERGRAPH TECHNIQUES (FEYNMAN'S RULES ON THE SUPERSPACE) ARE AN ANCIENT TOOL (70s-80s) ANALOGOUS TO BUST CASTING. MODERN TECHNIQUES (MOSTLY BY SELIGER) HAVE MADE THEM UNIMPORTANT. THEY ARE Omitted IN TEXTBOOKS POST 2000. WEINBERG'S QFT SERIES, WHICH IS NORMALLY VERY COMPREHENSIVE, ONLY GIVES A BUD TO THE SUBJECT. WHY SHOULD WE CARE?

1. **Curiosity** - My primary motivation
2. **Completeness** - Feynman: Good physicists can derive results in multiple ways
3. **Elegance** - Geometry is beautiful
4. **Review** - Return key methods by working with a more general system with rich structure.
5. **Applications** - Effective kahler pot, supergravity, ...

**Big picture**: We know that symmetry simplifies our lives by grouping together things which are really different manifestations of a single "thing".

**CA. Principle**: Particle in diff frame is the same

\( SU(3)_c : \ u_8, u_6, u_4 \) are the same

\( SU(2)_L : \ e, \nu, \tau \) are the same (up to flavor effects)

If we work in entire multiplets under the sym., much easier than working w/ component fields, often even when symmetry is broken! (cf flavor; spurion analysis; see also recent paper on constraint formulation.)

**This is what we observe** (4D reps)

\( \) SUSY is broken

**Sit in SUSY, we always work in components!**

A good analogy: **5D theories** [SU(3)_c w/ bosonic K.O.; SUSY: extend w/ fermionic K.O.]

Just like SUSY, we typically work with 4D reps: KK reps = 5D multiplet.

5D SUSY can even be broken: Branes, wrapping, susions

**Remark**: These cases where a full 5D formalism is beneficial

\( \rightarrow \) Just one flip? Yukawa! Can find finiteness arguments in 5D which are visible in 4D. We will do an analogous thing in superspace!

**REMARK**: One 'clear' shortfall: cannot canonically normalize in superspace (Fischler, p. 186)

This book is really just a pure joy to read, very geometric intuition w/ rigorous formalism.

**REPS**

- Any really good SUSY book.
- West, West Bogner, Mcvain are especially good.
- The bible of mathematical superspace: Buchweitz, Hobernko.
- Lecture notes by C. Sarmann.

**Begning caveat**: I have been very sloppy with minus signs! Do not trust them! (Fortunately they don't really matter for our purposes.)
**Path Integral Formalism**

\[ S = \int \mathcal{D}x \mathcal{L} \quad \text{SUPERFACEx} \quad \mathcal{S} = \int \mathcal{D}x \mathcal{D}\theta \mathcal{L} \]

**BUT:** You already know \( \mathcal{L}_1 \sim \frac{1}{2} \theta^2 \) + \( \frac{1}{2} \phi \dot{\phi} \), \( \theta^2 \mathcal{L}_2 \) + \( \frac{1}{2} \phi \dot{\phi} \mathcal{L}_3 \)

\[ S = S_0 + S_{int} \]

**FREE PART = ACTION, QUADRATIC, EXPLICITLY SOLUBLE**

* \( S_0 \) GIVES OUR FIELD PROPAGATORS.
  *INSERT*: JUST INVERT THE QUADRATIC OPERATOR ('mass + mass')

You already know:

\[ \langle 0 | T \left( \psi(x) \psi(y) \right) | 0 \rangle = \frac{i}{p^2 - m^2} \delta(x-y) \]

\[ \langle 0 | T \left( \bar{\psi}(x) \bar{\psi}(y) \right) | 0 \rangle = \frac{i}{p^2 - m^2} \delta(x-y) \]

Dirac operator acting on KG Green's func!

⇒ PART THAT WE SHOULD BE ABLE TO TREAT DIFFERENT CONFINEMENT IN A SUPERMULTIPLIET IN A UNIFIED WAY!

* \( S_{int} \) GIVES OUR INTERACTIONS.
  IN THE PATH INTEGRAL FORMALISM, WE CONSTRUCT THE GENERATING FUNCTIONAL

\[ Z[J, \ldots] = (2\pi)^{\frac{D}{2}} \int \mathcal{D}[\phi] \mathcal{D}[\psi] \exp \left[ L_0 + S_{int} + \text{SOURCE} \right] \]

*NOT REALY NEEDED IN TAO'S FORM.*

THEN WE CAN READ OFF INTERACTIONS BY TAKING FUNCTIONAL DERIVATIVES WITH THE SOURCE(s)

\[ G(x, y, x_0) = \left. \frac{\delta^2 Z[J]}{\delta J(x_0) \delta J(y)} \right|_{J = 0} \]

PART TAIN LEAPE WE CAN ALSO \( \phi \)

\[ Z[J, \ldots] = \exp \left[ i \text{S}_{\text{int}} \left[ \phi \right] \right] Z_0 \]

\[ Z_0 \left[ J, \ldots \right] = \int \delta(\text{fields}) \exp \left[ i S_0 + i \phi \right] \]
THIS IS ALL VERY EASY TO SEE DiAGRAMMICALLY if YOU ARE ALL VERY FAMILIAR w/ it.
You also know that from \( \Gamma \) we can define more sophisticated objects
like \( \hat{\Gamma} \) (generating functional of connected diagrams) \& \( \Gamma \) (effective
action \& generating functional of \( \Gamma \) diagrams). THE USEFUL THING TO KEEP
IN MIND IS THAT WE CAN ACTUALLY USE \( \Gamma \) calculated \& some loop order, TO
WRITE A TREE-LEVEL "QUANTUM RESUMMED" ACTION.
WE'LL GET TO THIS \( \circ \) THE END OF MY TALK.

GOAL: SUPERSYMMETRIZE ALL THIS! (\# TREAT MANIFESTLY SU(2)/U(1) WAY)
\[
\frac{d}{dx} \rightarrow \frac{d}{d(\theta + \phi)} = \frac{1}{2} \frac{d}{d\theta} \frac{d}{d\phi} \equiv \frac{1}{2} \frac{d}{d\theta} \frac{d}{d\phi} - \frac{1}{2} \frac{d}{d\phi} \frac{d}{d\theta}
\]

**REMARK:** IT IS EVEN MORE ELEGANT TO \( \# \) WORK IN A COMPLEXIFIED
SUPERSPACE \( \mathbb{C} \times \mathbb{R} \# \) TREAT "\( \mathbb{R} \# "
SUPERSPACE AS A SUBSPACE.

- STRONG \# TAKES 
- \# WORKSHED COORDS
- \( \theta^\# = \theta, \phi^\# = \phi \), BUT PHYSICAL
- WORKSHED IS \( \mathbb{R} \# \# \) SURFACE.

I EXPECT YOU ALL TO BE FAMILIAR w/ GRASSMANNIAN COORDINATES!
[BY NOW, JUST, PATH INTEGRAL FOR FERMIONS, BEST METHOD, ...]

IF NOT:
1. GET YOUR SHIT TOGETHER.
2. JUST RECALL THAT THEY HAVE FUNDAMENTAL PROPERTIES
   - ANTICOMMUTING
   - INTEGRATION \& DIFFERENTIATION ARE THE SAME

**SUPERSPACE** IS A IMAGINARY LAND WHERE FERMIONS LIVE AS HIGHER-DIMENSIONAL
"SHADOWS" OF SCALARS. LET US CONSIDER ONLY THE SIMPLEST CHIRAL
SUPERMULTIPLET, ON THE \( \theta = \bar{\theta} = 0 \) SLICE OF SUPERSPACE ONE HAS A
THEORY OF COMPLEX SCALARS.

**LOOK \# THROUGH:** THIS IS LIKE CSABA'S BRANE EFFECTIVE ACTION!

BUT AS ONE OF THESE COMPLEX SCALARS \"PROPAGATES\" (Via \"IMAGINATION\"
Into THE BULK \# # KEEPS INTO THE BULK) \# \# IN OTHER SCALARS \# #, IT MAGICALY
BECOMES A FERMION (WAL) \# AND REVEALS ITS TRUE NATURE AS A
**SUPERFIELD.**

"PROPAGATION" IN GRASSMANNIAN DIRECTIONS = SUSY TRANSFORMATION.

\( \# \) "PROPAGATION" IN SU(2) (GAUGE) IS A GUAGE TRANSFORM
\( \# \) OR "PROP" IN SU(2) is a PLASER TRANSFORM.
WHAT WE EXPECT: SUPERGRAVITY GIVE PROPAGATION THROUGH SUPERSPACE
WE CAN PICK OUT COMPONENTS TO GET INDIVIDUAL MINKOWSKI SPACE DIAGRAMS.

to.
VECTORS BOSON SCATTERING OR SPINOR SCATTERING
[ bugs multiply we pick external states
[IR, G, or W(→) ] and graph tells us amplitude.

That's kind of basic. The real flavor is at loop level.
Chord analogy: using euclidean loop diagram's automatically
'long' IR tower.

In such, supergraphs will automatically sum over internal fermion
and boson states. In particular, fermion-boson cancellations are
manifest in 'true' divergence structure can be seen directly.

alright! let's get to work.

We will restrict ourselves to theories of chiral superfields
(ex. ignore VSF, supergravity, etc.) this turns out to capture most
of the formalism.

In fact, it is more complex than VSF (fun!), though
VSF has additional subtleties about gauge fixing.

So: if we could write down
\[ \bar{\chi} \gamma^\mu \chi \]
then we're almost done.

But you know that life isn't that kind.

\[ \int \bar{\chi} \gamma^\mu \chi K(\phi, \phi^+) \left[ \bar{\phi} \gamma^\mu \phi \right] + \left[ \bar{\phi} \gamma^\mu \phi \right] \]

\[ \text{Kähler term already set} \]
\[ \text{Kinetic terms} \]
\[ \text{Superpotential: needs work!} \]

Strategically: use some such wu-woo identities

to 

\[ \int \bar{\phi} \gamma^\mu \phi \rightarrow \int \bar{\chi} \gamma^\mu \chi \]

Can it be done? Yes!
Just need to construct an operator that
projects onto the VSF 'half superspace'.

By the way, physical meaning of this 'half superspace' should be clear: VSF doesn't have 3-gravitons, can't
propagate. There's a fantastic elegant (though nontrivial)
presentation can be found in Buchbinder & Kuzenko §2.5.
1. \( \int d^4x \, d^2\theta \, f(x, \theta, \bar{\theta}) = \int d^4x \, d^2\theta \, f_{\theta \bar{\theta}} \, \bar{\theta} \theta = \int d^4x \, f_{\theta \bar{\theta}} \)

Extra surface integral
that we want to include

expression without the \( d^2\theta \) integral.

\[ f = f + f_{\theta \theta} + f_{\bar{\theta} \bar{\theta}} + f_{\theta \bar{\theta}} \theta^2 + \ldots \]

Drop trace-like derivatives
in only the under the \( d^4x \) integral.

2. \( \int d^4x \, \frac{1}{4} \delta^2 f(x, \theta, \bar{\theta}) = \int d^4x \, \frac{1}{4} \delta^2 + f_{\theta \theta} \bar{\theta} \theta = \int d^4x \, f_{\theta \bar{\theta}} \)

\( D_d = \frac{\partial}{\partial \theta^d} + i \frac{1}{4} \epsilon^{d \mu \nu \rho} \partial_{\mu} \theta_{\nu} \theta_{\rho} \)

Such covariant derivatives

\[ \bar{D}_\mu = -\frac{1}{\theta^2} \epsilon_{\mu \nu \rho \sigma} \partial_{\nu} \theta_{\rho} \theta_{\sigma} \]

ASIDE: THIS IS ACTUALLY A GEOMETRIC OBJECT (along \( \epsilon\theta\bar{\theta} \))

- LEFT/RIGHT TRANSLATION IN SUPERSPACE (see Wess+Bagger)
- PUSH FORWARD OF VECTOR (L Algebra) OF A LIE GROUP MANIFOLD
  \( \rightarrow \) see, e.g., Buchbinder & Kuzenko p.170
- PROJECTION OPERATOR ONTO CHIRAL SUBSPACE
  \( \rightarrow \) eq. \( \delta \bar{\theta} = 0 \); we'll see this more formally
- HORIZONTAL LIFT OF 'SUPER' FIBER BUNDLE
  \( \rightarrow \) see eq. Gackler \& Schücker, Azcárraga \& Izquierdo

\[ \delta \theta : (1 + \epsilon) \]

\[ \int d^4x \, d^2\theta = \frac{1}{4} \int d^4x \, \delta^2 \]

\( \text{up to sign}\)

This gives us a guiding principle:

\( \delta \theta \)

A dial integrand \( (d^2\theta) \)

If you can pull out a \( \delta^2 \) from an integrand
then you can convert it into a \( d^2\theta \)

It is 'obvious' that similarly pulling out a \( \theta^2 \rightarrow d^2\theta \).

OK, good. BUT HOW DO WE GO ABOUT PULLING OUT \( \delta^2 \)S
out of our asses?

SOLUTION: Another magical identity.
\[
\begin{align*}
\nabla^2 = \frac{D_x^2 D_y^2}{16 D^2}
\end{align*}
\]

**Correct**: I'm not being careful with signs (I still have W references use GR metric!). But signs won't be too important for what we're interested in.

\[
PF/ E^2 D^2 \Phi = E_x E_y D^2 D_x D_y \Phi
= E_x \left( \frac{D_x^2}{D^2} \right) D_x \Phi
= \left( \frac{D_x^2}{D^2} \right) D_x \Phi
\]

\[
= \frac{\left( D_x D_y \right)^2}{D^2} D_x \Phi
\]

Since extra term vanishes via \( \Phi = 0 \)

\[
+ \frac{1}{4} \left( \frac{D_x^2}{D^2} \right) \frac{D_x^2}{D^2} \Phi
= \frac{1}{4} \left( \frac{D_x^2}{D^2} \right) \frac{D_x^2}{D^2} \Phi
\]

\[
= + \frac{1}{16} \Phi
\]

**Let's put this to work - quadratic part \( S_{\text{free}} \), fix mass term in \( W \)**

Consider \( W_2 = \frac{1}{2} m \Phi^2 \)

\[
\begin{align*}
\int d^n x \, d^2 \theta \Phi \Phi
= \int d^n x \, d^2 \theta \Phi \left( \frac{-D_x^2}{16 D^2} \right) \Phi
= \frac{1}{4} \int d^n x \, d^2 \theta \left( \frac{-D_x^2}{4 D^2} \right) \Phi \Phi
\end{align*}
\]

So including the canonical Kähler potential

\[
S_{\text{free}} = \int d^n x \, d^2 \theta \Phi \Phi + \frac{1}{2} m \Phi \left( \frac{-D_x^2}{4 D^2} \right) \Phi + \frac{1}{2} m \Phi \left( \frac{-D_x^2}{4 D^2} \right) \Phi
\]

Now, as we said before the free action gives \( W_{\text{source}} \) our propagators

\[
Z_{[jj]} = \int \mathcal{D} \Phi e^{i S_{\text{free}} + i S_{\text{source}}}
\]

\[
= \int \mathcal{D} \Phi j \Phi \left( \frac{-D_x^2}{4 D^2} \right) \Phi + \text{h.c.}
\]

Since \( S \) is also a Kähler.
We need to know how to take functional derivatives again, there is a subtlety for \( \mathfrak{z} \) because they are constrained.

For an unconstrained superfield \( \mathfrak{z} \),

\[
\frac{\delta \mathfrak{z}(x', \theta', \bar{\theta}')}{\delta \mathfrak{z}(x, \theta, \bar{\theta})} = \mathfrak{z}^a(x-x') \mathfrak{z}^b(\theta-\theta') \mathfrak{z}^c(\bar{\theta}-\bar{\theta}')
\]

Note that \( \mathfrak{z} \) functions in superspace are also very simple:

\[
\mathfrak{z}^a(x) = \tilde{\theta}^a
\]

What about chiral superfield? (of boson and fermion \( \mathbf{2} \rightarrow \mathbf{2} \))

\[
\begin{align*}
\mathfrak{z}^+(x', \theta, \bar{\theta}) &= \int d^4x \, d^2\theta \, d^2\bar{\theta} \, \mathfrak{z}^+(x, \theta, \bar{\theta}) \, \delta^8(2-2') \iff \mathfrak{z}^+(x-x') \mathfrak{z}^+ (\theta-\theta') \mathfrak{z}^+ (\bar{\theta}-\bar{\theta}') \\
&= \int d^4x \, d^2\theta \, \frac{\delta^2}{4} \, \delta^8(2-2') \, \mathfrak{z}^+
\end{align*}
\]

\[
\frac{\delta \mathfrak{z}^+(x)}{\delta \mathfrak{z}^+(x)} = \frac{1}{4} \delta^2 \delta^8(2-2')
\]

(again maybe overall sign depending on conventions)

Sanity check:

\[
\frac{\delta}{\delta \mathfrak{z}^+(x, \theta, \bar{\theta})} \int d^4x' \, d^2\theta' \, d^2\bar{\theta}' \, f(\mathfrak{z}(x', \theta', \bar{\theta}')) = \int d^4x' \, d^2\theta' \, d^2\bar{\theta}' \, f'(\mathfrak{z}) \, \frac{\delta \mathfrak{z}^+(x', \theta', \bar{\theta}')}{\delta \mathfrak{z}^+(x, \theta, \bar{\theta})}
\]

This is the key technique, can integrate by parts since \( f \) is known:

\[
\frac{\delta}{\delta \mathfrak{z}^+(x)} \int d^4x' \, d^2\theta' \, d^2\bar{\theta}' \, f'(\mathfrak{z}) \, \delta^8(2-2') = f'(\mathfrak{z})
\]

We want to see this in action in the partition function \( Z \) in the source term [of \( W \) term]

\[
\frac{\delta}{\delta \mathfrak{z}^+(x, \theta, \bar{\theta})} \int d^4x' \, d^2\theta' \, d^2\bar{\theta}' \, f(\mathfrak{z}(x', \theta', \bar{\theta}')) = \int d^4x' \, d^2\theta' \, d^2\bar{\theta}' \, \frac{\delta^2}{4} \frac{\delta}{\delta \mathfrak{z}^+(x, \theta, \bar{\theta})} \, f(\mathfrak{z}') \, \delta^8(2-2')
\]

\[
\begin{align*}
&= \int d^4x' \, d^2\theta' \, d^2\bar{\theta}' \, \left( \frac{D^2 \mathfrak{z}^2}{16} \right) \frac{\delta}{\delta \mathfrak{z}^+(x, \theta, \bar{\theta})} \, f(\mathfrak{z}') \, \delta^8(2-2') \\
&= \frac{\delta}{\delta \mathfrak{z}^+(x, \theta, \bar{\theta})} \left( \frac{D^2 \mathfrak{z}^2}{16} \right) \mathfrak{z}^+(x, \theta, \bar{\theta}) \, \delta^8(2-2')
\end{align*}
\]

\[
\mathfrak{z}^+(x) = \mathfrak{z}^+(\phi)
\]
So let's solve the free motion propagator for $\propto_{SF}$

Write:

$$X = \left( \frac{1}{\xi} \right) = \left( \frac{1}{\tilde{\xi}} \right)$$

field $(\propto_{SF} + \propto_{SF}^{-})$

$$K = \frac{1}{4\pi} \left( \frac{\partial^2}{\partial^2} \right)$$

source

$$Z_0[K] = \int \mathcal{D}^2 X \ e^{i \int d^2 \tilde{X} \ \frac{1}{2} \tilde{X} \tilde{A} \ X} + \tilde{X} K + \tilde{E} X$$

we take

$$\tilde{X} = \left( \frac{1}{\xi} \frac{1}{\tilde{\xi}} \right)$$

$$A = \left( \begin{array}{cc} \frac{1}{4\partial^2/\alpha} & \frac{m}{4\partial^2/\alpha} \\ \frac{m}{4\partial^2/\alpha} & 1 \end{array} \right)$$

$$A^{-1} = \frac{\partial}{\partial^2 + m^2} \left( \begin{array}{cc} 1 & \frac{m}{4\partial^2/\alpha} \\ \frac{m}{4\partial^2/\alpha} & 1 \end{array} \right)$$

it's easy to check $AA^{-1} = 1$ using $1 = \partial^2/\alpha^2$.

Now we can use our usual Gaussian integral trick

$$\int d^2 z \ e^{-\pi \tilde{X} A \tilde{z} + \tilde{z} \tilde{A} \ z + \tilde{z} \tilde{u}_i + \tilde{u}_i \tilde{z}} \ d \exp \left( \tilde{u}_i A^{-1} \tilde{u}_i \right)$$

$$\Rightarrow Z_0[K] = e^{i W_0[K]}$$

$$W_0[K] = \int d^2 \tilde{z} \int \frac{1}{\partial^2 + m^2} J + \frac{1}{2} \int d^2 \tilde{z} \int \frac{m^2 \partial^2}{\partial^2 + m^2} J + \int \frac{m^2 \partial^2}{\partial^2 + m^2} J$$

\[\xi : \text{see}\ Wigg\ P.72\]

\[\text{use: } D^2 B^2 D^2 = 16 D^2 B^2\]
\[B^2 D^2 B^2 = 16 D^2 B^2\]

$$\frac{1}{1 - \frac{m^2 \partial^2}{\partial^2} \frac{16 \partial^2}{m^2}} = 1 - \frac{m^2 \partial^2}{16 \partial^2} \left[ 1 - \frac{m^2 \partial^2}{\partial^2} + \cdots \right]$$

$$= 1 + \frac{m^2 \partial^2}{16 \partial^2 (\partial^2 + m^2)}$$

\[\text{but signs are all ambiguous}\]
WE END UP WITH A SET OF PROPAGATORS (FOURIER TRANSFORMING MINK. CoORDs)
(\langle ... \rangle = \langle 01T ... 10 \rangle ) \quad \{ \text{GORBAR - ROZK - SZEKEL} \}

\begin{align*}
\langle \Phi(z) \Phi(z') \rangle & = \frac{1}{p^2 - m^2} \delta^4(\theta - \theta') \equiv \Delta_p \\
\langle \Phi(z) \Phi(z') \rangle & = \frac{(m^4) B^2}{p^2 (p^2 - m^2)} \delta^4(\theta - \theta') \quad \text{where} \quad \theta \to iP^+ \\
\langle \Phi(z) \Phi(z') \rangle & = \frac{(m^4) B^2}{p^2 (p^2 - m^2)} \delta^4(\theta - \theta')
\end{align*}

THESE ARE OUR PROPAGATORS!

- THERE ARE 3 VARIETIES \( \Phi, \Psi, \bar{\Phi}, \bar{\Psi} \)
  \footnotesize{\text{of FERMION PROPAGATORS: } 2\Phi, 2\Psi, 4\Psi}
  \footnotesize{\text{in fact, can see the usual names.}}

\[ \Phi \quad \Psi \quad \bar{\Phi} \quad \bar{\Psi} \]

- THE WEIRD \( \frac{m^4}{4! D_2} \frac{1}{p^2} \) IS JUST A PROJECTION
  \footnotesize{onto a chiral subspace, eg enforces chiral superfield condition}

- CAN WE WRITE THIS IN COMPONENTS?
  \footnotesize{eq. 3 from Nuss + Ragoer p.63 \text{ - need motivates more machinery.}}
  \footnotesize{we know we can write \( \Phi = \Phi(x, \theta) \), \( \Sigma = x + i \theta \bar{\sigma} \)}

\begin{align*}
\langle \Phi(y, \theta) \Phi(y', \theta') \rangle & = \langle \left( \Phi(y) + i \theta \bar{\sigma} \Phi(y) \right) \left( \Phi(y') + i \theta \bar{\sigma} \Phi(y') \right) \rangle \\
& = \delta^4(\theta - \theta') \langle \Phi(y) \Phi(y') \rangle + 2 \theta \bar{\sigma} \delta^3(\theta - \theta') \langle \Phi(y) \Phi(y') \rangle \\
& = -i m (\theta - \theta')^2 \Delta_p (y - y') \\
& = -i m \delta^3(\theta - \theta') \Delta_p (y - y')
\end{align*}

\( \text{Then } \frac{D_2}{4!} \delta^4(\theta - \theta') \text{ for full subspace + xeral projection.} \)

\( \text{WHERE } - F^+ = \frac{2m \sigma}{m \sigma} = m \Phi \)
Having done the heavy lifting, let’s consider the \( \frac{\lambda}{3!} \phi^3 \) vertex.

Ignoring the tadpole term which can be absorbed into a field redefinition, this gives us the most general renormalizable theory of \( \Delta \phi^4 \).

\[
S_{\text{int}} \left[ \frac{\lambda}{3!} \int (\phi(3)) (\phi(2)) (\phi(1)) = \frac{\lambda}{3!} \int d^4x \int d^2\theta \frac{\delta^3}{\delta\phi(3)} \frac{\delta^2}{\delta\phi(2)} \frac{\delta^2}{\delta\phi(1)} \right]
\]

\[
= \lambda \int d^4x \int d^2\theta \left[ \frac{1}{4} \int d^2\theta \frac{\delta^2}{\delta\phi(3)} \frac{\delta^2}{\delta\phi(2)} \frac{\delta^2}{\delta\phi(1)} \right]
\]

\[
= \lambda \int d^8z \int \frac{\delta^4}{\delta\phi(3)} \left[ \frac{\delta^2}{\delta\phi(2)} \frac{\delta^2}{\delta\phi(1)} \right]
\]

\[
\text{General lesson: } 2 \text{ propagator legs come w/ } \frac{\delta^4}{\delta\phi(3)} \text{ in 3 pt int.}
\]

\[
\text{of mass term: } 1 \text{ propagator leg comes w/ } \frac{\delta^4}{\delta\phi} \text{ in 2 pt int.}
\]

\[
\text{Recall please are } \Delta \phi^4 \text{ projects!}
\]

\[
\text{We can already guess what they do:}
\]

\[
\text{Pick out fermions in Yukawa interaction}
\]

\[
\text{Q: what about scalar potential quard form?}
\]

\[
\text{Recall } V(\phi) = \lambda^2 \phi^4 \text{ is form of 4 pt scalar is constrained by super to relate to Yukawa.}
\]

\[
\text{Superfield:}
\]

\[
\Delta \phi^4 \text{ act on the attached propagator}
\]

But tree-level diagrams are boring anyway.

Trivial to do in components, easier since we usually only care about specific external status.
Feynman Rules for $\lambda S F$

- Propagators are on p. 9
- Vertices from the Superpotential
  \[ \lambda \text{ point vertex } (x^{\alpha 0}) \]
  Gets $(n-1)$ factors of $B^3/4$ on attached propagators
- Integrate over loop momenta $i$ for 0 vertices
- (usual factors of $-1$ from ghosts in gauge fix)

Vector Superfield Remark

VSF's are easier since their interactions w/ VSF live in the Kähler potential $\rightarrow$ don't need to finagle VSF $\Rightarrow$ unconscr. SF

But: kinetic terms live in field strength VSF.

This all follows the above tricks, except that one has to do all the usual gauge fixing. We won't bother.

Raison d'être: loops!

We are interested in constructing the effective action $\Gamma$ in a loop expansion (leading order).

How do: amputate external legs
Multiply by sources $\phi, \bar{\phi}$ (omt $B^3/4, B^2/4$)

This will allow us to integrate by parts.

Eq. 1-loop correction to 2 point function $W/M = 0$

Useful Identities:

\[ \delta^4(x_i - x_j) \bar{D}_1(-p) = -D_2(-p) \delta^4(x_i - x_j) \]
\[ \bar{D}^2 D^2 \delta^4(x_i - x_j) = 16 \]

[Hint: write $\delta^4(x_i - x_j) = (x_i - x_j)^4 (x_i - x_j)^{2} x_j]
\[ \Gamma[\Phi, \bar{\Phi}] = \frac{\lambda^2}{2} \int \frac{d^4p}{(2\pi)^4} \frac{d^4p'}{(2\pi)^4} \bar{\Phi}(-p, \theta) \Phi(p, \theta') \]

\[ \times \int d^4k \delta^4(\theta - \theta') \left( \frac{1}{(p + k)^2} \right) \frac{1}{k^2} \delta^4(\theta - \theta') \left( \frac{5^2}{4} \right) \]

\[ = \frac{\lambda^2}{2} \int \frac{d^4p}{(2\pi)^4} \frac{d^4p'}{d^4p} \bar{\Phi}(-p, \theta) \Phi(p, \theta) \]

\[ \times \int d^4k \frac{1}{k^2(p + k)^2} \frac{1}{16} D^2(k) E^2(k) \delta^4(\theta - \theta') \]

\[ = \frac{\lambda^2}{2} \int \frac{d^4p}{(2\pi)^4} \bar{\Phi} \Phi \int d^4k \frac{1}{k^2(p + k)^2} \sim \frac{\log A}{16\pi^2} \]

What about the other diagrams? (in the \( \mu \to 0 \) limit)

\[ \Phi \quad \Phi \quad \Phi \quad \Phi \]

\[ \Phi \quad \phi \quad \Phi \]

Where we've written chiral \( \Theta \)

These are proportional to \( \delta^2(\theta - \theta') = \delta^2(\phi - \phi') = \delta(0) = 0 \)

\( \Rightarrow \) No mass renormalization

Check: \( \frac{\phi \bar{\phi}}, \frac{\Phi \bar{\Phi}} \) internal propagators have funny factors of \( 5^2/4 \), but these just reinforce chirality so that we do indeed get \( \delta^2(\theta - \theta') \).

Why is \( \delta^2(p) = 0 \)? For Grassmann \( \varphi = \delta^2(\theta) = \theta^2 \).

\( \Rightarrow \) Ward cancellation then.

\( \Rightarrow \) No miraculous cancellations between fermions + bosons.
NEW SOME INTERESTING REMARKS

1. REGULARIZATION OF SUPERSYMMETRIC THEORIES

WE SAW THAT $g^2$ AT 1 LOOP IS UV DIVERGENT. ONE OF THE BENEFITS OF THE SUPERFIELD FORMALISM
IS THAT SUSY IS MANIFEST AT EVERY STAGE, SO IT WOULD BE A SHAME TO MESS IT ALL UP WHEN WE REGULATE OUR INTEGRALS.

IN OTHER WORDS, CHOOSING A BAD REGULATOR WILL VIOLATE SUPERSYMMETRY IN RECURRENCE CALCULATIONS. (THIS IS WHY WE ALWAYS USE DUA REG - WOULDN'T SCREW UP LORENTZ OR INTEGRAL GAUGE SYM.) WE WANT TO PRESERVE OUR WARD IDENTITIES.

IN SUSY, DUA REG DOESN'T WORK! CHANGING D MAKES IT HARD TO KEEP # BOSONS = # FERMIONS EQUAL. THE CURRENT BEST STRATEGY IS DIMENSIONAL REDUCTION (DDRED) BY SIEGEL. ('79)

DO ALL $Y$ ALGEBRA (IGNED DUE TO ALGEBRA) IN 4D MOMENTUM INTEGRALS IN (2+6) DM.

THIS IS NOT PERFECT! SUSY NOT MANIFEST (WILL!

HAVE TO SACRIFICE FREEZING IDENTITY + E. ANOMALIES
BUT HELPS AT LEAST UP TO 2 LOOPS.
CURRENT STATUS: hep-p://9303-21-8

REMARK: CAN DO NDA ON SUSY GRAPHS

see: West 81 4-5 82 Gates 86 6 (R.99) hep-p://41/082200

1 LOOP GRAPH, N VECTORS, P PROPAGATORS: $L+V = P+1$
1 REAL PROP ($\Phi^2, \Phi^2$), E EXT LINES ($\bar\Phi$, $\Phi$)

VECTORS: $V$ FACTORS OF $V^2 \sim k^2$
PROPAGATORS: $P$ FACTORS OF $k^2$ W/ ADDITIONAL FACTORS OF $D^{1/2} \sim k^{-1}$
LOOPS: $L^2 \sim k^4$ BUT USES UP $D^2 \sim k^2$ FACTORS
EXT LINES: $E$ FACTORizes ONE $D^2 \sim k$ PER VERT (NO $D^2/4$ ON EXT LINE)

SUMMARY: $D = 4L - 2L - 2P + 2V - C + E = 2 - C - E$
The effective action is supported on a single l-th integral. We saw this in our sample calculation.

Consider any loop in a diagram:

\[ \gamma^{l-1} \quad \text{a vertex has a } g^l(\theta_1 - \theta_3) \]

\[ \sim g^l(\theta_1 - \theta_2) g^l(\theta_2 - \theta_3) \ldots g^l(\theta_n - \theta_1) \]

These have a bunch of D's acting on them. Can reduce to

\[(D^2)^l, (D^2)^k, g^l(\theta_1 - \theta_2), (D^2)^l, (D^2)^k, g^l(\theta_2 - \theta_3) \ldots\]

Using \( D^2 D^2 = 16 D^2 \)

\( D^2 D^2 D^2 = 16 D^2 \)

Can simplify to \( l, k \leq 30, 1 \)

Now do integration by parts to get rid of all derivatives acting on, e.g., \( g^l(\theta_n - \theta_1) \). (Recall total derivatives of Grassmannian quantities vanish upon integration.

Then use \( g^l(\theta_n - \theta_1) \) to do \( \int d\theta_n \) integral.

Iterate this process until there's only one integral left: \( \int d\theta \ (D^2)^k, \ldots \). This is only nonzero when \( l = k = 1 \).

Terms with wrong D structure vanish!

Some contributions to \( l = 0 \Rightarrow \) nonrenormalization.

Eq. superpotential coupler terms cannot arise

B/c they are supertrace SUS integrals

\( \Rightarrow \) no divergence renormalization of these quantities

\( \Rightarrow \) get one-loop finiteness

\( \Rightarrow \) never mentioned holomorphic!
3. CANCELLATION THM (from #2)

TO UNDERSTAND THIS BETTER, LET'S SEE IT EXPLICITLY.

CLAIM: CLOSED CHIRAL LOOPS VANISH.

\[ \langle \phi \phi \rangle \cdots \langle \phi \phi \rangle \text{ LOOPS} = 0. \]

STRATEGY: RETURN TO THERMAL GREENS

\[ \frac{1}{2} \int d^4 \theta \int \frac{e^{iD^2}}{D^2 + m^2} J = \frac{1}{2} \int d^4 \phi d^4 \theta \int \frac{e^{iD^2}}{D^2 + m^2} J \]

\[ \Rightarrow \langle \phi \phi \rangle = \frac{m}{D^2 + m^2} \delta^2(\theta - \theta') \]

Then chiral loop \( \sim \int \frac{d^4 \theta}{D} \delta^2(\theta_1, \theta_2) \cdots \delta^2(\theta_n, \theta_1) \)

\[ = \int \frac{d^4 \theta}{D} \delta^2(\theta_1, \theta_2) \delta^2(\theta_1, -\theta_1) \cdots \delta^2(\theta_1, -\theta_1) \delta^2(\theta_1, \theta_1) \]

\[ = \delta(2z_1 - 2z_2) \delta(2z_1 - 2z_2) \delta(2z_1 - 2z_2) \]

\[ \therefore \text{DIAGRAM VANISHES} \]

\[ \Rightarrow \text{CANCELLATION BETWEEN BOSONS + Fermions} \]

\[ \text{(diagram)} \]

\[ \Rightarrow 0 \]

Also: \( \Rightarrow 0 \), no tachyons!

COR: EFFECTIVE POTENTIAL VANISHES FOR CLASSICAL SUSY THM

EFFECTIVE POTENTIAL IS X-MPD PART OF \( \Gamma \)

\[ V = \int d^4 \phi d^4 \theta \frac{e^{iD^2}}{D^2 + m^2} \langle \phi \phi \rangle, D\langle \phi \phi \rangle, \cdots \]

But if SUSY PRESEERVED CLASSICALLY, \( V \)S HAVE NO \( \theta \) DEPENDENCE \( \Rightarrow \delta \langle \phi \rangle = 0. \)

(WE, OF COURSE, KNEW THIS FROM SUSY ALGEBRA ALREADY.)

CAN WE STILL DO SUPERTEPERS W/ BROKEN SUSY?

YES! \[ \text{SUPERTEPERS FOLLOW} \]

\[ \text{THEOREM} \]

\[ \text{SEE EG. SCHOUTE, Z. PHYS C, } 28, 545 - 553 \ (1985) \]
ONE FINAL REMARK.

THE USEFULNESS OF SUPERGRAF TECHNIQUES ARE EXEMPLIFIED IN THE CALCULATION OF THE EFFECTIVE KAHLER POTENTIAL.

Recall $W$ IS NOT RENORMIALIZED (via many arguments)
$\Rightarrow$ BUT $\phi$ WAVEFUNCTION RENORMALIZATION

$\Rightarrow$ eg. of NSUSY $\beta$ function vs. holomorphic $\beta$ function.

GENERALLY ONE LOOP CORRECTIONS CAN BE VERY IMPORTANT IN SUSY: eg. STABILIZING MODULI SPACES.
FOR EXAMPLE, IN METAStABLE SUSY Models ONE HAS TO THOROUGHLY CALCULATE THE GOLDBERG-MEINEREK EFFECTIVE POTENTIAL TO DETERMINE THE STABILITY OF ONE'S METAStABLE STATE. THIS IS DONE IN CONCLUSIONS.

ONE CAN ALSO DO THIS IN $W$ SUPERGRAFs.
$\Rightarrow$ EFFECTIVE KAHLER POTENTIAL

$\Rightarrow$ IT IS NOW OBVIOUS WHY THIS SHOULD BE A $K$ CORRECTION
($W = 0$, *NO* other picks up on $\beta\phi$)

I CLOSED FORM 1-LOOP GENERAL FORMULA (HEP-TH/9605149)

$$K_{\text{eff}} = \frac{-1}{2\pi i} \text{Tr} \left[ H \left( \frac{\partial H}{\Lambda^2} \right) \right]$$

$\uparrow$

SUGGEST REF. HAB, eg. $M \sim M_{\text{sup}}$.

doesn't supertrace!

$$V_{\text{scalar}} = K_{\text{eff}} |W_\phi|^2$$ (see appendix)

ONE MAJOR DRAWBACK: MANIFESTLY SUPERGRAF-METRIC
SO FORMULA IS ONLY USEFUL IN SMALL SUSY LIMIT.
(EG. SEE, eg. ORIGINAL SUSY PAPERS, APP 5 OR INFLATIONIST-SEIBERG NOTES ON SUSYING FOR DISCUSSION.)

[Would SUSY field contents help here?]

SEE ALSO: UPDATING PAPER BY BEN KAIN + UNDERGRADS
FOR KAHLER ANALYSIS OF SUSY MODELS & $\phi$. 