BSM JOURNAL CLUB SPRING 2013

SEIBERG - WITTEN MARIO

SEE: th/9701069 & TERNING ch 13

1. $\mathcal{N}=2$
2. MODULI SPACE & SINGULARITIES
3. 'T HOOFT POLYAKOV MONOPOLE ← BRING IT ALL TOGETHER
(4.) CONFINEMENT IN $\mathcal{N}=1$

$\underline{\mathcal{N}=2 \text{ SUSY}} \rightarrow SU(2)_R$

TRIVIAL EXTENSION

$$\{Q^a_\alpha, \bar{Q}_{\dot\beta b}\} = 2\sigma^\mu_{\alpha\dot\beta} \boxed{\delta^a_b}$$

$$\left.\begin{array}{l} \{Q^a_\alpha, Q^b_\beta\} = 2\sqrt{2}\, \varepsilon_{\alpha\beta}\varepsilon^{ab} Z \\ \{\bar{Q}, \bar{Q}\} = \text{———}''\text{———} \end{array}\right\}$$ NONTRIVIAL EXTENSION

w/ $Z = a\left(n_e + \frac{4\pi i}{g^2} n_m\right)$ CENTRAL CHARGE

from where? Q's ARE NOETHER CHARGES
0^{th} COMPONENT of FERMIONIC PART of SUPERCURRENT

REPS: MASSLESS - SAME AS IN $\mathcal{N}=1$

GO TO FRAME $P_\mu = (M, \vec{0})$

$$\leadsto \{Q^a_\alpha, \bar{Q}_{\dot\alpha b}\} = \begin{pmatrix} 0 & 0 \\ 0 & 4M \end{pmatrix} \delta^a_b$$

$I, J \in \{1, \dots, \mathcal{N}\}$

DEF: $a^I = \frac{1}{2\sqrt{M}} Q^I_2 \qquad a^{I\dagger} = \frac{1}{2\sqrt{M}} \bar{Q}^I_{\dot 2}$

& NICE ANTICOMM. RELATIONS:

$$\{a^I, a^{J\dagger}\} = \delta^{IJ} \qquad \{a^I, a^J\} = \{a^{I\dagger}, a^{J\dagger}\} = 0$$

MASSIVE : $(\mathcal{N}=2)$: $\{a_\alpha, a^\dagger_\beta\} = \delta_{\alpha\beta}(M + \sqrt{2}Z)$
$\{b_\alpha, b^\dagger_\beta\} = \delta_{\alpha\beta}(M - \sqrt{2}Z)$

IF $M = \sqrt{2}Z$, THE b RAISING/LOWERING OPS HAVE A TRIVIAL ALGEBRA

→ SHORT MULTIPLET

(THIS IS EFFECTIVELY A MASSLESS REP, WHILE MASSIVE REPS GENERICALLY HAVE MORE STATES)

REMARK: $\exists$ $\mathcal{N}=2$ SUPERSPACE FORMULATION W/ EXTRA θ's.

$\mathcal{N}=2$ VECTOR MULTIPLET

$$\Phi = (\underbrace{A, \psi}_{\chi SF}, \underbrace{\lambda, A_\mu}_{VSF})$$

$\mathcal{N}=2$ VEC PLET

$\mathcal{N}=1$ COMPONENTS

$\mathcal{N}=2$ HYPERMULTIPLET (MASSIVE)

IF $M = \sqrt{2}Z$, THEN THE SHORT MULTIPLET IS A PAIR OF $\mathcal{N}=1$ χSF: $(Q, \tilde{Q})$ W/ $\tilde{Q} = Q^\dagger$.
(for LONG HYPER $(Q, \tilde{R})$ W/ $\tilde{R} \neq Q^\dagger$).

NOW WRITE $\mathcal{L}$ of $\mathcal{N}=2$ GAUGE THY:

$$\mathcal{L} = \frac{1}{4\pi} \mathrm{Im}\left[\mathrm{Tr} \int d^2\theta\, d^2\tilde{\theta}\, \frac{1}{2}\tau\, \Psi^2\right]$$

$$\underset{\substack{\uparrow \\ \text{HIGHER ORDER}}}{\mathcal{L}_{NR}} = \frac{1}{4\pi} \mathrm{Im}\, \mathrm{Tr} \int d^2\theta\, d^2\tilde{\theta}\, \underbrace{\mathcal{F}(\Psi)}_{\text{PREPOTENTIAL}}$$

$$= \frac{1}{8\pi} \mathrm{Im}\left[\int d^2\theta\, \mathcal{F}_{ab}(\Phi)\, W^{a\alpha} W^b{}_\alpha + \int d^2\theta\, d^2\bar{\theta}\, (\Phi^\dagger e^{2gV})^a \mathcal{F}_a(\Phi)\right]$$

$\mathcal{F}_{ab} = \frac{\partial^2 \mathcal{F}}{\partial\Phi^a \partial\Phi^b}$ where Φ is AN $\mathcal{N}=1$ XSF

NOTE τ is 2nd DERIVATIVE of $\mathcal{F}$

THIS IS A NONPERTURBATIVE STATEMENT.

<u>SO FOCUS ON $SU(2)$ $\mathcal{N}=2$ PURE SYM</u>

$\uparrow$ rank 1 (dim H)

recall A is SCALAR

$$\mathcal{L} = \frac{1}{g^2} \mathrm{Tr}\Big[-\frac{1}{4}F^2 + \frac{g^2\Theta_{YM}}{32\pi} F\tilde{F} + (D_\mu A)^\dagger (D^\mu A) - \frac{1}{2}[A^\dagger, A]^2$$
$$- i\lambda\sigma^\mu D_\mu\bar{\lambda} - i\bar{\psi}\bar{\sigma}^\mu D_\mu\psi - i\sqrt{2}[\lambda,\psi]A^\dagger - i\sqrt{2}[\bar{\lambda},\bar{\psi}]A\Big]$$

IN PARTICULAR:

$$V(A) = [A^\dagger, A]^2$$

CONSIDER <u>MODULI SPACE</u>:

IF $A = A^i H_i$ w/ $H_i \in$ ~~CASIMIR~~ CARTAN, then $V(A) = 0$

IN $SU(2)$, $\langle A\rangle = \frac{1}{2} a\,\sigma_3$ describes vacuum manifold

... A LITTLE MORE COMPLICATED, ACTUALLY: WEYL REFLECTION

IN PARTICULAR $A \to -A$

WANT TO IDENTIFY DIFF $\langle A \rangle$ THAT ARE GAUGE IDENTICAL (ie PHYSICS IS IDENTICAL), ie WANT TO MOD OUT BY THIS WEYL REFLECTION.

SO THE REAL MODULI SPACE PARAMETER COORD. IS

$$u = Tr(A)^2 = \frac{1}{2}a^2$$

ACTUALLY: PREPOTENTIAL GIVES NONTRIVIAL METRIC — in full QUANTUM ACTION.

$$g_{ab} = \mathrm{Im}\, \partial_a \partial_b \mathcal{F} \quad \Leftrightarrow \left(ds^2 = \mathrm{Im}\, d\left(\frac{\partial \mathcal{F}}{\partial a}\right) da \right)$$

$a, b \in \{1, \dots, \text{rank } G\}$

MODULI SPACE IS DESCRIBED BY

- u (OR a) ← COORDS
- $\partial \mathcal{F} / \partial a$ ← GIVES METRIC.

WE'RE GOING TO STUDY $\left(a, \partial \mathcal{F}/\partial a\right)$ SPACE

YOU'VE HEARD OF S-DUALITY & T-DUALITY IN EM THEORIES

T-DUALITY: $\tau \to \tau + 1$ IS AN INVARIANCE

($\Theta \to \Theta + 2\pi$)

~~S-DUALITY : IN USUAL QED.~~

S-DUALITY : IN USUAL QED n_e ARE PERTURBATIVE
n_m ARE NON-PERTURBATIVE

$$\mathcal{L} \sim F^2 + A_\mu J^\mu$$
$$(\rho, \vec{J})$$

CAN ADD FIELDS w/ ELEC CHARGE
BUT NOT w/ MAG CHARGE w/o ~~ADDING~~ BREAKING BIANCHI

DUAL

$\exists$ THY w/ n_M PERTURBATIVE
n_e NON PERTURBATIVE
$g \to 1/g \quad (\tau \to 1/\tau)$

So: T DUALITY $\tau \to \tau + 1$
S DUALITY $\tau \to 1/\tau$ $\Big\}$ $SL(2,\mathbb{Z}) : \tau \to \dfrac{a\tau + b}{c\tau + d}$

HOW DOES $SL(2,\mathbb{Z})$ ACT ON $\left(a, \partial f/\partial a\right)$

eg T: $\tau \to \tau + 1$ w/ $\tau = \partial^2 f/\partial a$

$$\frac{\partial f}{\partial a} = \int (\tau + 1)\, da \quad \leftrightarrow \quad \frac{\partial f}{\partial a} \to \frac{\partial f}{\partial a} + a$$

$a \to a$. ie: $T : \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} a \\ \partial f/\partial a \end{pmatrix}$

So T-DUALITY: $\begin{pmatrix}1&1\\0&1\end{pmatrix}$

$$\partial f/\partial a \to \partial f/\partial a + a$$
$$a \to a$$

S-DUALITY: $\begin{pmatrix}0&1\\-1&0\end{pmatrix}$

$$\partial f/\partial a \to a$$
$$a \to -\partial f/\partial a \quad (\equiv -a_D)$$

USING ANOMALOUS $U(1)_R$ & SPURION ANALYSIS FOR τ

$$\left[\begin{array}{l}\tau \xrightarrow{U(1)_R} \tau - n F\tilde{F} \\ U(1)_R \to n F\tilde{F}\end{array}\right]$$
SPURIOUS SYM.

So YOU OBTAIN: [the QUANTUM PREPOTENTIAL]

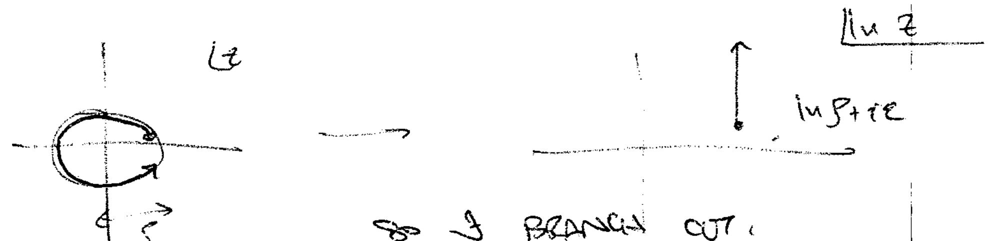
$$\mathcal{F} = \underbrace{\frac{i}{2\pi} A^2 \ln \frac{A^2}{\Lambda^2}}_{\text{PERTURBATIVE}} + \underbrace{\sum_{k=1}^{\infty} \mathcal{F}_k \left[\frac{\Lambda}{A}\right]^{4k} A^2}_{\text{INSTANTONS}}$$

So WHAT'S THE IDEA?

- LOGARITHM KINDA SUCKS $\qquad a = \langle A \rangle \gg \Lambda$

$$\frac{\partial \mathcal{F}}{\partial a} = \frac{2ia}{\pi} \ln \frac{a}{\Lambda} + \frac{ia}{\pi}$$

HAVE TO INTRODUCE MONODROMY of $\ln z$ w/ $z \in \mathbb{C}$



So $\exists$ BRANCH CUT.

MONODROMY is $\mathbb{Z}$ (multiples of 2π)

$u = \frac{1}{2}a^2$; $\ln u \to \ln u + 2\pi i$

$\ln a \to \underbrace{\ln a + \pi i}$ or $a \to -a$

$a e^{\pi i} = -a$

MONODROMY of $\begin{pmatrix} \partial f/\partial a \to -\partial f/\partial a + 2a \\ a \to -a \end{pmatrix}$

BUT SPACE IS SINGULARITY (MONODROMY)

$\downarrow$

PROCESS OF INTEGRATING THINGS OUT WASN'T VALID: MASSIVE DOF BECAME MASSLESS

STARTED w/ (A, Ψ, λ, A_r)

$\uparrow$

HIGGS. $SU(2) \to U(1)$, CAN IMAGINE A MASSIVE A_r BECOMES MASSLESS? NOT POSSIBLE!

REF. FOR NONABELIAN MONODROMY

Why? MODULI SPACE IS u. EXPECT $u=0 \to$ MASSLESS GAUGE BOSONS (no vev). ~~BUT OF~~

BUT: EXPECT AT LEAST 2 SINGULARITIES, REMNANT of $U(1)_R$: $u \to -u$.

$\rightarrow$ so CAN'T BE GAUGE BOSON BECOMING MASSLESS.

SO WHAT'S BECOMING MASSLESS?

↳ turns out $\exists$ monopole in $SU(2)$ SYM and mass $\sim Z$.

SEIBERG-WITTEN GUESSED THESE BECOME MASSLESS IN THE MODULI SPACE.

IS THIS COMPATIBLE w/ $\mathcal{N}=1$?
TO GO FROM $\mathcal{N}=2 \rightarrow \mathcal{N}=1$: $\Psi = (\Phi, V)$
ADD, SAY $m^2 = \mathrm{Tr}\, \Phi^2$