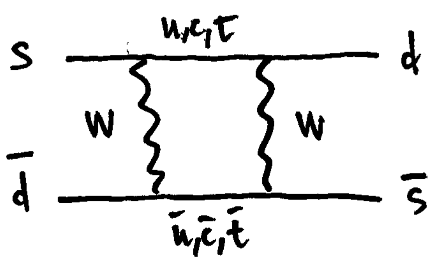
The Supersymmetric Flavor Problem

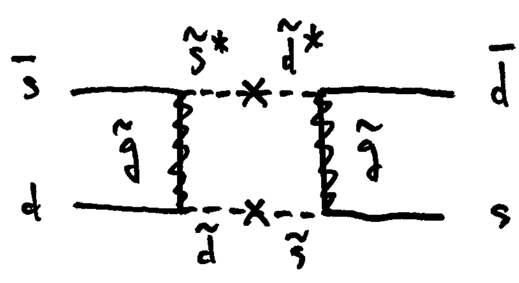
In the Standard Model the flavor symmetry is broken by the yukawa couplings. By rotating the fields, the flavor symmetry breaking is put into the charged current interactions and there are no Flavor Changing Neutral Currents at tree level in the Standard Model.

But on loop level FCNCs are generated and in low energy phenemenology it manifests as eg. $K^0\bar{K}^0$, $D^0\bar{D}^0$, mixing (see DC's journal club on $D^0\bar{D}^0$ mixing)

For the Kaon system we find mixing from diagrams of the sort



In supersymmetric theories with flavor violating massterms we get contributions of the sort



From experiments we find that (for approximatly universal squark masses $m_{\tilde{q}}$)

$$\frac{m^2_{\tilde{s}\tilde{d}}}{m^2_{\tilde{q}}} \lesssim \frac{m_{\tilde{q}}}{50\,\mathrm{TeV}}$$

Thus the squark mass matrices must be nearly diagonal.

Now the mass terms might potentially get big contributions from interactions of the type

$$\int d^4\theta \; \Sigma^\dagger \Sigma \, \frac{c_{ij}}{M_{Pl}^2} \, Q_i^\dagger Q_j \; ,$$

Σ susy breaking with $\langle \Sigma \rangle = \theta^2 \Lambda_H^2$, Q_i squark field.

From the above we find squark mass terms

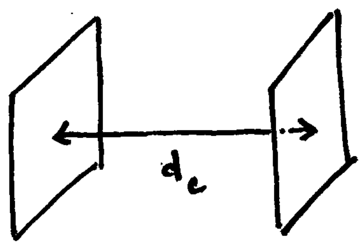
$$\Lambda_H^4 \, \frac{c_{ij}}{M_{Pl}^2} \, \tilde{q}_i^* \tilde{q}_j$$

Now c_{ij} might not be flavor diagonal and we have a potential problem. We need to find a way to suppress such couplings, to "sequester" them, "hiding" them away.

Note that there are two different types of Planck mass suppressed operators. First, operators coming from the UV-complete theory, such as string theory. Second, operators arising from exchange of supergravity field in the low energy theory (of string theory)

The second type is harmless since gravity is flavor blind (it of course couples in a flavor preserving way). The first type is potentially dangerous since we expect the UV-complete theory to break flavor symmetry. In fact it must to generate yukawas.

The solution comes from going to a higher dimensional theory. We consider embedding the MSSM on a 3-brane into a higher dimensional space. Then we imagine that supersymmetry is preserved locally on the brane but broken else where in the bulk eg. by breaking supersymmetry on some other brane, having branes at angles, dimensional reduction, gaugino condensation etc.



MSSM, Q_i $\cancel{\text{SUSY}}$, Σ
visible   hidden

Then susy breaking is propagated from the hidden sector to the visible sector.

The contribution from the first type of fields with masses $m \sim M_{Pl}$ that violates flavor is then exponentially suppressed $\sim e^{-md_c}$ as long as we consider large extra dimensions with compactification scale $\mu_c = \frac{1}{d_c} \ll M_{Pl}$.

The harmless type two fields will infact also contribute vanishing to the squark masses but we will find contributions to gaugino and squark masses at one-loop and two-loop level, via "anomaly mediation".

4D effective field theory

We now want to write down a 4D effective field theory of supergravity coupled to the MSSM, Q, and the susy breaking sector Σ.

Supergravity

on-shell field content e_μ^a spin-2, ψ_μ^α spin-$3/2$.

off-shell we have to introduce auxiliary fields A_μ, F_Φ.

Why the auxiliary fields takes this form can be understood when writing the theory as a superconformal theory, which I don't have time to go into the details of here.

Notice that F_Φ is the only field that can take on supersymmetry breaking vev without destroying Lorentz invariance.

To explain F_Φ we will make a short detour into Brans-Dicke theory and its supersymmetric version.

Brans–Dicke theory

The Einstein Hilbert action is not invariant under local Weyl-transformations $g_{\mu\nu} \to \tilde{g}_{\mu\nu} = \Omega^2(x)\, g_{\mu\nu}$ (not global either) Though it can be made invariant by the introduction of a compensator:

$$S_{EH} = \frac{M_{Pl}^2}{2} \int d^4x \sqrt{g}\, R \longrightarrow S_{BD} = \frac{M_{Pl}^2}{2} \int d^4x \sqrt{g} \left(\phi^2 R - \frac{1}{6} (\nabla\phi)^2 \right)$$

Now S_{BD} is infact invariant under the transformation

$$g_{\mu\nu} \to \tilde{g}_{\mu\nu} = \Omega^2(x)\, g_{\mu\nu} \;, \quad \phi \to \tilde{\phi} = \Omega^{-1}(x)\, \phi$$

We get back S_{EH} by letting $\phi = 1$.

In the supersymmetric version we introduce a chiral superfield compensator $\Phi = (\phi, \chi, F_\Phi)$ and the non-conformal supergravity is obtained by putting $\Phi = (1, 0, F_\Phi)$.

Going back to the 4D effective field theory the action turns out to be

$$\mathcal{L} = \sqrt{g}\Big\{ \int d^4\theta \, f(Q^\dagger, e^V Q)\, \Phi^\dagger \Phi + \Big[\int d^2\theta \left(\Phi^3 W(Q) + \tau(Q) \mathcal{W}_\alpha^2 \right) + \text{h.c}\Big]$$
$$- \frac{1}{6} f(\tilde{q}^*, \tilde{q})\, R \quad + \text{auxiliary fields and fermions} \Big\}$$

The factors of Φ are new compared to global supersymmetry and we now write f instead of K. The function f is related to the Kähler potential through $f = -3M_{Pl}^2 e^{-\frac{K}{3M_{Pl}^2}}$.

This gives us the right theory in the limit $M_{Pl} \to \infty$

$$f \to -3M_{Pl}^2 + K + O\left(\frac{1}{M_{Pl}^2}\right) \quad \text{as} \quad M_{Pl} \to \infty ,$$

which gives the right term in front of R and the right kinetic terms for Q.

Note that one can do a Weyl transformation $g_{\mu\nu} \to e^{+\frac{K}{3M_{Pl}^2}} g_{\mu\nu}$ and solve for auxiliary fields to get

$$\mathcal{L} = \sqrt{g}\Big\{ \frac{M_{Pl}^2}{2} R + K_{i\bar{j}} D_\mu \tilde{q}^{i*} D^\mu \tilde{q}^j - \mathcal{V}(\tilde{q}^*, \tilde{q})$$
$$- \operatorname{Re}\tau(\hat{q}) FF - i \operatorname{Im}\tau(\hat{q}) F\tilde{F} + \text{fermions} \Big\} ,$$

$$\mathcal{V} = e^{\frac{K}{3M_{Pl}^2}} \cdot \left(D_i W \overline{D_j W} K^{i\bar{j}} - 3|W|^2 \right) + \frac{g^2}{2}\left(K_i t^a q^i \right)^2$$

with $D_i W = W_i + K_i W$ though this form is hardly useful.

Supersymmetry breaking mass terms

We see that there are three possible places where we can find mass terms:

i) vev for F_Φ

ii) coupling of Σ and Q in f

iii) coupling to curvature R

iii) is definitely not feasible in our universe where the cosmological constant is tiny

i) actually vanishes by miraculous cancellation or by noting the fact that Φ can be rotated away by the transformation $Q\Phi \to \Phi$.

ii) this seems to be the only possibility!

Thus to guarantee the vanishing of flavor violating squark masses we have to remove all direct couplings of Σ and Q inside f.

Now we know a theory where this is guaranteed: in a sequestered theory where all mass terms are killed when we decompactify the theory letting $\mu_c, M_{pl} \to \infty$!

Thus a sequestered theory is of the form

$$f = -3M_{Pl}^2 + f_{vis} + f_{hid}$$

$$W = W_{vis} + W_{hid}$$

$$\tau w^2 = \tau_{vis} w_{vis}^2 + \tau_{hid} w_{hid}^2$$

which is guaranteed by the construction of the theory.

Note that in this theory the Kähler potential takes the form

$$K = -3M_{pl}^2 \log\left(1 - \frac{f_{vis}}{3M_{pl}^2} - \frac{f_{hid}}{3M_{pl}^2}\right)$$

Now plugging this into V we indeed find no mass terms which seems like a miraculous thing in the low energy 4D effective theory!

Old solution (very implausible)

We take $K = |\Sigma|^2 + Q^{\dagger} e^{V} Q$ (rather than in f) which gives us for f:

$$f = -3M_{pl} e^{-\frac{K}{3M_{pl}}} = -3M_{pl} + |\Sigma|^2 + Q^{\dagger} e^{V} Q - \frac{2}{3M_{pl}^2} |\Sigma|^2 Q e^{V} Q + \ldots$$

Now the coupling in f is flavorblind but this is a very strange assumption since f is supposed to be generated by the UV-complete theory which is certainly not flavor blind.

Anomaly Mediated Supersymmetry Breaking

Although we found no tree-level $\cancel{susy}$ masses we do find them at loop level.

Now when Σ is decoupled and only influence the visible sector through gravity (Φ) we have the low energy 4D action:

$$\mathcal{L}_{eff} = \int d^4\theta\, Q^\dagger e^V Q\, \Phi^\dagger \Phi + \left[\int d^2\theta \left(\Phi^3 (mQ^2 + yQ^3) + \frac{1}{g^2} W_\alpha^2\right) + h.c.\right]$$

Performing the redefinition $Q\Phi \to Q$ we find

$$\mathcal{L}_{eff} = \int d^4\theta\, Q^\dagger e^V Q + \left[\int d^2\theta \left(m\Phi Q^2 + yQ^3 + \frac{1}{g^2} W_\alpha^2\right) + h.c.\right]$$

Now except for the μ-term the action is classically invariant under the rescaling.

But at the quantum level the theory suffers from an anomaly and under the rescaling and we have $\tau \to \tau - 2b_0 \ln \Phi$

From the term τW_α^2 we then find a mass term for the gaugino $m_\lambda = -b_0 g^2 F_\Phi$.

We can say more by noticing that when regulating the theory we always have to introduce a mass scale which will feel Φ. Thus the renormalized theory will take the form:

$$\mathcal{L}_{eff} = \int d^4\theta \, Z\left(\frac{\mu}{\Lambda_{UV}\Phi}, \frac{\mu}{\Lambda_{UV}\Phi^\dagger}\right) Q^\dagger e^V Q$$

$$+ \left[\int d^2\theta \, y Q^3 + \tau\left(\frac{\mu}{\Lambda_{UV}\Phi}\right) W_\alpha^2 + h.c. \right]$$

We will now show that $Z \ni |F_\Phi|^2 \theta^2 \bar{\theta}^2$ thus providing us with a squark mass term from the kinetic term above.

Now Φ will only appear in Z as $\Phi^\dagger\Phi = |\Phi|^2$, we can then taylor expand $Z(\mu/\Lambda_{UV}|\Phi|)$ around $|\Phi| = 1 + \ldots$

We then find, see Appendix below

$$\ln Z\left(\frac{\mu}{\Lambda_{UV}|\Phi|}\right) = \ln Z\left(\frac{\mu}{\Lambda_{UV}}\right) - \frac{1}{2}\gamma F_\phi \theta^2 - \frac{1}{2}\gamma F_\phi^* \bar{\theta}^2$$
$$+ \frac{1}{4}|F_\phi|^2 \theta^2\bar{\theta}^2 \left(\frac{\partial\gamma}{\partial g}\beta_g + \frac{\partial\gamma}{\partial y}\beta_y\right)$$

so that our factor in the kinetic term becomes

$$Z\left(\frac{\mu}{\Lambda_{UV}|\phi|}\right) = \exp\left\{ \ln Z - \frac{1}{2}\gamma F_\phi \theta^2 - \frac{1}{2}\gamma F_\phi^* \bar{\theta}^2 \right.$$
$$\left. + \frac{1}{4}|F_\phi|^2 \theta^2\bar{\theta}^2 \left(\frac{\partial\gamma}{\partial g}\beta_g + \frac{\partial\gamma}{\partial y}\beta_y\right)\right\}$$

Now again do a redefinition of your fields

$\exp\left(\frac{1}{2}\ln Z - \frac{1}{2}\gamma F_\phi \theta^2\right) Q \to Q$ thus we get

$$Z\left(\frac{\mu}{\Lambda_{UV}|\phi|}\right) \to \exp\left(\frac{1}{4}|F_\phi|^2\theta^2\bar{\theta}^2\left(\frac{\partial\gamma}{\partial g}\beta_g + \frac{\partial\gamma}{\partial y}\beta_y\right)\right)$$

We taylor expanding this we find a squark mass term

$$m_{\tilde{q}}^2 = -\frac{1}{4}|F_\phi|^2 \left(\frac{\partial\gamma}{\partial g}\beta_g + \frac{\partial\gamma}{\partial y}\beta_y\right) \qquad \square$$

<u>Appendix</u> (taylor expanding)

$$|\phi|^2 = 1+\theta^2 F + \bar\theta^2\bar F + \theta^2\bar\theta^2 |F|^2$$

$$\sqrt{|\phi|^2} = 1 + \frac{1}{2}(\theta^2 F + \bar\theta^2 \bar F + \theta^2\bar\theta^2|F|^2) - \frac{1}{8}\cdot 2\,\theta^2\bar\theta^2|F|^2$$

$$= 1 + \frac{1}{2}(\theta^2 F + \bar\theta^2\bar F) + \frac{1}{4}\theta^2\bar\theta^2|F|^2$$

$$\frac{1}{\sqrt{|\phi|^2}} \approx 1 - \left[\frac{1}{2}\theta^2 F + \frac{1}{2}\bar\theta^2\bar F + \frac{1}{4}\theta^2\bar\theta^2|F|^2\right] + 2\cdot\frac{1}{4}\theta^2\bar\theta^2|F|^2$$

$$= 1 - \frac{1}{2}\theta^2 F - \frac{1}{2}\bar\theta^2\bar F + \frac{1}{4}\theta^2\bar\theta^2|F|^2$$

$$\ln Z\left(\frac{\mu}{|\phi|}\right) = \ln Z\left(\mu - \frac{1}{2}\theta^2 F\mu - \frac{1}{2}\bar\theta^2\bar F\mu + \frac{1}{4}\theta^2\bar\theta^2|F|^2\mu\right)$$

$$= \ln Z + \frac{d}{d\mu}\ln Z\cdot\left(-\frac{1}{2}\theta^2 F\mu - \frac{1}{2}\bar\theta^2\bar F\mu + \frac{1}{4}\theta^2\bar\theta^2|F|^2\mu\right)$$

$$+ \frac{1}{2}\frac{d^2}{d\mu^2}\ln Z\cdot 2\cdot\frac{1}{4}\theta^2\bar\theta^2|F|^2\mu^2$$

$$= \ln Z - \frac{1}{2}\theta^2 F\mu\frac{d}{d\mu}\ln Z - \frac{1}{2}\bar\theta^2\bar F\mu\frac{d}{d\mu}\ln Z$$

$$+ \frac{1}{4}\theta^2\bar\theta^2|F|^2\underbrace{\left(\mu\frac{d}{d\mu} + \mu^2\frac{d^2}{d\mu^2}\right)}_{\mu\frac{d}{d\mu}\left(\mu\frac{d}{d\mu}\right)}\ln Z$$

$$= \ln Z - \frac{1}{2}\theta^2 F\gamma - \frac{1}{2}\bar\theta^2\bar F\gamma + \frac{1}{4}\theta^2\bar\theta^2|F|^2\frac{d}{d\ln\mu}\gamma$$

References

Randall and Sundrum "Out of This World Supersymmetry breaking"
hep-th/9810155

Luty "2004 TASI lectures on Supersymmetry breaking"
hep-th/0509029

Terning "Modern Supersymmetry"

Nieuwenhuizen "Supergravity"
Physics Reports 68, 189-398 (1981)

Martin "A Supersymmetry Primer"
hep-ph/9709356

Everything to be said in this notes is actually found in chapter 15 and 16 of Ternings book.