The Birds and the Bs

A Case Study of $B_s \rightarrow \mu^+ \mu^-$ in the MSSM

Flip Tanedo

In collaboration with A. Dedes, J Rosiek.
Domination Game

Kobe and the Lakers ruled Game 1. Daily

- TrueHoop » Magic meltdown » Nelson's
- Experts » Simmons » Scouts Inc. » Horrifying
- Denton: Superman's mission » Magic lost
- Adande: Mamba's moment » Bynum v. Howard
- Magic page » Lakers page » Poll: Who
- Blogs: Magic Daily » Forum Blue & Gold

SERIES DETAILS
(1) LAKERS LEAD (3) MAGIC, 1-0

<table>
<thead>
<tr>
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<th>3</th>
<th>4</th>
<th>5</th>
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<td>MAGIC</td>
<td>75</td>
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<td>LAKERS</td>
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Final

- Magic
- Lakers

Recap »
Box Score »
Play By Play »
String Theory
Unification & $\sqrt{s}$
Supersymmetry
EWSB
Hadronic Physics
brown muck of
Cosmology
LHC Energy... eventually
GeV
... not to scale

$B \rightarrow \mu\mu$
TASI 2009

"small"

Physics of the Large and Small

"large"

String Theory

Unification & Vs

Supersymmetry

EWSB

LHC Energy... eventually

brown muck of Hadronic Physics

Cosmology

... not to scale

... not to scale

GeV

$10^{19}$

$10^{16}$

$10^3$

$10^2$

$10^1$

$10^0$

$10^{-6}$

Flip Tanedo, Cornell University/CIHEP

The Birds and the Bs: $B \rightarrow \mu \mu$

3/16
String Theory
Unification & $\mathcal{N}$s
Supersymmetry
EWSB
Hadronic Physics
Cosmology

... not to scale

TASI 2009

Physics of the Large and Small

"small"

"large"

... meh
Complimentarity & Flavor Physics

The Birds and the Bs: $B \rightarrow \mu\mu$
B-mesons: state-of-the-art flavor laboratories

<table>
<thead>
<tr>
<th>Meson</th>
<th>Mass</th>
<th>Mean lifetime</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^0_d$</td>
<td>5.280 GeV</td>
<td>$1.53 \times 10^{-12}$ s</td>
</tr>
<tr>
<td>$B^0_s$</td>
<td>5.370 GeV</td>
<td>$1.44 \times 10^{-12}$ s</td>
</tr>
</tbody>
</table>

B-factories ‘traditionally’ run at $\Upsilon(4S)$ resonance, which produce $B_d$, but not $B_s$.

- **B-mesons** have just the right mass and width to allow us to measure their $CP$ phase.
- Asymmetric **$B$-factories** allow us to measure the different branching ratios of $B$ and $\overline{B}$ mesons.

**Strategy**: Search for BSM in FCNC $B$-decays.
The March of the Penguins

Penguin diagram
Allows FCNC sub-diagram to occur on-shell.
The March of the Penguins

Penguin diagram
Allows FCNC sub-diagram to occur on-shell.

$b \rightarrow \mu \mu$

Flip Tanedo, Cornell University/CIHEP

The Birds and the Bs: $B \rightarrow \mu \mu$
The March of the Penguins

Penguin diagram
Allows FCNC sub-diagram to occur on-shell.

\[ b \rightarrow u_i \rightarrow s \]
The March of the Penguins

new physics

Penguin diagram
Allows FCNC sub-diagram to occur on-shell.

$B \rightarrow \mu \mu$
The March of the Penguins

Where do we look for penguins?
The March of the Penguins

Where do we look for penguins? Antarctica.
The March of the Penguins

Where do we look for penguins? Antarctica.

Very little background, penguin is dominant fauna.
Where do we look for SUSY penguins? $B_s \rightarrow \mu^+ \mu^-$. Very little background, penguin is dominant process.
Very little background

The Standard Model background is suppressed by...

- **Loop**: no tree-level contribution, \((16\pi^2)^{-1}\)
- **FCNC**: ‘GIM’ suppression, \(|V^\dagger V|_{bs}\)
- **Helicity**: Lepton mass insertion, \(m_\mu/M_{B_s}\)

<table>
<thead>
<tr>
<th>Channel</th>
<th>Expt.</th>
<th>Bound (90% CL)</th>
<th>SM Prediction</th>
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<tbody>
<tr>
<td>(B_{s}^0 \rightarrow \mu^+\mu^-)</td>
<td>CDF II</td>
<td>(&lt; 4.7 \times 10^{-8})</td>
<td>((4.8 \pm 1.3) \times 10^{-9})</td>
</tr>
<tr>
<td>(B_{d}^0 \rightarrow \mu^+\mu^-)</td>
<td>CDF II</td>
<td>(&lt; 1.5 \times 10^{-8})</td>
<td>((1.4 \pm 0.4) \times 10^{-10})</td>
</tr>
<tr>
<td>(B_{s}^0 \rightarrow \mu^+e^-)</td>
<td>CDF II</td>
<td>(&lt; 2.0 \times 10^{-7})</td>
<td>(\approx 0)</td>
</tr>
<tr>
<td>(B_{d}^0 \rightarrow \mu^+e^-)</td>
<td>CDF II</td>
<td>(&lt; 6.4 \times 10^{-8})</td>
<td>(\approx 0)</td>
</tr>
</tbody>
</table>

Clean dilepton signal, only hadronic uncertainty is \(f_B\). ‘Ideal’ for LHC.
Penguin is the dominant process

In the MSSM, the **Higgs-penguin** mediated $B_s \rightarrow \mu^+ \mu^-$ diagram is sensitive to $\tan \beta$. Recall: $\tan \beta = \frac{v_u}{v_d}$.

$$y_{b,\ell} = \frac{m_{b,\ell}}{v_d} \propto \frac{1}{\cos \beta} \xrightarrow{\tan \beta \gg 1} \tan \beta$$

Amplitude is enhanced by $\tan^3 \beta$.
The March of the Penguins

The standard model background...
The March of the Penguins

The standard model background... and SUSY at large tan $\beta$

$$Br(B_s \to \mu\mu) \approx 5 \cdot 10^{-7} (\tan \beta / 50)^6 (300 \text{ GeV} / M_{A_0})^4$$

Motivation: Grand unification, mSUGRA + $(g - 2)_{\mu}$
But what about low $\tan \beta$?

... and low $\tan \beta$?
(Heuristic plot, not to scale!)

$\text{Br}(B \to \mu\mu)$

$\tan \beta$

MSSM at large $\tan \beta$

Standard Model
But what about low tan $\beta$?

No photon penguin by Ward identity.

- Higgs penguin no longer dominant
- One has to consider interference with other diagrams
- Possibility: cancellation below SM prediction?
Scan over MSSM parameter space with respect to SM prediction and experimental limit, taking into account existing experimental bounds.

Mass insertion parameterizes flavor violation: \( \delta^{IJ}_{QXY} = \frac{(M_Q^2)_{XU}^{IJ}}{\sqrt{(M_Q^2)_{XX}^{IJ}(M_Q^2)_{YY}^{IJ}}} \)

Funnel region: Pseudoscalar and axial contributions cancel, scalar contribution is negligible; e.g. models where MSSM is extended with an additional light CP-odd Higgs.
LHCb ‘benchmark’ process

Potential...
‘Signal’ in 1Y
‘Discovery’ in 3Y

Implications on LHCb upgrade?
($B_s$ or $B_d$?)

$\text{Br}(B \rightarrow \mu \mu) \times 10^{-9}$

$5\sigma$

$3\sigma$

Standard Model Prediction

Lenzi arXiv:0710.5056
General purpose detectors...

**ATLAS Sensitivity**

Policicchio and Crosetti arXiv:0710.1206

- **Br(B_s → μμ)** \( \times 10^{-9} \)
- **Standard Model Prediction**
- **BG only, 90% CL**

**Integrated Luminosity, fb^{-1}**

- 0
- 10
- 20
- 30
- 40
- 130

**5σ**

**3σ**
Conclusion: Lessons

Theory
- There is life outside of Minimal Flavor Violation (MFV)
- ... though perhaps only minimal life?
- We can model-build beyond MFV; e.g. Flavorful SUSY
- Our numerical code is available

Experiment
- Keep an eye out for a measurement of $B_s \rightarrow \mu \mu$
- Non-discovery at SM limit could hit at low $\tan \beta$, beyond-MFV
- Need to think about LHCb upgrade scenarios
Range of input parameters for numerical scan

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Min</th>
<th>Max</th>
<th>Step</th>
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<tr>
<td>Ratio of Higgs vevs</td>
<td>$\tan \beta$</td>
<td>2</td>
<td>30</td>
<td>varied</td>
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<tr>
<td>CKM phase</td>
<td>$\gamma$</td>
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<td>$\pi$</td>
<td>$\pi/25$</td>
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<tr>
<td>CP-odd Higgs mass</td>
<td>$M_A$</td>
<td>100</td>
<td>500</td>
<td>200</td>
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<tr>
<td>SUSY Higgs mixing</td>
<td>$\mu$</td>
<td>-450</td>
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<tr>
<td>$SU(2)$ gaugino mass</td>
<td>$M_2$</td>
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<tr>
<td>Gluino mass</td>
<td>$M_3$</td>
<td>$3M_2$</td>
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<td>SUSY scale</td>
<td>$M_{\text{SUSY}}$</td>
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<td>1000</td>
<td>500</td>
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<tr>
<td>Slepton Masses</td>
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<td>$M_{\text{SUSY}}/3$</td>
<td>$M_{\text{SUSY}}/3$</td>
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<td>Left top squark mass</td>
<td>$M_{\tilde{t}_L}$</td>
<td>200</td>
<td>500</td>
<td>300</td>
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<tr>
<td>Right bottom squark mass</td>
<td>$M_{\tilde{b}_R}$</td>
<td>200</td>
<td>500</td>
<td>300</td>
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<tr>
<td>Right top squark mass</td>
<td>$M_{\tilde{t}_R}$</td>
<td>150</td>
<td>300</td>
<td>150</td>
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<tr>
<td>Mass insertion</td>
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<td>-1</td>
<td>1</td>
<td>$1/10$</td>
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<tr>
<td></td>
<td>$\delta_{dLR}^{13}, \delta_{dLR}^{23}$</td>
<td>-0.1</td>
<td>0.1</td>
<td>$1/100$</td>
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</table>
Constraints used in numerical scan

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<tr>
<th>Quantity</th>
<th>Current Measurement</th>
<th>Experimental Error</th>
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<tr>
<td>$m_{\chi^0_1}$</td>
<td>&gt; 46 GeV</td>
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<td>$m_{\chi^\pm_1}$</td>
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<td>$m_{\tilde{b}}$</td>
<td>&gt; 89 GeV</td>
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<td>$m_{\tilde{t}}$</td>
<td>&gt; 95.7 GeV</td>
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<td>$m_h$</td>
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<td>\Delta M_D</td>
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<tr>
<td>$\Delta M_{B_d}$</td>
<td>3.337 \cdot 10^{-13}$ GeV</td>
<td>0.033 \cdot 10^{-13}$ GeV</td>
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<tr>
<td>$\Delta M_{B_s}$</td>
<td>116.96 \cdot 10^{-13}$ GeV</td>
<td>0.79 \cdot 10^{-13}$ GeV</td>
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<tr>
<td>$\text{Br}(B \rightarrow X_s\gamma)$</td>
<td>3.34 \cdot 10^{-4}</td>
<td>0.38 \cdot 10^{-4}</td>
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<tr>
<td>$\text{Br}(K_L \rightarrow \pi^0\nu\bar{\nu})$</td>
<td>&lt; 1.5 \cdot 10^{-10}</td>
<td></td>
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<tr>
<td>$\text{Br}(K^+ \rightarrow \pi^+\nu\bar{\nu})$</td>
<td>1.5 \cdot 10^{-10}</td>
<td>1.3 \cdot 10^{-10}</td>
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<tr>
<td>Electron EDM</td>
<td>&lt; 0.07 \cdot 10^{-26}</td>
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<tr>
<td>Neutron EDM</td>
<td>&lt; 0.63 \cdot 10^{-25}</td>
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Calculation: Effective Operators

The effective Hamiltonian can be written as

\[ \mathcal{H} = \frac{1}{(4\pi)^2} \sum_{X,Y=L,R} \left( C_{VXY} O_{VXY} + C_{SXY} O_{SXY} + C_{TX} O_{TX} \right) \]

Writing flavor indices \( I, J, K, L \), the operators are

\[ O_{VXY}^{IJKL} = (q^J \gamma^\mu P_X q^I)(\ell^L \gamma_\mu P_Y \ell^K) \]
\[ O_{SXY}^{IJKL} = (q^J P_X q^I)(\ell^L P_Y \ell^K) \]
\[ O_{TX}^{IJKL} = (q^J \sigma^{\mu\nu} P_X q^I)(\ell^L \sigma_{\mu\nu} \ell^K) \]
Calculation: Factorization

The hadronic and leptonic parts of the matrix element factorize:

\[ \langle \ell, \ell' | \mathcal{H}_{\text{eff}} | B(p) \rangle = \sum_{i=\text{ops}} \langle \ell, \ell' | \mathcal{O}_i^L | 0 \rangle \langle 0 | \mathcal{O}_Q^i | B(p) \rangle \]

Definition of the decay constant, \( f_B \)

\[ \langle 0 | \bar{b} \gamma_{\mu} P_{L,R} s | B(p) \rangle = \pm \frac{i}{2} p_{\mu} f_B \]

\[ \rightarrow \langle 0 | \bar{b} P_{L,R} s | B(p) \rangle = \pm \frac{i M_B f_B}{2 m_b + m_s} \]

Note that there are no tensor \( (\bar{b} \sigma^{\mu\nu} s) \) operators by antisymmetry.

\( f_B \) contains all the hadronic muck; look it up from non-perturbative methods (i.e. lattice).

Leptonic decay: don’t have to worry about jets, inclusive decays, etc.
We can now write the amplitude in terms of form factors

\[ \mathcal{M} = F_S \bar{\ell}\ell + F_P \bar{\ell}\gamma_5\ell + F_V p^\mu \bar{\ell}\gamma_\mu\ell + F_A p^\mu \bar{\ell}\gamma_\mu\gamma_5\ell \]

In terms of the Wilson coefficients, these are

\[ F_S = \frac{i}{4} \frac{M_{Bs}^2 f_{Bs}}{m_b + m_s} \left( C_{SLL} + C_{SLR} - C_{SRR} - C_{SRL} \right) \]

\[ F_P = \frac{i}{4} \frac{M_{Bs}^2 f_{Bs}}{m_b + m_s} \left( -C_{SLL} + C_{SLR} - C_{SRR} + C_{SRL} \right) \]

\[ F_V = -\frac{i}{4} f_{Bs} \left( C_{VLL} + C_{VLR} - C_{VRR} - C_{VRL} \right) \]

\[ F_A = -\frac{i}{4} f_{Bs} \left( -C_{VLL} + C_{VLR} - C_{VRR} + C_{VRL} \right) \]
Calculation: Branching Ratio

\[ B(B^0_s \to \ell^- \ell^+_K) = \frac{\tau_{B^0_s}}{16\pi} \frac{|M|^2}{M_{B^0_s}} \sqrt{1 - \left( \frac{m_{\ell_K} + m_{\ell_L}}{M_{B^0_s}} \right)^2} \sqrt{1 - \left( \frac{m_{\ell_K} - m_{\ell_L}}{M_{B^0_s}} \right)^2} \]

\[ |M|^2 = 2|F_S|^2 \left[ M^2_{B^0_s} - (m_{\ell_L} + m_{\ell_K})^2 \right] + 2|F_P|^2 \left[ M^2_{B^0_s} - (m_{\ell_L} - m_{\ell_K})^2 \right] \]

\[ + 2|F_V|^2 \left[ M^2_{B^0_s} (m_{\ell_K} - m_{\ell_L})^2 - (m_{\ell_K}^2 - m_{\ell_L}^2)^2 \right] \]

\[ + 2|F_A|^2 \left[ M^2_{B^0_s} (m_{\ell_K} + m_{\ell_L})^2 - (m_{\ell_K}^2 - m_{\ell_L}^2)^2 \right] \]

\[ + 4 \text{Re}(F_S F^*_V)(m_{\ell_L} - m_{\ell_K}) \left[ M^2_{B^0_s} + (m_{\ell_K} + m_{\ell_L})^2 \right] \]

\[ + 4 \text{Re}(F_P F^*_A)(m_{\ell_L} + m_{\ell_K}) \left[ M^2_{B^0_s} - (m_{\ell_L} - m_{\ell_K})^2 \right] . \]
Calculation: $B_s \rightarrow \mu^+ \mu^-$ at low $\tan \beta$

For the case $\ell_K = \ell_L = \mu$, the amplitude-squared is

$$|\mathcal{M}|^2 \approx 2M_{Bq}^2 \left( |F_S|^2 + |F_P + 2m_\mu F_A|^2 \right),$$

where we have also taken $m_\mu/M_B \rightarrow 0$.

The minima of this comes from two cases,

(1) $F_P + 2m_\ell F_A \approx 0, F_P \gg F_S$

(2) $|F_S| \approx |F_P| \approx |F_A| \approx 0.$