GOLDSTONE FERMION DARK MATTER

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SUSY, 31 August 2011

The WIMP Miracle

Contains factors of M_{Pl}, s_0, $\cdots$

$$\Omega_{\mathrm{DM}} h^2 \approx 0.1 \left(\frac{x_f}{20}\right)\left(\frac{g_*}{80}\right)^{-\frac{1}{2}}\left(\frac{\langle \sigma v\rangle_0}{3\times 10^{-26}\ \mathrm{cm}^3/\mathrm{s}}\right)$$

$$\sim \left\langle \frac{\alpha^2 v}{(100\ \mathrm{GeV})^2}\right\rangle$$

Within orders of magnitude!

Reality: direct detection vs Ωh^2

$$\boxed{\sigma_{\text{ann.}} \sim 0.1 \text{ pb}}$$

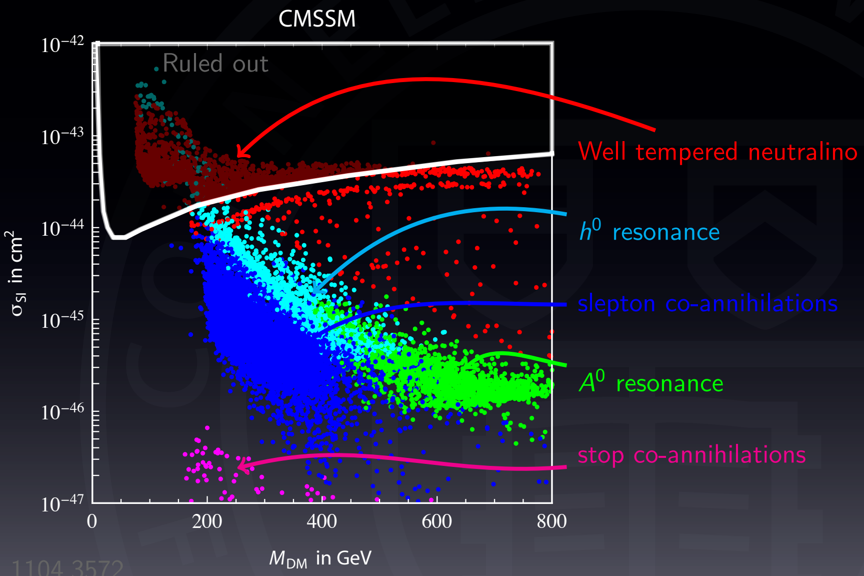
$$\boxed{\sigma_{\text{SI}} \sim 7.0 \times 10^{-9} \text{ pb}}$$

50 GeV WIMP

Typical strategy: pick parameters such that σ_{SI} is suppressed, then use tricks to enhance $\sigma_{\text{ann.}}$.

- Tune the neutralino composition ($\widetilde{B}$ vs. $\widetilde{W}$, $\widetilde{H}$)
- Coannihilations (accidental slepton degeneracy)
- Resonant annihilation

Reality: direct detection vs Ωh^2



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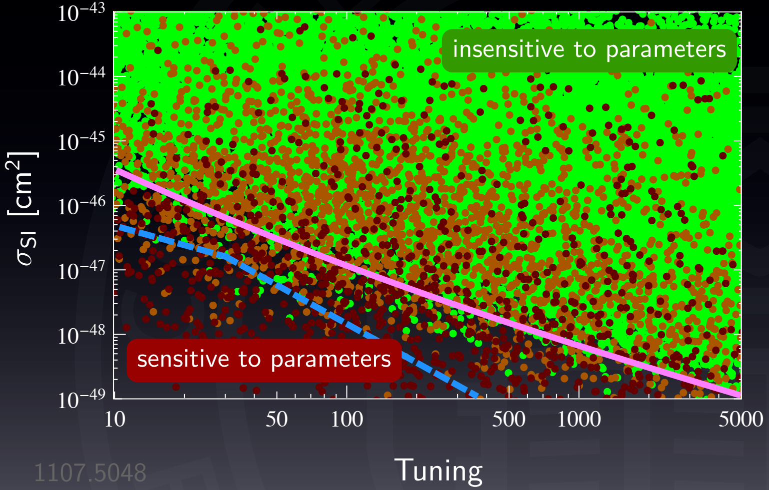
MSSM Dark Matter and Tuning



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Motivation I: a natural WIMP

Typical MSSM WIMP: σ_{SI} too large

Want to naturally suppress direct detection while maintaining 'miracle' of successful abundance.

If LSP is part of a **Goldstone multiplet**, $(s + ia, \chi)$, additional suppression from derivative coupling.

- Like a weak scale axino, but unrelated to CP
- Like singlino DM, but global symmetry broken in SUSY limit

Motivation I: a natural WIMP

Annihilation: *p*-wave decay to Goldstones

$$\frac{1}{f}\bar{\chi}\gamma^\mu\gamma^5\chi\partial_\mu a \quad \Rightarrow \quad \langle \sigma v\rangle \approx \left(\frac{m_\chi^2}{f^4}\right)\left(\frac{T_f}{m_\chi}\right) \approx 1\ \text{pb}$$

Direct detection: CP-even Goldstone mixing with Higgs

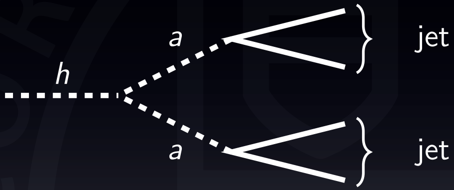
$$\text{mixing} \sim \frac{m_\chi v}{f^2} \sim 0.01 \quad \Rightarrow \quad \sigma_{\text{SI}} = \left(\frac{m_\chi v}{f^2}\right)^2 \sigma_{\text{SI}}^{\text{MSSM}}$$

$$\sim \mathcal{O}(10^{-45}\ \text{cm}^2)$$

Motivation II: Buried Higgs

Idea: Light Higgs buried in QCD background

~~Global symmetry~~ at $f \sim 500$ GeV with coupling $\frac{1}{f^2} h^2 (\partial a)^2$



0906.3026, 1012.1316, 1012.1347

Can we bury the Higgs through a decays,
but dig up dark matter in χ?

The Goldstone Supermultiplet

sGoldstone   Goldstone boson   Goldstone fermion

$$A = \frac{1}{\sqrt{2}}\left(s + i\, a \right) + \sqrt{2}\theta\, \chi + \theta^2 F$$

Carries the low-energy degrees of freedom of the UV fields,

$$\Phi_i = f_i e^{q_i A/f} \qquad\qquad f^2 = \sum_i q_i^2 f_i^2$$

Neglecting terms which simultaneously break SUSY and U(1):

$\cancel{SUSY} \Rightarrow$ explicit s mass, $m_\chi \approx q_i \langle F_i \rangle / f$, a massless

a mass through small supersymmetric explicit $\cancel{U(1)}$ terms

Interactions: Kähler potential

Our non-linear realization of the global U(1) leads to interactions of the Goldstone fields in through the kinetic (Kähler) terms:

$$\frac{\partial^2 K}{\partial A \partial A^\dagger} = 1 + b_1 \frac{q}{f}(A + A^\dagger) + \cdots \qquad b_1 = \frac{1}{qf^2}\sum_i q_i^3 f_i^2$$

Note the manifest shift-invariance. This leads to:

$$\mathcal{L} = \left(1 + b_1 \frac{\sqrt{2}}{f} s + \cdots\right)\left(\frac{1}{2}(\partial s)^2 + \frac{1}{2}(\partial a)^2 + \frac{i}{2}\bar{\chi}\gamma^\mu \partial_\mu \chi\right)$$
$$+ \frac{1}{2\sqrt{2}}\left(b_1 \frac{1}{f} + b_2 \frac{\sqrt{2}}{f^2} s + \cdots\right) \boxed{\left(\bar{\chi}\gamma^\mu\gamma^5\chi\right)\partial_\mu a} + \cdots$$

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b_1 controls the annihilation cross section.

Interactions: scalar mixing

MSSM fields are uncharged under the global U(1), but may mix with the Goldstone multiplet through higher-order terms in K:

$$K = \frac{1}{f}\left(A + A^\dagger\right)\left(c_1 H_u H_d + \cdots\right) + \frac{1}{2f^2}\left(A + A^\dagger\right)^2\left(c_2 H_u H_d + \cdots\right)$$

The new scalar interactions take the form

$$\mathcal{L} \supset \left[\frac{1}{2}(\partial a)^2 + \frac{1}{2}\boxed{\bar{\chi}\not{\partial}\chi}\right]\left(1 + \boxed{c_h \frac{v}{f} h} + \cdots\right)$$

Where c_h is a function of the c_i and the Higgs mixing angles.

$c_h \to (m_h/m_s)^2$ in the large m_s limit.

We neglect mixing with the heavy higgses.

Interactions: kinetic mixing

The higher order terms in K also induce kinetic $\widetilde{H}$–χ mixing.

$$\mathcal{L} \supset i\epsilon_u \boxed{\bar{\chi}\gamma^\mu\partial_\mu \widetilde{H}_u^0} + i\epsilon_d \boxed{\bar{\chi}\gamma^\mu\partial_\mu \widetilde{H}_d^0} + \text{h.c.}$$

where $\epsilon \sim v/f$.

In the large μ limit, χ has a small $\widetilde{H}$ component on the order of $vm_\chi/f\mu$.

Mixing with other MSSM fields is suppressed. Assuming MFV,

$$K = \frac{1}{f}\left(A + A^\dagger\right)\left(\frac{Y_u}{M_u}\bar{Q}H_u U + \cdots\right)$$

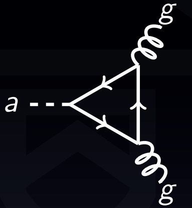
where the scalse $M_{u,d,\ell}$ are unrelated to f or v and can be large and dependent on the UV completion

Interactions: anomaly

Fermions Ψ charged under global U(1) and Standard Model

$$\mathcal{L}_{\text{an}} \supset \frac{c_{\text{an}}}{f\sqrt{2}}\left(\boxed{a G^a_{\mu\nu}\widetilde{G}^a_{\mu\nu}} + 2\bar{\chi} G^a_{\mu\nu}\sigma^{\mu\nu}\gamma^5\lambda^a \right)$$

$$c_{\text{an}} = \frac{\alpha}{8\pi}\sqrt{2}\sum_i^{N_\Psi}\left(\frac{y_i f}{m_{\Psi_i}}\right) = \frac{\alpha}{8\pi} q_\Psi N_\Psi$$



Where we have assumed degeneracy of m_Ψ and Yukawas $y = m_\Psi q_\Psi / f\sqrt{2}$

$\mathrm{U}(1)\,\mathrm{SU}(3)_c^2$

$\mathrm{U}(1)\,\mathrm{U}(1)_{\text{QED}}^2$

Integrating out λ^a generates χ couplings to gluons

$$\mathcal{L} \supset -\left(\frac{c_{\text{an}}^2}{2M_\lambda f^2}\right)\boxed{\bar{\chi}\chi\, GG} - i\left(\frac{c_{\text{an}}^2}{2M_\lambda f^2}\right)\boxed{\bar{\chi}\gamma^5\chi\, G\widetilde{G}}$$

This contributes to direct detection and collider operators.

Interactions: explicit breaking

Include explicit $\cancel{U(1)}$ spurion $R_\alpha = \lambda_\alpha f$ with $\lambda_\alpha \ll 1$

$$W_{\cancel{U(1)}} = f^2 \sum_\alpha R_{-\alpha} e^{aA/f}$$

Perserve SUSY $\Rightarrow$ at least two spurions with opposite charge.

This generates $m_a = m_\chi = m_s$ and couplings

$$\mathcal{L} \supset \underbrace{-\frac{m_a}{2\sqrt{2}f}(\alpha+\beta)}_{\delta}\, i\, \boxed{a\bar{\chi}\gamma^5\chi} + \underbrace{\frac{m_a}{8f^2}(\alpha^2+\alpha\beta+\beta^2)}_{\rho}\, a^2\bar{\chi}\chi$$

By integration by parts this is equivalent to a shift in the b_1 coefficient from the Kähler potential

Parameter space scan

Abundance: $\langle \sigma v \rangle \approx \frac{b_1{}^4}{8\pi}\frac{T_f}{m_\chi}\frac{m_\chi^2}{f^4} \approx 1$ pb

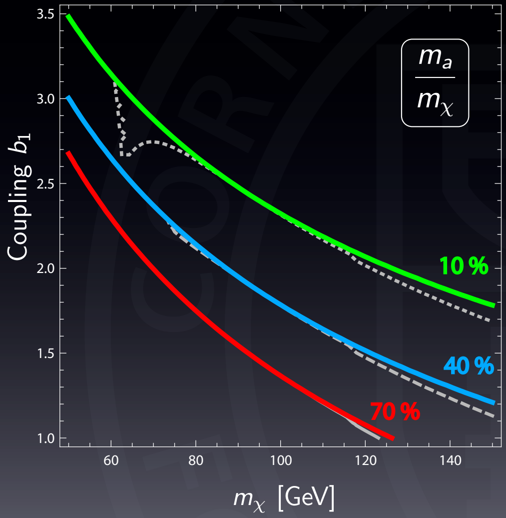
p-wave: $b_1 \gtrsim 1$, all other parameters take natural values

Parameter	Description	Scan Range
f	Global symmetry breaking scale	500 GeV – 1.2 TeV
m_χ	Goldstone fermion mass	50 – 150 GeV
m_a	Goldstone boson mass	8 GeV – $f/10$
b_1	$\chi\chi a$ coupling	$[0, 2]$
c_{an}	Anomaly coefficient	0.06
c_h	Higgs coupling	$[-1, 1]$
δ	Explicit breaking $ia\bar{\chi}\gamma^5\chi$ coupling	$3/2$

$$\mathcal{L} \supset \left[\frac{1}{2}(\partial a)^2 + \frac{1}{2}\bar{\chi}\not{\partial}\chi\right] c_h \frac{v}{f} h + \frac{b_1}{2\sqrt{2}f}\left(\bar{\chi}\gamma^\mu\gamma^5\chi\right)\partial_\mu a + \frac{c_{an}}{f\sqrt{2}} aG\widetilde{G} + i\delta a\bar{\chi}\gamma^5\chi$$

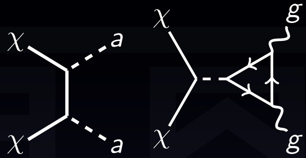
Contours of fixed Ω

$$\Omega h^2 = 0.11$$



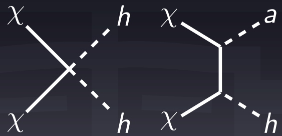
Dominant contribution

Kähler, anomaly, ~~U(1)~~



Subleading

Mixing with Higgs



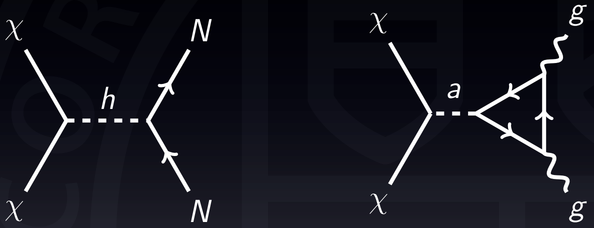
Negligible

$\chi\chi \to s \to aa,\ \chi\chi \xrightarrow{t,u} hh$

Direct Detection

Relevant couplings from EWSB and anomaly:

$$\mathcal{L} \supset \frac{c_h v}{2f}\bar{\chi}\partial\!\!\!/\chi h - \frac{c_{an}{}^2}{2M_\lambda f^2}\bar{\chi}\chi GG - \frac{i\,c_{an}{}^2}{2M_\lambda f^2}\bar{\chi}\gamma^5\chi G\widetilde{G}$$



Effective coupling to nucleons: $\mathcal{L} = G_{\text{nuc}}\bar{N}N\bar{\chi}\chi$,

$$G_{\text{nuc}} = c_h\frac{\lambda_N}{2\sqrt{2}}\left(\frac{m_\chi m_N}{m_h^2 f^2}\right) + \frac{4\pi c_{an}{}^2}{9\alpha_s}\frac{m_N}{M_\lambda f}\left(1 - \sum_{i=u,d,s} f_i^{(N)}\right)$$

Direct Detection

Higgs exchange typically dominates by a factor of $\mathcal{O}(10^3)$.

$$\sigma_{\text{SI}}^{\text{H}} \approx 3 \cdot 10^{-45}\ \text{cm}^2 c_h^2 \left(\frac{115\ \text{GeV}}{m_h}\right)^4 \left(\frac{700\ \text{GeV}}{f}\right)^4 \left(\frac{m_\chi}{100\ \text{GeV}}\right)^2 \left(\frac{\mu_\chi}{\text{GeV}}\right)^2 \left(\frac{\lambda_N}{0.5}\right)^2$$

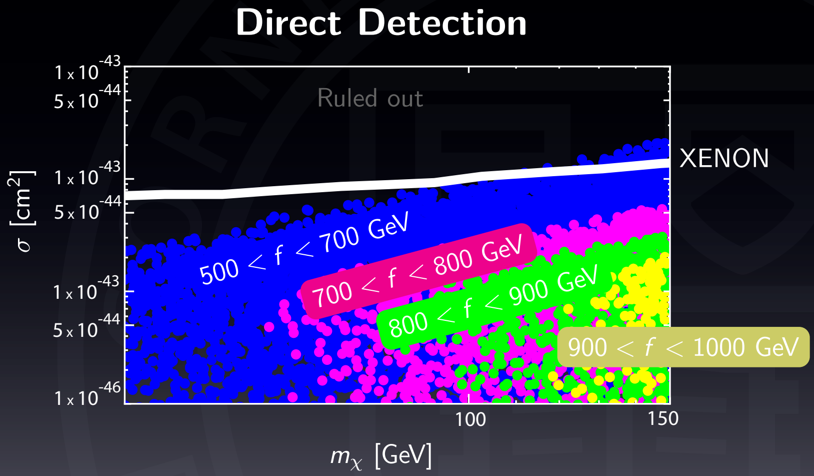
Compare this to the MSSM Higgs with $\mathcal{L} = \frac{1}{2} c g \bar{\chi}\chi h$:

$$\sigma_{\text{SI}}^{\text{MSSM}} \sim \frac{c^2 g^2}{2\pi} \frac{\lambda_N^2 \mu^2 m_N^2}{m_h^2 v^2} \approx c^2 \times 10^{-42}\ \text{cm}^2$$

Natural suppression: $(m_\chi v/f^2)^2$ due to Goldstone nature

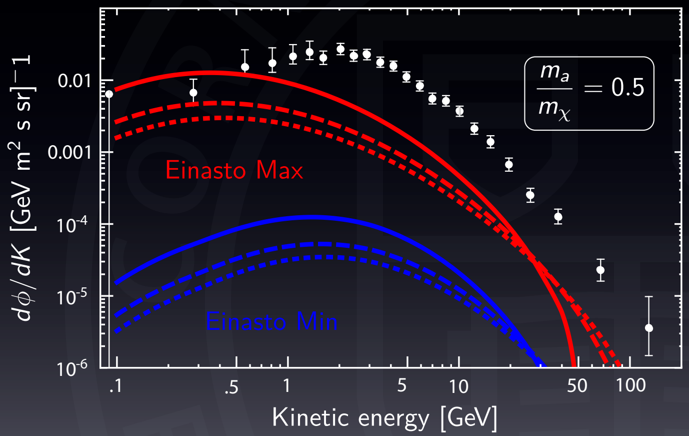
Is it enough to avoid current direct detection bounds?

Parameter space scan



Indirect detection: $\bar{p}$ flux vs. PAMELA

$f = 700$ GeV, $Q_\Psi = 2$, $\delta = \frac{3}{2}$, $N_\Psi = 5$



Dotted: $m_\chi = 150$ GeV, $b_1 = 1$

Dashed: $m_\chi = 100$ GeV, $b_1 = 1.5$

Solid: $m_\chi = 50$ GeV, $b_1 = 3$

Using Einasto DM Halo profile in 1012.4515, 1009.0224

Indirect detection: Fermi-LAT

γ-ray line search: 30 – 200 GeV

- Upper bound $\langle \sigma v \rangle_{\gamma\gamma} < 2.5 \times 10^{-27}$ cm^3/s
- $\chi\chi \to a \to \gamma\gamma$ via anomaly
- For SU(5) fundamentals, $\langle \sigma v \rangle_{\gamma\gamma} \sim 2 \times 10^{-3} \langle \sigma v \rangle_{gg}$
- $\mathcal{O}(10)$ smaller than bound even for extreme parameters

Diffuse γ-ray spectrum: 20 – 100 GeV

- Bounds $\chi\chi$ to charged particles, π^0s
- $\chi\chi \to a \to gg$ via anomaly
- $\mathcal{O}(10)$ smaller than bound

Photo-production from DM annihilation: spheroidal galaxies

- Low mass DM $m_\chi \lesssim 60$ GeV, constrains bb decays
- GF: annihilation σ always at least a factor of 3 lower

http://fermi.gsfc.nasa.gov/science/symposium/2011/program

Collider production

Collider production through gluons. ISR monojet signature is sensitive to $\sigma_{\text{SI}}^{N} \sim 10^{-46}$ cm^2 at the LHC with 100 fb^{-1}.

The dim-7 anomaly operators are too small:

$$\mathcal{L} \supset -\frac{c_{\text{an}}{}^2}{2M_\lambda f^2}\bar{\chi}\chi GG - \frac{i c_{\text{an}}{}^2}{2M_\lambda f^2}\bar{\chi}\gamma^5\chi G\widetilde{G}$$

$gg \to a^* \to \chi\chi$ may be within 5σ reach with 100 fb^{-1}

1005.1286, 1005.3797, 1008.1783, 1103.0240, 1108.1196

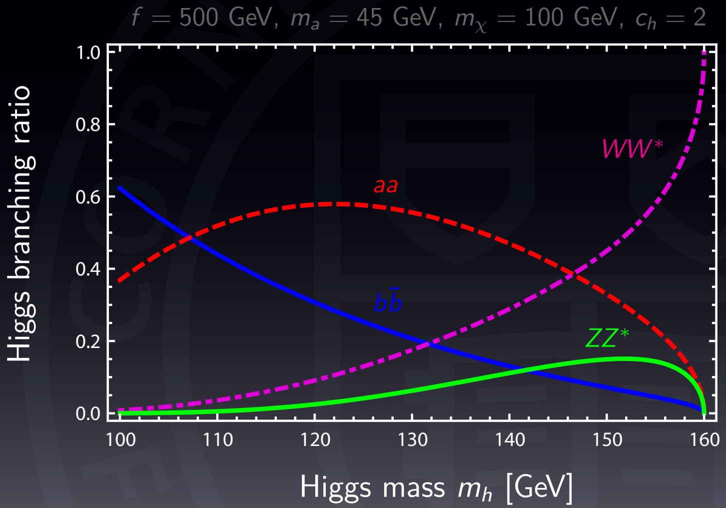
Cascade decays: LOSP $\to \chi$ through

- $\bar{\chi}G\lambda$ anomaly
- χ–$\widetilde{H}$ kinetic mixing

Decays typically prompt, *a* reconstruction is difficult for light masses. Heavy fermions Ψ in anomaly may appear as "fourth generation" quarks

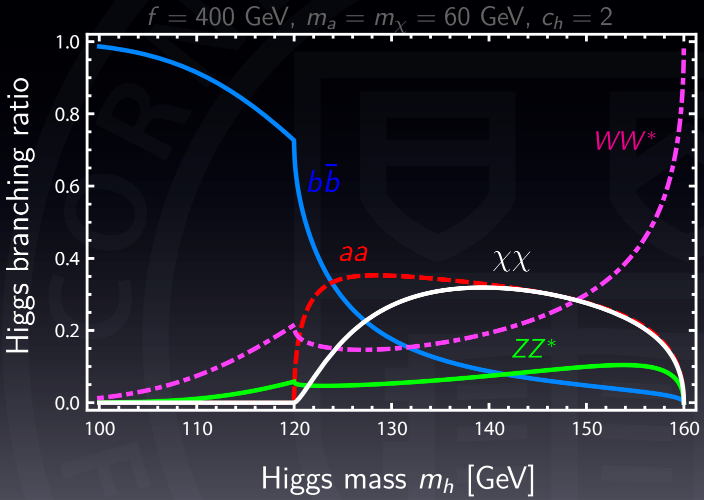
Non-standard Higgs decays

Hard to completely bury the Higgs. LEP: Br(SM)$\gtrsim$ 20% $\Rightarrow m_h \gtrsim 110$ GeV



Non-standard Higgs decays

Partially buried & invisible: Suppressed SM channels, MET, $\Gamma_{tot} < 1$



Conclusions

Executive summary: Goldstone Fermion dark matter

- SSB: global U(1) $\Rightarrow$ Goldstone boson a and fermion χ
- χ is LSP and DM, a gives 'buried' Higgs channel

Simple extension of MSSM with natural WIMP dark matter

- Kähler $\chi\chi a$ interaction controls abundance
- Higgs mixing, anomaly controls direct detection
- Novel collider signature: partially buried/invisible Higgs

Further directions:

- p-wave Sommerfeld enhancement (can push m_a, m_χ to 10 GeV)
- Non-abelian generalization

Extra Slides

Examples of Linear Models

Simplest example:

$$W = yS\left(\bar{N}N - \mu^2\right) + \underbrace{N\bar{\phi}\phi}_{\text{anomaly}} + \underbrace{SH_uH_d}_{\text{mixing}} + \underbrace{W_{\text{explicit}}}_{\text{explicit}\,\cancel{U(1)}}$$

Example with $|b_1| \geq 1$:

$$W = \lambda XYZ - \mu^2 Z + \frac{\tilde{\lambda}}{2}Y^2N - \tilde{\mu}\bar{N}N$$

$q_Z = 0$, $q_N = -q_{\bar{N}} = -2q_Y = 2q_X$. Goldstone multiplet:

$$A = \sum_i \frac{q_i f_i \psi_i}{f} = \frac{q_Y}{f}\left(Yf_Y - Xf_X + 2\bar{N}F_{\bar{N}}\right)$$

$$b_1 = \frac{-f_X^2 + f_Y^2 + 8f_{\bar{N}}^2}{f_X^2 + f_Y^2 + 4f_{\bar{N}}^2}$$

Tamvakis-Wyler Theorem

Phys. Lett B 112 (1982) 451; Phys. Rev. D 33 (1986) 1762

Global symmetry: $W[\Phi_i] = W[e^{i\alpha q_i}\Phi_i]$ so that

$$0 = \frac{\partial W[e^{i\alpha q_i}\Phi_i]}{\partial\alpha} = \sum_j W_j q_j \Phi_j,$$

Taking a derivative $\partial/\partial\Phi_i$ gives:

$$0 = \frac{\partial}{\partial\Phi_i}\left(\sum_j W_j q_j \Phi_j\right)\Bigg|_{\langle\Phi\rangle} = \sum_j W_{ij} q_j f_j + W_i q_i$$

SUSY NLΣM

Phys. Lett. B 87 (1979) 203

Expand Kähler potential, drop total derivatives, integrate out F:

$$\begin{aligned}\mathcal{L} = \;& K'' \left(\frac{i}{2} \partial\chi\sigma\bar{\chi} + |\partial\phi|^2 \right) \\ & + \frac{K'''}{4} i\chi\sigma\bar{\chi}\partial\left(\phi - \phi^*\right) \\ & + \frac{1}{4}\left(K'''' - \frac{(K''')^2}{K''} \right) \chi^2\bar{\chi}^2\end{aligned}$$

These terms can be understood in terms of geometric properties of the vacuum manifold, see e.g. hep-th/0101055.

SUSY Breaking and χ mass

We assume that soft $\cancel{\text{SUSY}}$ terms that also explicitly break the global U(1) are negligible. Neglect D-term mixing with λ^a, then fermion mass matrix is W_{ij}. Tamvakis-Wyler:

$$\sum_j W_{ij} q_j f_j = -q_i W_i = -q_i F_i$$

so that $\chi = \sum_i q_i f_i \psi_i / f$ mass depends on how U(1)-charged F-terms in the presence of soft $\cancel{\text{SUSY}}$ terms.

If W has an unbroken R symmetry, then $R[\chi] = -1$ which prohibits a Majorana mass. However, while soft scalar masses preserve R, A-terms are holomorphic and generally break R symmetries to contribute to m_χ.

SUSY Breaking and χ mass

The A-term contribution to m_χ is equivalent to F-term mixing between U(1) charged fields and the ~~SUSY~~ spurion, X. This was recently emphasized in 1104.0692 as an irreducible $\mathcal{O}(m_{3/2})$ contribution to the Goldstone fermion

For concreteness, consider gravity mediation with $m_{\text{soft}} \sim F/M_{\text{Pl}}$.

$$K = \sum_i Z(X, X^\dagger)\Phi_i^\dagger \Phi_i$$

Analytically continue into superspace hep-ph/9706540

$$\Phi \to \Phi' \equiv Z^{1/2}\left(1 + \frac{\partial \ln Z}{\partial X} F\theta^2\right)\Phi$$

Canonical normalization generates A-terms:

$$\Delta\mathcal{L}_{\text{soft}} = \left.\frac{\partial W}{\partial \Phi}\right|_{\Phi=\phi} Z^{-1/2}\left(-\frac{\partial \ln Z}{\partial \ln X}\frac{F}{M}\right)$$

SUSY Breaking and χ mass

$$\Delta\mathcal{L}_{\text{soft}} = \left.\frac{\partial W}{\partial \Phi}\right|_{\Phi=\phi} Z^{-1/2}\left(-\frac{\partial \ln Z}{\partial \ln X}\frac{F}{M}\right)$$

Completely incorporates F-term mixing of the form $FF_i^\dagger\Phi_i$. The χ mass is determined by the induced F_i obtained by minimizing

$$V = \left|\frac{\partial W}{\partial \phi_i}\right|^2 + A_i\frac{\partial W}{\partial \phi_i}\phi_i + \text{h.c.} + m_i^2|\phi_i|^2$$

Assuming $A_i, m_i < f_i$, generic size is $|F_i| \approx A_i f_i$ so that $m_\chi \sim A_i q_i$. Often the A-terms are suppressed relative to other soft terms, so it's reasonable to expect χ to be the LSP.

Contributions from soft scalar masses are on the order of m_i^2/f_i which can easily be suppressed.

Direct detection: nucleon matrix elements

Nucleon matrix elements can be parameterized via

Phys. Rev. D38 2869, Phys. Lett. B219 347, 0801.3656, 0907.417

$$m_i \langle N | \bar{q}_i q_i | N \rangle = f_i^{(N)} m_N$$

The heavy quark contribution via gluons can be calculated by the conformal anomaly, Phys. Lett. B78 433

$$f_j^{(N)} m_N = \frac{2}{27} \left(1 = \sum_{q=u,d,s} f_q^{(N)} \right) \qquad j = c, b, t$$

Relevant quantity in Higgs exchange: c_q, diagonalized Yukawa

$$\lambda_N = \sum_{q=u,d,s} c_q f_q^{(N)} + \frac{2}{27} \left(1 = \sum_{q=u,d,s} f_q^{(N)} \right) \sum_{q'=c,b,t} c_{q'}$$

Direct Detection

Some details:

$$G_{\chi N} = c_h \frac{\lambda_N}{2\sqrt{2}} \left(\frac{m_\chi m_N}{m_h^2 f^2} \right) + \frac{4\pi {c_{an}}^2}{9\alpha_s} \frac{m_N}{M_\lambda f} \left(1 - \sum_{i=u,d,s} f_i^{(N)} \right)$$

For reduced mass $\mu_\chi = (m_\chi^{-1} + m_N^{-1})^{-1}$,

$$\sigma_{\text{SI}}^{\text{Higgs}} = \frac{4\mu_\chi^2}{A^2\pi} \left[G_{\chi p} Z + G_{\chi n}(A - Z) \right]$$

$$\sigma_{\text{SI}}^{\text{H}} \approx 3{\cdot}10^{-45}\ \text{cm}^2 c_h^2 \left(\frac{115\ \text{GeV}}{m_h} \right)^4 \left(\frac{700\ \text{GeV}}{f} \right)^4 \left(\frac{m_\chi}{100\ \text{GeV}} \right)^2 \left(\frac{\mu_\chi}{1\ \text{GeV}} \right)^2 \left(\frac{\lambda_N}{0.5} \right)^2$$

$$\sigma_{\text{SI}}^{\text{glue}} \approx 2 \cdot 10^{-48}\ \text{cm}^2 \left(\frac{700\ \text{GeV}}{M_\lambda} \right)^2 \left(\frac{700\ \text{GeV}}{f} \right)^4 \left(\frac{N_\Psi}{5} \right)^4 \left(\frac{q_\Psi}{2} \right)^4 \left(\frac{\mu}{1\ \text{GeV}} \right)^2$$

using $c_{an} = \alpha_s q_\Psi N_\Psi / 8\pi$

Why are the $\chi\chi \to aa$ annihilations p-wave?

If the initial state is a particle-antiparticle pair with zero total angular momentum and the final state is CP even, then the process must vanish when $v = 0$.

Under CP a particle/antiparticle pair picks up a phase $(-)^{L+1}$. When $v = 0$ momenta are invariant and thus the initial state gets an overall minus sign. Since final state is CP even, the amplitude must vanish in this limit. For Dirac particles P is sufficient, but for Majorana particles CP is the well-defined operation.

This is why $\chi\chi \to G\widetilde{G}$ is s-wave while $\chi\chi \to aa$ is p-wave.

Nuclear matrix element and matching

The nucleon matrix element at vanishing momentum transfer:

$$M_N = \langle \Theta^\mu_\mu \rangle = \langle N | \sum_{i=u,d,s} m_i \bar{q}_i q_i + \frac{\beta(\alpha)}{4\alpha} G^a_{\alpha\beta} G^a_{\alpha\beta} | N \rangle$$

from: Shifman, Vainshtein, Zakharov. Phys. Lett 78B (1978)

$\beta = -9\alpha^2/2\pi + \cdots$ contains only the light quark contribution, M_N is the nucleon mass. The GG matches onto the nucleon operator $\bar{N}N$.

$$M_N f^{(N)}_{i=u,d,s} = \langle N | m_i \bar{q}_i q_i | N \rangle \qquad f^{(N)}_g = 1 - \sum_{i=u,d,s} f^{(N)}_i$$

Nuclear matrix element and matching

$$\frac{\beta(\alpha)}{4\alpha} G^a_{\alpha\beta} G^a_{\alpha\beta} \longrightarrow M_N \left(1 - \sum_{i=u,d,s} f_i^{(N)}\right) \bar{N} N$$

Where $f_{u,d}^{(N)} \ll f_s^{(N)} \approx 0.25$. For a detailed discussion, see 0801.3656 and 0803.2360.

Image Credits and Colophon

- 'Zombie arm' illustration from
  http://plantsvszombies.wikia.com
- Beamer theme Flip, available online
  http://www.lepp.cornell.edu/~pt267/docs.html
- All other images were made by Flip using TikZ and Illustrator