FLIGHT OF THE WARPED PENGUINS

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Warped Penguins

- Randall-Sundrum & flavor anarchy
- Defying anarchy in $\mu \to e\gamma$
- UV finite 5D loops
- Remarks on current work

Lepton Flavor Violation

$$\text{Br}\left(\mu \to e\gamma\right)_{\text{SM}} = 0$$

Current bound: $\text{Br}\left(\mu \to e\gamma\right) < 1.2 \times 10^{-11}$
MEGA, LAMPF

Later this year from MEG:
$\text{Br}\left(\mu \to e\gamma\right) < 1.5 \times 10^{-12}$

Part III 2006-2007

Ben Allanach's Essay Prompt

ESSAY 74, The Phenomenology of Extra Dimensions. Extra dimensional models provide an interesting playground for model building and investigating collider signatures.

Candidates are invited to provide an overview of one of the following extra-dimensional models: ADD (Arkani-Hamed, Dimopoulos and Dvali), UED (Universal Extra Dimensions) or Randall-Sundrum I.

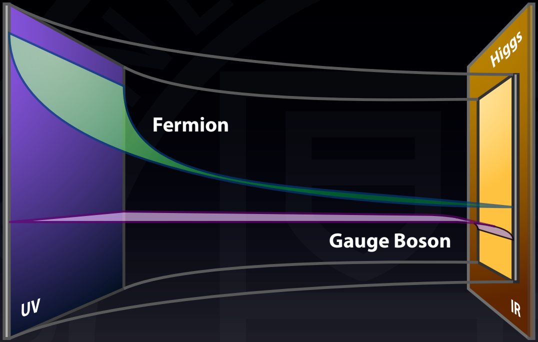
The candidate should include a calculation of a matrix element squared for a collider signature of the model.

Reminder: Randall-Sundrum

$$ds^2 = \left(\frac{R}{z}\right)^2 \left(dx^2 - dz^2\right)$$

Randall, Sundrum (99);

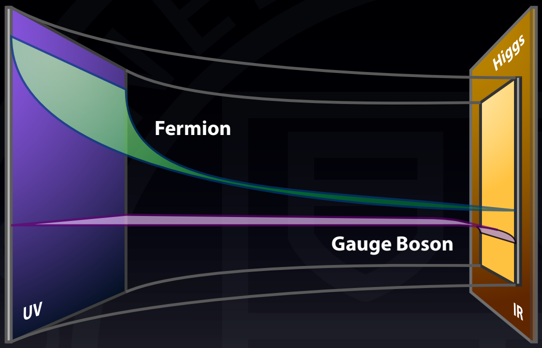
Reminder: Randall-Sundrum



$$ds^2 = \left(\frac{R}{z}\right)^2 \left(dx^2 - dz^2\right)$$

Randall, Sundrum (99); Davoudiasl, Hewett, Rizzo (99); Grossman, Neubert (00); Gherghetta, Pomarol (00); **Bulk Higgs:** Agashe, Contino, Pomarol (04); Davoudiasl, Lille, Rizzo (05), Agashe, Okui, Sundrum (08)

Reminder: Yukawa matrices



$$Y_{ij}^{(4D)} = f_i Y_{ij}^* f_j \qquad f_i = \sqrt{\frac{1-2c_i}{1-(R/R')^{1-2c_i}}}$$

Flavor: Huber, Shafi (03); Burdman (03); Kalil, Mohapatra (04); Agashe, Perez, Soni (04); Chen (05); Agashe, Blechman, Petriello (06); Davidson, Isidori, Uhlig (07); Csáki, Falkowski, Weiler (08); Chen, Yu (08); Agashe, Okui, Sundrum (08); Chen, Mahanthappa, Yu (09), ...

Anarchic Flavor in RS

Definition: anarchic matrix

All entries $\mathcal{O}(1)$ with arbitrary phase. The product of anarchic matrices is also anarchic. Assumption: true in all preferred bases.

$$Y_{ij}^{(4D)} = f_i Y_{ij}^* f_j \qquad f_i = \sqrt{\frac{1-2c_i}{1-(R/R')^{1-2c_i}}}$$

The 5D parameters Y_{ij}^* are anarchic matrices,

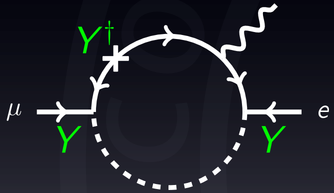
$$Y_{ij}^* = Y_* Ⓐ_{ij}$$

Mass hierarchy $m_i = f_i Y_{ii}^* f_i v$ set by exponentially small overlap of the zero-mode fermions with the Higgs vev; controlled by the fermion bulk masses, $c_i \sim 0.51 - 0.8$.

Lepton Flavor Violation

Controlled by two dominant parameters

Flavor is dominantly controlled by: Y_* and M_{KK}

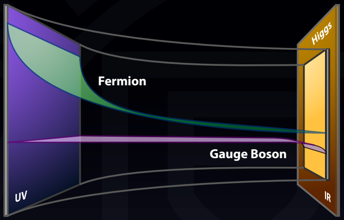


$$\mathcal{M}_{\text{loop}} \sim \left(\frac{1}{M_{KK}}\right)^2 f_L {Y_*}^3 f_{-E}$$

$$\sim \left(\frac{1}{M_{KK}}\right)^2 {Y_*}^2 m$$

Lepton Flavor Violation

Two dominant parmeters



$$\mathcal{M}_{\text{tree}} \sim \left(\frac{1}{M_{\text{KK}}}\right)^2 \left(\frac{1}{Y_*}\right)$$

If we increase Y_*, must maintain SM mass spectrum

$\Rightarrow$ push fermion profiles to UV

$\Rightarrow$ Less overlap with the FCNC part of the Z

Complementary tree- and loop-level bounds

Possible tension between tree- and loop-level processes

- Tree-level bound: $\left(\frac{3\text{ TeV}}{M_{\text{KK}}}\right)^2 \left(\frac{2}{Y_*}\right) < 0.5$, 1.6 (Custodial)
- Penguin bound: $\left| aY_*^2 + b \right| \left(\frac{3\text{ TeV}}{M_{\text{KK}}}\right)^2 \leq 0.015$

What the heck is this?

Can test anarchic flavor ansatz.

Operator analysis of $\mu \to e\gamma$

Match to 4D EFT:

$$R'^2 \frac{e}{16\pi^2} \frac{v}{\sqrt{2}} f_{L_i} \left(a_{k\ell} Y_{ik} Y^\dagger_{k\ell} \; Y_{\ell j} + b_{ij} Y_{ij} \right) f_{-E_j} \bar{L}_i^{(0)} \sigma^{\mu\nu} E_j^{(0)} F_{\mu\nu}^{(0)}$$

- Y_{ij} is a spurion of $\mathrm{U}(3)^3$ lepton flavor
- Indices on a_{ij} and b_{ij} encode bulk mass dependence

Flavor structure

- $a_{ij} Y_{ik} Y^\dagger_{k\ell} Y_{\ell j}$ gives a generic contribution
  Depends 'only' on Y_* and M_{KK}
- New: $b_{ij} Y_{ij}$ is aligned up to structure of b_{ij}
  $f_i Y_{ij} f_j \sim m_{ij}$, so this term is almost diagonal in the mass basis
  This depends on the *particular* flavor structure of the anarchic Y

Alignment vs FCNC

Definition: anarchic matrix, Ⓐ

All entries $\mathcal{O}(1)$ with arbitrary phase. The product of anarchic matrices is also anarchic. Assumption: this is true in all preferred bases.

$$R'^2 \frac{e}{16\pi^2}\frac{v}{\sqrt{2}} f_{L_i} \left(a_{k\ell} Y_{ik} Y^\dagger_{k\ell} Y_{\ell j} + b_{ij} Y_{ij} \right) f_{-E_j} \bar{L}_i^{(0)} \sigma^{\mu\nu} E_j^{(0)} F_{\mu\nu}^{(0)}$$

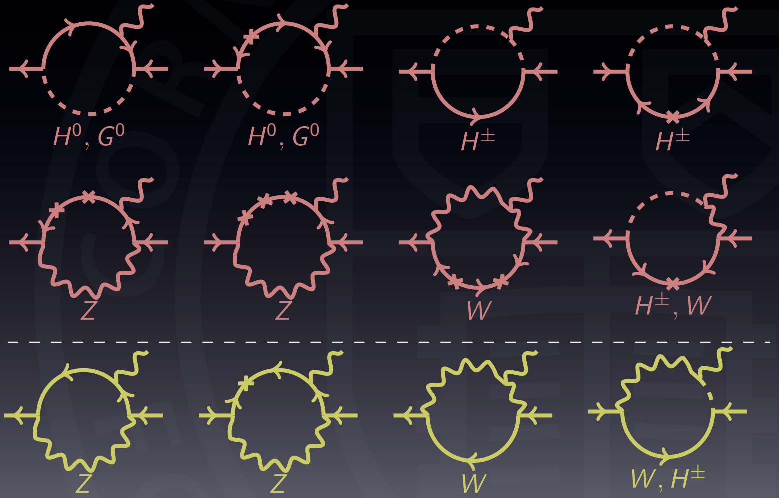
Compare to zero mode mass matrix: $m_{ij} = f_{L_i} Y^*_{ij} f_{-E_j} v$

- b terms have the flavor structure of 4D mass terms, up to bulk masses
- **Alignment**: b_{ij} term almost diagonalized in the mass basis
- $\Rightarrow$ Flavor structure important, defying anarchy

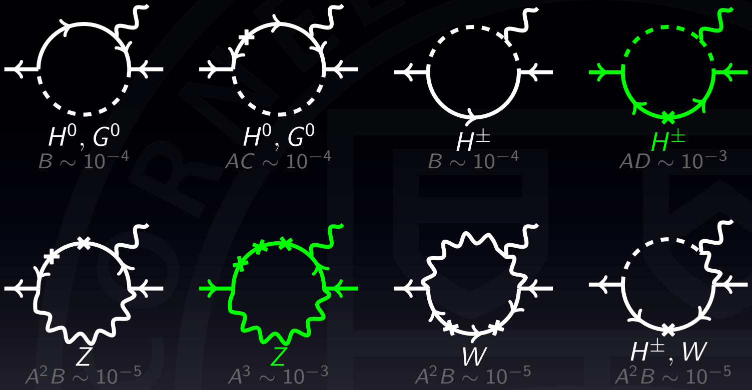
Alignment in RS: Agashe, Perez, Soni '04; Agashe, Azatov, Zhu '08

A bunch of diagrams: *a* and *b* coefficients

$$R'^2 \frac{e}{16\pi^2} \frac{v}{\sqrt{2}} f_{L_i} \left(a_{k\ell} Y_{ik} Y^\dagger_{k\ell} Y_{\ell j} + b_{ij} Y_{ij} \right) f_{-E_j} \bar{L}_i^{(0)} \sigma^{\mu\nu} E_j^{(0)} F^{(0)}_{\mu\nu}$$



The structure of RS penguins: a coefficient



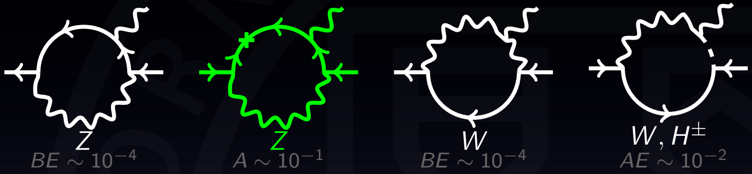
A. Mass insertion $\sim 10^{-1}$ per insertion (cross)

B. Equation of motion $\sim 10^{-4}$ (external arrows point same way)

C. Higgs/Goldstone cancellation $\sim 10^{-3}$ (H^0, G^0 diagram only)

D. Proportional to charged scalar mass $\sim 10^{-2}$

The structure of RS penguins: b coefficient



A. Mass insertion $\sim 10^{-1}$ per insertion (cross)
B. Equation of motion $\sim 10^{-4}$ (external arrows point same way)
E. No sum over internal flavors $\sim 10^{-1}$

Leading order diagrams

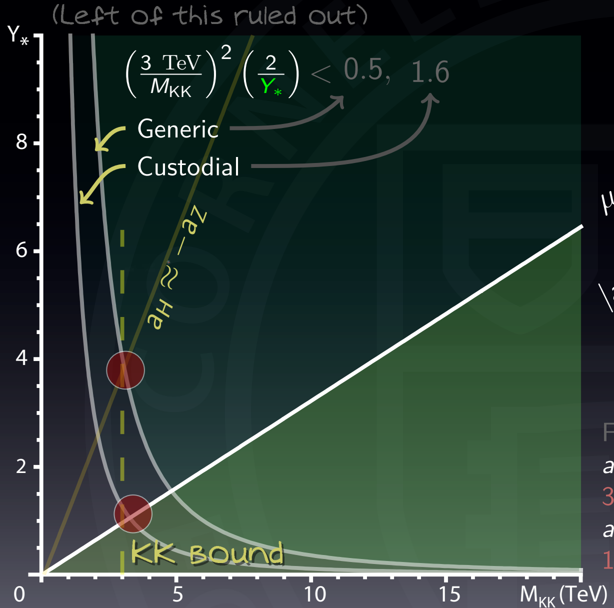
Three coefficients (a_H, a_Z, b) with arbitrary relative signs

Defined $aY_*^3 = \sum_{k,\ell} a_{k\ell} Y_{ik} Y_{k\ell}^\dagger Y_{\ell j}$ and $bY_* = \sum_{k,\ell} (U_L)_{ik} b_{k\ell} Y_{k\ell} (U_R^\dagger)_{\ell j}$

So, 'just calculate' these: (many details in paper)

- 5D position/momentum space: external zero modes
- Mass insertion approximation, but sum over all KK modes
- Gauge invariance: only identify $(p + p')^\mu$ coefficient

Representative Bounds: $b = 0$



$\mu \to e\gamma$, average

$|aY_*^2 + b| \left(\frac{3\text{TeV}}{M_{KK}}\right)^2 < .015$

For $M_{KK} = 3$ TeV, $b = 0$

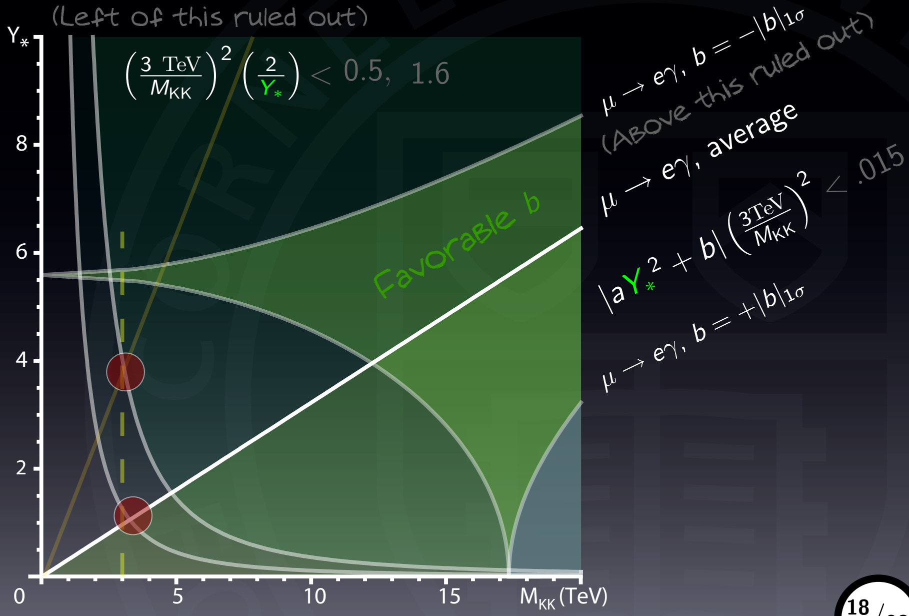
$a = .001$ and generic

$3.7 \lesssim Y_* \lesssim 4$

$a = .016$ and custodial

$1 \lesssim Y_* \lesssim 1$

Representative Bounds: $b \neq 0$



Finiteness: naïve dimensional analysis

4D Naïve: $\int d^4k\, \Delta_F \gamma^\mu \Delta_F \Delta_B \sim \log(\Lambda)$

Really log divergent? No, finite. Here's why:

- Gauge invariance: $q_\mu \mathcal{M}^\mu = 0$.
- Lorentz invariance: $\int d^4k\, \frac{\not{k}}{k^{2n}} = 0$.

Indeed, $\mathcal{M}_{4D} \sim \Lambda^{-2}$.
Suspect that $\mathcal{M}_{5D} \sim \Lambda^{-1}$.

5D Bulk, i.e. $d^4k \to d^5k$

Finiteness: bulk 5D fields

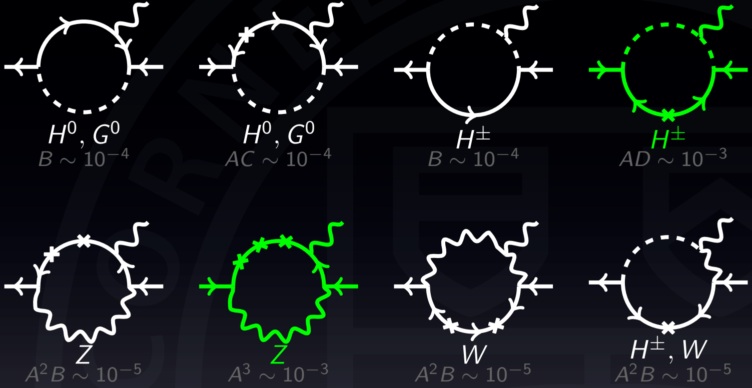
	Neutral	Charged
Loop integral (d^4k)	$+4$	$+4$
Gauge invariance ($p + p'$)	-1	-1
Bulk boson propagator	-1	-2
Bulk vertices (dz)	-3	-3
Overall z-momentum	$+1$	$+1$
Derivative coupling	0	$+1$
Mass insertion/EOM	-1	-1
Total degree of divergence	-1	-1

Note: this all carries over to the KK picture

Finiteness: brane-localized Higgs

	Neutral	Charged	W–$H^{\pm}$
Loop integral (d^4k)	$+4$	$+4$	$+4$
Gauge invariance ($p + p'$)	-1	-1	-1
Brane boson propagators	-2	-4	-2
Bulk boson propagator	0	0	-1
Bulk vertices (dz)	-1	0	-1
Derivative coupling	0	$+1$	0
Brane chiral cancellation	-1	0	0
Brane M_W^2 cancellation	0	-2	0
Total degree of divergence	-1	-2	-1

The structure of RS penguins: a coefficient



A. Mass insertion $\sim 10^{-1}$ per insertion (cross)

B. Equation of motion $\sim 10^{-4}$ (external arrows point same way)

C. Higgs/Goldstone cancellation $\sim 10^{-3}$ (H^0, G^0 diagram only)

D. Proportional to charged scalar mass $\sim 10^{-2}$

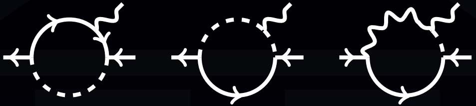
Finiteness: brane-localized Higgs

The **M_W^2 cancellation** comes from the form of the photon coupling to the brane-localized $H^{\pm}$:

$$\frac{(2k-p-p')^\mu}{[(k-p')^2-M_W^2][(k-p)^2-M_W^2]} = \frac{(p+p')^\mu}{(k^2-M_W^2)^2}\left[\frac{k^2}{k^2-M_W^2}-1\right]$$
$$= \frac{M_W^2(p+p')^\mu}{(k^2-M_W^2)^3} \sim \mathcal{O}(1/k^6)$$

We have used the fact that the $(p+p')^\mu$ coefficient gives the complete gauge-invariant contribution.

Finiteness: brane-localized Higgs



	Neutral	Charged	W–$H^{\pm}$
Loop integral (d^4k)	$+4$	$+4$	$+4$
Gauge invariance ($p+p'$)	-1	-1	-1
Brane boson propagators	-2	-4	-2
Bulk boson propagator	0	0	-1
Bulk vertices (dz)	-1	0	-1
Photon Feynman rule	0	$+1$	0
Brane chiral cancellation	-1	0	0
Brane M_W^2 cancellation	0	-2	0
Total degree of divergence	-1	-2	-1

Finiteness: brane-localized Higgs

The **chiral cancellation** comes from the UV structure of the sum of the two diagrams:

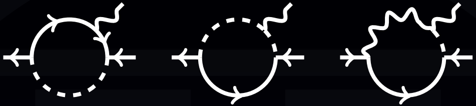
Fermion propagator goes like $\Delta \sim \not{k} + k\gamma^5$, numerator structures are

$$\mathcal{M}_a \sim \not{k}\gamma^\mu \not{k}\not{k} - k\gamma^\mu k\not{k} = k^2\left(\not{k}\gamma^\mu - \gamma^\mu \not{k}\right)$$
$$\mathcal{M}_b \sim \not{k}\not{k}\gamma^\mu \not{k} - \not{k}k\gamma^\mu k = k^2\left(\gamma^\mu \not{k} - \not{k}\gamma^\mu\right)$$

KK picture: appears as v/M_{KK} in the mass-basis Yukawa

See Agashe et al. '06

Finiteness: brane-localized Higgs



	Neutral	Charged	W–$H^{\pm}$
Loop integral (d^4k)	$+4$	$+4$	$+4$
Gauge invariance ($p + p'$)	-1	-1	-1
Brane boson propagators	-2	-4	-2
Bulk boson propagator	0	0	-1
Bulk vertices (dz)	-1	0	-1
Photon Feynman rule	0	$+1$	0
Brane chiral cancellation	-1	0	0
Brane M_W^2 cancellation	0	-2	0
Total degree of divergence	-1	-2	-1

Perturbativity and the 2-loop result

Yin-yang and double rainbow topologies. Insert a photon and odd number of mass insertions. Dotted line represents gauge or Higgs boson.

Purely bulk fields:

Loop integrals (d^4k)	$+8$
Gauge invariance ($p+p'$)	-1
Bulk boson propagators	-2
Bulk vertices (dz)	-5
Total degree of divergence	0

Log $\Lambda \Rightarrow$ large perturbative regime

Must do full calculation

Like 1-loop, hard to determine brane Higgs power counting. It may not be unreasonable to expect 1-loop cancellations to carry over to 2-loop.

The disappearing KK term

5D Lorentz invariance: must take the $M_n = nM_{\text{KK}}$ and $\Lambda = \lambda M_{\text{KK}}$ cutoffs together. Otherwise might lose leading term!

$$\mathcal{M}_{H^0} = \frac{gv}{16\pi^2} f_\mu f_{-e} \bar{u}_e (p+p')^\mu u_\mu \times \frac{1}{M^2}\left[c_0 + \mathcal{O}\left(\frac{v}{M}\right)^2\right]$$

$$c_0 = -\lambda^2 \sum_{n=1}^{N}\sum_{m=1}^{N} \frac{\lambda^2\left(n^2+m^2\right)+2n^2m^2}{4\left(n^2+\lambda^2\right)^2\left(m^2+\lambda^2\right)^2} \equiv -\frac{1}{\lambda^2}\sum_{n=1}^{N}\sum_{m=1}^{N}\hat{c}_0(n,m),$$

$$\hat{c}_0(n,n) \longrightarrow \left(\frac{n}{\lambda}\right)^2 \quad \text{for } n \ll \lambda$$

$$\hat{c}_0(n,n) \longrightarrow \left(\frac{n}{\lambda}\right)^0 \quad \text{for } n \approx \lambda$$

$$\hat{c}_0(n,n) \longrightarrow \left(\frac{\lambda}{n}\right)^4 \quad \text{for } n \gg \lambda.$$

Dominant contribution from $n \approx \lambda$. Taking $\lambda \to \infty$ for fixed n will lose this term! This is not a non-decoupling effect, just EFT.

Flight of the Warped Penguins

Future directions with local collaborators

1. Bulk Higgs models (integrals are much nastier)
2. $b \to s\gamma$ (operator mixing with $b \to sg$)

H^0, G^0, $H^\pm$

No Goldstone cancellation!

Conclusion

Calculation of $\mu \longrightarrow e\gamma$ in a warped extra dimension:

- Near tension between loop- and tree-level bounds on Y_*, M_{KK}
- $b_{ij} Y_{ij}$ is highly non-anarchic
- Finite at one-loop, suspect perturbative
- Certain features more transparent in 5D

Thanks!

Gauge invariance

This is a **dipole operator** and the Ward identity forces the gauge invariant amplitude to take the form

$$\mathcal{M} = \epsilon_\mu \mathcal{M}^\mu \sim \epsilon_\mu \bar{u}_{p'} \left[(p + p')^\mu - (m_\mu + m_e)\gamma^\mu \right] u_p$$

Thus it is sufficient to calculate the coefficient of the $(p + p')^\mu$ term in $\mathcal{M}^\mu$ to determine the overall gauge invariant amplitude.

Diagrams which are not 1PI, such as external photon emissions, are gauge redundant to the 1PI diagrams.

Lavoura '03

The standard $\mu \to e\gamma$ EFT

Traditional parameterization for the $\mu \to e\gamma$ amplitude

$$\frac{-iC_{L,R}}{2m_\mu}\bar{u}_{L,R}\,\sigma^{\mu\nu}\,u_{R,L}F_{\mu\nu},$$

For the case of RS,

$$C_{L,R} = \left(aY_*^3 + bY_*\right)R'^2\frac{e}{16\pi^2}\frac{v}{\sqrt{2}}2m_\mu f_{L_{2,1}}f_{-E_{1,2}}$$

$$\mathrm{Br}(\mu \to e\gamma) = \frac{12\pi^2}{(G_F m_\mu^2)^2}(|C_L|^2 + |C_R|^2) < 1.2\cdot 10^{-11}.$$

Trick: $C_L^2 + C_R^2 \geq 2C_L C_R$

$$\mathrm{Br}(\mu \to e\gamma) \geq 6\,\left|aY_*^2 + b\right|^2\frac{\alpha}{4\pi}\left(\frac{R'^2}{G_F}\right)^2\frac{m_e}{m_\mu}$$

Mixed 5D position/momentum space

Mixed position/momentum space: (p^μ, z)

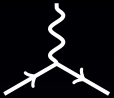
Due to the explicit z-dependence of the geometry and the localization of the Higgs, it is natural to work in mixed space.

$$\int đ^d k \frac{i}{k^2} e^{-ik\cdot(x-x')} \Rightarrow \int đ k_z \frac{i}{k^2 - k_z^2} e^{ik_z(z-z')}$$

- Usual momentum space in Minkowski directions
- Propagator dimension: $[\Delta_{5\text{D}}] = [\Delta_{4\text{D}}] + 1$
- Each vertex: perform dz overlap integral $\sim 1/k$
- External states carry zero-mode z-profile

5D Feynman rules

See our paper for lots of appendices on performing 5D calculations.



$$= ig_5 \left(\frac{R}{z}\right)^4 \gamma^\mu$$

$$= ie_5 (p_+ - p_-)_\mu$$

$$= \frac{i}{2} e_5 g_5 \, v \, \eta^{\mu\nu}$$

$$= i \left(\frac{R}{R'}\right)^3 Y_5$$

$$= \Delta_k(z, z')$$

$$= -i\eta^{\mu\nu} G_k(z, z')$$

$$= \epsilon^\mu(q) f_A^{(0)}$$

$$= \frac{f_c}{\sqrt{R'}} \left(\frac{z}{R}\right)^2 \left(\frac{z}{R'}\right)^{-c} u(p)$$

$$g_5^2 = g_{SM}^2 R \ln R'/R \qquad e_5 f_A^{(0)} = e_{SM} \qquad Y_5 = RY$$

Analytic expressions

$$\mathcal{M}(1\mathrm{MI}H^\pm) = \frac{i}{16\pi^2}(R')^2 f_{c_L} Y_E Y_N^\dagger Y_N f_{-c_E} \frac{ev}{\sqrt{2}} \cdot 2I_{1\mathrm{MI}H^\pm}$$

$$\mathcal{M}(3\mathrm{MI}Z) = \frac{i}{16\pi^2}(R')^2 f_{c_L} Y_E Y_E^\dagger Y_E f_{-c_E} \frac{ev}{\sqrt{2}} \left(g^2 \ln \frac{R'}{R}\right)\left(\frac{R'v}{\sqrt{2}}\right)^2 \cdot I_{3\mathrm{MI}Z}$$

$$\mathcal{M}(1\mathrm{MI}Z) = \frac{i}{16\pi^2}(R')^2 f_{c_L} Y_E f_{-c_E} \frac{ev}{\sqrt{2}} \left(g^2 \ln \frac{R'}{R}\right) \cdot I_{1\mathrm{MI}Z}.$$

Written in terms of dimensionless integrals. See paper for explicit formulae.

Finiteness in the KK picture

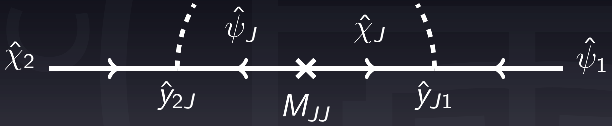
Power counting for the brane-localized Higgs

Charged Higgs: same M_W^2 cancellation argument as 5D
Neutral Higgs: much more subtle!

A basis of chiral KK fermions:

$$\chi = \left(\chi^{(0)}_{L_i}, \chi^{(1)}_{R_i}, \chi^{(1)}_{L_i}\right) \qquad\qquad \psi = \left(\psi^{(0)}_{R_i}, \psi^{(1)}_{R_i}, \psi^{(1)}_{L_i}\right)$$

Worry about the following type of diagram:



The (KK) mass term in the propagator can be $\sim \Lambda$.
Have to show that the mixing with large KK numbers is small.

Finiteness in the KK picture

Power counting for the brane-localized Higgs

A basis of chiral KK fermions:

$$\psi = \left(\psi^{(0)}_{R_i}, \psi^{(1)}_{R_i}, \psi^{(1)}_{L_i}\right) \qquad \chi = \left(\chi^{(0)}_{L_i}, \chi^{(1)}_{R_i}, \chi^{(1)}_{L_i}\right)$$

Mass and Yukawa matrices (gauge basis, $\psi M \chi + \mathrm{h.c.}$):

$$M = \begin{pmatrix} m^{11} & 0 & m^{13} \\ m^{21} & M_{\text{KK},1} & m^{23} \\ 0 & 0 & M_{\text{KK},2} \end{pmatrix} \qquad y \sim \begin{pmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

The zeroes are fixed by gauge invariance.

$$\hat{y}_{1J}\hat{y}_{J2} = 0$$

Indices run from $1, \ldots, 9$ labeling flavor and KK number

Finiteness in the KK picture

Power counting for the brane-localized Higgs

$$\psi = \left(\psi_{R_i}^{(0)}, \psi_{R_i}^{(1)}, \psi_{L_i}^{(1)}\right)$$
$$\chi = \left(\chi_{L_i}^{(0)}, \chi_{R_i}^{(1)}, \chi_{L_i}^{(1)}\right)$$

$$M = \begin{pmatrix} m^{11} & 0 & m^{13} \\ m^{21} & M_{\text{KK},1} & m^{23} \\ 0 & 0 & M_{\text{KK},2} \end{pmatrix}$$

Rotating to the mass basis, $\epsilon \sim v/M_{\text{KK}}$:

$$\hat{y} \sim \begin{pmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & \epsilon & 1 \\ 1 & & \\ \epsilon & & \end{pmatrix}$$

Now we have $y_{1J} y_{J2} \sim \epsilon$, good!

Finiteness in the KK picture

Power counting for the brane-localized Higgs

$$\psi = \left(\psi_{R_i}^{(0)}, \psi_{R_i}^{(1)}, \psi_{L_i}^{(1)}\right)$$
$$\chi = \left(\chi_{L_i}^{(0)}, \chi_{R_i}^{(1)}, \chi_{L_i}^{(1)}\right)$$

$$M = \begin{pmatrix} m^{11} & 0 & m^{13} \\ m^{21} & M_{\text{KK},1} & m^{23} \\ 0 & 0 & M_{\text{KK},2} \end{pmatrix}$$

Rotating to the mass basis, $\epsilon \sim v/M_{\text{KK}}$:

$$\hat{y} \sim \begin{pmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & \epsilon & 1 \\ 1 & & \\ \epsilon & & \end{pmatrix} \to \begin{pmatrix} 1 & 1+\epsilon & -1+\epsilon \\ 1+\epsilon & & \\ 1-\epsilon & & \end{pmatrix}$$

Must include 'large' rotation of m^{21} and m^{13} blocks representing mixing of chiral zero modes into light Dirac SM fermions. This mixes wrong-chirality states and does not affect the mixing with same-chirality KK modes.

Indeed, $\mathcal{O}(1)$ factors cancel: $y_{1J} y_{J2} \sim \epsilon$, good!