Physics 121: Intermediate Electricity and Magnetism

Final Exam

Text: Introduction to Electrodynamics (3rd edition)
By: David J. Griffiths

Time: pick-up, 1:00 Pm, Tuesday, June 12th
Turn in: 1:00 m, Wednesday, June 13th.

Rules:

1) **Open to textbook, homework and class note only, finish the problem by yourself.**
2) **Honor code rules apply.**

Problem #1 (3%)

Lorentz and Coulomb gauge

Explain qualitatively why we can choose different gauges in electrodynamics. Write down the Coulomb and Lorentz gauge conditions.

Problem #2 (4%)

Bremsstrahlung

What happens if a moving charge is stopped? Sketch the angular distribution of radiation for $v \ll c$ and $v \sim c$. Also sketch the case for when the velocity and acceleration are perpendicular to each other as in synchrotron radiation for $v \sim c$.

Problem #3 (3%)

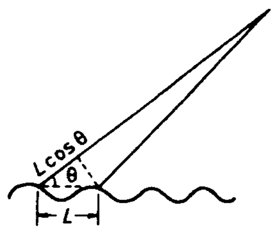
In Lienard-Wiechert potential, where does the factor $(1-\mathbf{r}\bullet v/c)$($\mathbf{r}$ is the unit vector from the point charge to the observation point) come from? Why should one consider this for a "point charge" potential?

Problem #4 (15%)

In a region of free space, the magnetic field is described by $\mathbf{B} = B_0 e^{ax}[\sin(ky-\omega t)]\,\mathbf{e_z}$(a is negative for x>0). (a) Calculate $\mathbf{E}$; (b) Find the speed of propagation v of this field; c) Is it possible to generate such a field? If so, how?

Problem #5 (10%)

A charge particle is constrained to move with constant velocity v in the x-direction (with $y=y_0$, $z=0$ fixed). It moves above an infinite perfectly conducting metal sheet at $y=0$ that undulates with "wavelength" L along the x-direction. A distant observer is located in the $z = 0$ plane and detects the electromagnetic radiation emitted at angel θ (the angle between the velocity and a vector draw from the charge to the observer as shown in the figure on next page). What is the wavelength λ of the radiation detected by the observer?



Problem #6 (10%)
Consider the situation shown in Fig. 1, where a perfectly conducting thin wire connects two small metallic balls. Suppose the charge density is given by

$$\rho(x,t) = [\delta(z-a) - \delta(z+a)]\delta(x)\delta(y)Q\cos(\omega_0 t).$$

The current flows between the metallic balls through the thin wire.
a, Q and ω_0 are constants.

(a) When is the dipole approximation valid?
(b) Calculate $dP/d\Omega$, the average power emitted per unit solid angle in the dipole approximation.
(c) The dipole radiation model is often used to describe light emission from atoms and molecules of length scale less than 1 nm. Will the dipole approximation be valid for visible light? Why? Will the dipole approximation be valid for x-ray? Why?

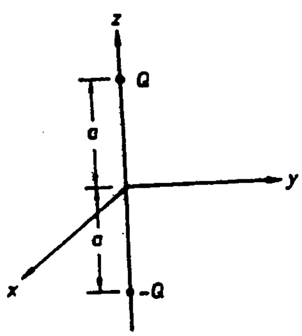


Fig.1

Problem #7 (10%)
As observed in an inertial frame S, two spaceships are traveling in opposite directions along straight, parallel trajectories separated by a distance d as shown in Fig. 2. The speed of each ship is $c/2$, where c is the speed of light.

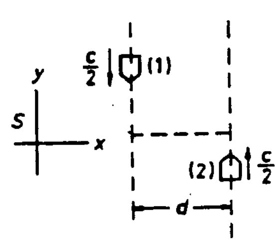


Fig.2.

(a) At the instant (as viewed from *S*) when the ships are at the points of closest approach (indicated by the dotted line in Fig.2) ship (1) ejects a small package which has speed 3c/4 (also as viewed from *S*). From the point of view of an observer in ship (1), at what angle must the package be aimed in order to be received by ship (2)? Assume the observer in ship (1) has a coordinate system whose axes are parallel to those of *S* and, as shown in the Fig. 2, the direction of motion is parallel to the *y* axis.
(b) What is the speed of the package as seen by the observer in ship (1)?

Problem #8 (15%)
Among the quantities of mass, energy, charge and velocity, which one is invariant under Lorentz transformation? Show that E^2-B^2 and **E•B** are invariant under Lorentz transformation.

Problem # 9 (15%)

(a) A muon at rest lives 10^{-6} sec, and its mass is 100 MeV/c^2. How energetic must a muon be to reach the Earth's surface if it is produced high in the atmosphere (say ~ 10^4 m up)?
(b) Suppose to a zeroth approximation that the Earth has a 1 gauss magnetic field pointing in the direction of its axis, extending out to 10^4 cm. How much, and in what direction, is a muon of energy *E* normally incident at the equator deflected by the field (neglect gravity, and take *E* from (a))?
(c) Very high energy protons in cosmic rays can lose energy through collision with 3 K radiation (cosmological background) in the process $p + \gamma \rightarrow p + \pi$. How energetic need a proton be to be above threshold for this reaction? (Express your answer in terms of masses of p and π, and the photon energy E_λ)

Problem #10 (15%)
A cylinder shaped electron bunch at SLAC linac (see Fig.3) moves at a constant speed of v ~ c. The energy of the electron is 50 GeV ($\gamma = 10^5$) and the pulse length is L = cτ, with τ = 1ps (10^{-12} s). a) explain qualitatively why a large number of about 2.5 x 10^{10} electrons can be packed into such a short beam in the laboratory frame, given the tremendously strong Coulomb repulsion among the electrons; b) draw and explain the electric and magnetic fields in the rest frame of the charge, and in the laboratory frame; c) For σ_r near 1 μm, estimate the order of magnitude in electric and magnetic field strength at 100 μm away from the beam axis. Will the field change if the beam is expanded to σ_r =10 μm but with the total number of charge fixed? Will it change if the electron bunch is compressed to 100 fs (10^{-13} s) but with the total number of charge fixed?

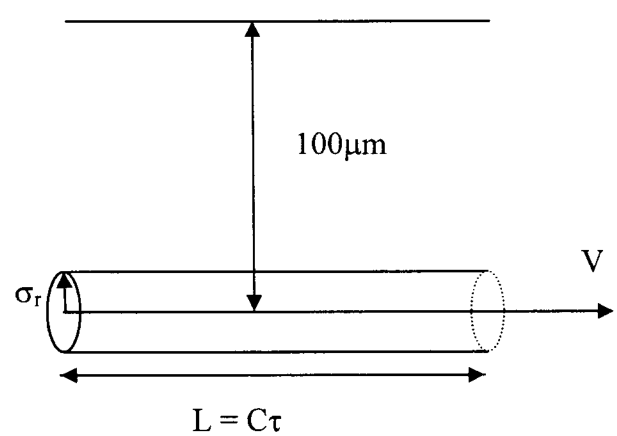


Fig. 3