NOTE: FOR ALL GRADING ISSUES, PLEASE CONTACT FLIP TANEDO AT flipt@stanford.edu

1. π^- CAPTURED IN S-WAVE $\Rightarrow L = 0$

$S_{\pi^-} = 0$, $S_d = 1$ $\Big\} \rightarrow \boxed{J = 1}$ (TOTAL INITIAL ANGULAR MOMENTUM)

$\boxed{P = -}$ (TOTAL INITIAL PARITY : $P_{\pi^-} = (-)$, $P_d = (+)$)

THESE QUANTITIES MUST BE CONSERVED IN STRONG & EM INTERACTIONS.

(a) $\longrightarrow \pi^0\, n\, n$ ← 3 PARTICLE FINAL STATE, CAN THINK OF AS A π^0 + (nn) SYSTEM, i.e. SUM OF 2 ORBITAL ANGULAR MOMENTA L_π AND L_n.

	π^0	n	n
SPIN:	0	1/2	1/2
PARITY:	−	+	+

CONSIDER THE PAIR OF NEUTRONS: IDENTICAL FERMIONS
→ MUST BE NEGATIVE UNDER INTERCHANGE OF PARTICLES
i.e. $(-1)^{L_n + S_n + 1} = (-1)$
$\Rightarrow L_n + S_n$ is EVEN

$S_n = 0, 1$ ← $\frac{1}{2} \otimes \frac{1}{2} = 0 \oplus 1$ in FANCY LANGUAGE
$\Rightarrow L_n = 0, 1$
$J_n = 0, (0,1,2)$

L_π IS ARBITRARY, SO ANGULAR MOMENTUM IS CONSERVED. ✓
(i.e. $J = J_n + L_n$)

PARITY IS GIVEN BY THE PRODUCT OF EACH OF THE PARTICLES' INTRINSIC PARITIES TIMES FACTORS FROM ANGULAR MOMENTUM.

$P = (-)(+)(+)(-)^{L_\pi}(-)^{L_n} \Rightarrow L_\pi + L_n = \text{EVEN}$ (we want $P = -$)

WE WANT THE FOLLOWING TO HOLD:

$$\begin{cases} L_n + S_n = \text{EVEN} \\ L_\pi + L_n = \text{EVEN} \\ J = 1 \end{cases}$$

THIS CAN BE TRUE FOR $S_n = L_n = L_\pi = 1$ SINCE J is GIVEN BY $1 \otimes 1 = 0 \oplus 1 \oplus 2$.

... SO NAIVELY THIS LOOKS OKAY... BUT

IT TURNS OUT THAT $L_\pi = 1$ is NOT ALLOWED! A HEURISTIC MOTIVATION: THE INTERACTION IS ESSENTIALLY CHARGE TRANSFER & OFFERS NO WAY TO SEND THE S-WAVE π^- INTO A P-WAVE π^0. (no points deducted for this subtle detail!)

(b) $\longrightarrow \gamma \; n \; n$ ⇆ AGAIN, 3 PARTICLE FINAL STATE. ORBITAL ANGULAR MOMENTUM CAN BE THOUGHT OF AS COMING FROM THE 'SUM' OF L_γ AND L_n

SPIN:	1	1/2	1/2
PARITY:	−	+	+

AS BEFORE, $S_n + L_n = \text{EVEN}$

SO WE CONSIDER $S_n = 0, 1$
$L_n = 0, 1$

$$\underbrace{\qquad}_{J_n = 0,1,2}$$

J_γ IS ARBITRARY

$\Rightarrow J = J_n + J$ CAN BE SET TO 1 ✓

$P = (-)(+)(+)(-)^{L_\gamma}(-)^{L_n}$

CAN TAKE $L_n = L_\gamma = 1$. ✓

⇒ this interaction is possible.

2. η GIVEN BY $J^{PC} = \underbrace{0^{-+}}_{CP=-}$, $I = 0$, $I_3 = 0$

(a) $\rightarrow \pi^0 \; \pi^0$
$0^{-+} \; 0^{-+}$ ✗ VIOLATES P, CP

(b) $\rightarrow \pi^+ \; \pi^-$ ✗ VIOLATES P
$0^- \; 0^-$

(c) $\rightarrow \pi^0 \; \pi^0 \; \pi^0$ ✓ ALLOWED: EM; CANNOT BE STRONG SINCE THIS VIOLATES G-PARITY. SANITY CHECK: IF THIS WERE STRONG, IT WOULD DOMINATE OVER EM DECAY MODES, SUCH AS $\eta \rightarrow \gamma\gamma$; BUT CHECKING THE PDG 2004 BOOKLET, WE SEE THIS IS NOT TRUE.
$0^{-+} \; 0^{-+} \; 0^{-+}$

(d) $\rightarrow \gamma \; \gamma$
$1^{--} \; 1^{--}$

$1 \otimes 1 = 0 \oplus 1 \oplus 2$

{CONTINUED ON NEXT PAGE}

d) $\longrightarrow \gamma\gamma$
$\quad 1^{--}\ 1^{--}$

THIS IS A TRICKY PROBLEM!
NO POINTS DEDUCTED FOR THIS PROBLEM,
BUT IT BEHOOVES YOU TO THINK ABOUT
THIS A LITTLE!

$1 \otimes 1 = 0 \oplus 1 \oplus 2$

BUT, BY SYMMETRY, $L=0$!
i.e. WHAT AXIS CAN THERE BE ANY ORBITAL ANGULAR MOMENTUM ABOUT?

PHYSICAL INTUITION: PHOTONS DON'T FORM BOUND STATES, SO FINAL STATE ($\gamma\gamma$) CAN HAVE NO ORBITAL ANGULAR MOMENTUM.

SO, NAIVELY, $P = (-)(-) = +$ $\Rightarrow$ PARITY VIOLATION

HOWEVER, THIS IS NOT THE CASE (!!)

CONSIDER THE $\gamma\gamma$ FINAL STATE: 

WRITE THE FINAL SPIN STATE AS PROJECTIONS ON THE $+\hat{z}$ AXIS.

i.e. $= |\downarrow\uparrow\rangle$

SINCE $L=0$, WE WANT THE SINGLET STATE $|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$ FOR J CONSERVATION

(THIS IS ACTUALLY A LITTLE DISHONEST, SEE BELOW.)

IN TERMS OF HANDEDNESS (CHIRALITY), NOTE THAT $|\downarrow\uparrow\rangle = |RR\rangle$, $|\uparrow\downarrow\rangle = |LL\rangle$

HENCE OUR SINGLET STATE IS $|RR\rangle - |LL\rangle$, AND IT IS CLEAR THAT THIS IS $(-)$ UNDER PARITY INVERSION. $\Rightarrow$ $\boxed{\text{DECAY IS POSSIBLE.}}$
(& EM)

SO WHAT GIVES?

argument from K. Giovanetti's (James Madison U.) PARTICLE PHYSICS COURSE WEBSITE

THE FOLLOWING DISCUSSION IS A LITTLE MORE FORMAL, BUT STILL SOMEWHAT UNCONVINCING. IF YOU ASK ME.

$$|\uparrow\uparrow\rangle = |2\ 2\rangle$$
$$|\uparrow\downarrow\rangle = \sqrt{1/6}\,|2\ 0\rangle + \sqrt{1/2}\,|1\ 0\rangle + \sqrt{1/3}\,|0\ 0\rangle$$
$$|\downarrow\uparrow\rangle = \sqrt{1/6}\,|2\ 0\rangle - \sqrt{1/2}\,|1\ 0\rangle + \sqrt{1/3}\,|0\ 0\rangle$$
$$|\downarrow\downarrow\rangle = |2\ {-2}\rangle$$

WE NOW NOTE THAT THE SINGLET CHIRALITY STATE ABOVE IS $|RR\rangle - |LL\rangle = |1\ 0\rangle$, i.e IT HAS TOTAL SPIN 1. WE THEN ARGUE (I read as: "hand wave") and say $L=S$ to allow COUPLING TO THE $J=0$ init. state.

ANYWAY, NOW OUR PREVIOUS FORMULA FOR PARITY WORKS OUT: $P = (-)(-)(-)^{\ell} = (-)$ & PARITY IS CONSERVED, AS DESIRED.

NOTE: NOTICE THAT THE $|RR\rangle + |LL\rangle$ CHIRALITY STATE IS A MIXTURE OF THE $|2\,0\rangle$ AND $|0\,0\rangle$ STATES, ALLOWING $J=1$ INITIAL STATES $\rightarrow \gamma\gamma$.

FOR A DETAILED ANALYSIS (& A CREDIBLE ONE!) SEE C.N. YANG PHYS. REV. 77 p. 241 (1950)
ATTACHED AT THE END OF THIS DOCUMENT & AVAILABLE ON COURSEWORK.

(e) $\rightarrow$ $\gamma \quad \gamma \quad \gamma$ $\quad 1^{--} \; 1^{--} \; 1^{--}$ ✗ VIOLATES C, CP
(WE COULD HAVE GUESSED A PRIORI THAT THIS IS NOT ALLOWED BECAUSE $\rightarrow \gamma\gamma$ IS ALLOWED)

(f) $\rightarrow$ $\pi^0 \quad \gamma$ $\quad 0^{-+} \; 1^{--}$ ✗ VIOLATES C, P, ANGULAR MOMENTUM

(g) $\rightarrow$ $\pi^0 \quad \gamma \quad \gamma$ $\quad 0^{-+} \; 1^{--} \; 1^{--}$ ✓ ALLOWED : EM
note: VIOLATES I BUT NOT I_3.

3. $\rho(770)$: $\rho^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$ $\quad I = 1, \; I^3 = 0 \rightsquigarrow |1\,0\rangle$

$\pi^0 \pi^0$ FINAL STATE $I = 0, \; I^3 = 0 \rightsquigarrow |0\,0\rangle$

LOOK AT CLEBSCH-GORDAN COEFFICIENTS FOR $|1\,0\rangle$ AND NOTE THAT THERE IS NO MIXING WITH THE $|0\,0\rangle$ STATE. HENCE THIS DECAY IS NOT ALLOWED.

4. (a) SINCE THE $K^*(892)$ IS IN THE TRIPLET S STATE, IT IS AN EXCITED STATE OF A ^1S MESON.

IT CAN DECAY INTO A GLUON ($J^P = 1^-$) AND A K ($J^P = 0^-$) WITH SOME ORBITAL ANGULAR MOMENTUM. (THE GLUON PRODUCES A π.)

$\Rightarrow$ STRONG INTERACTION

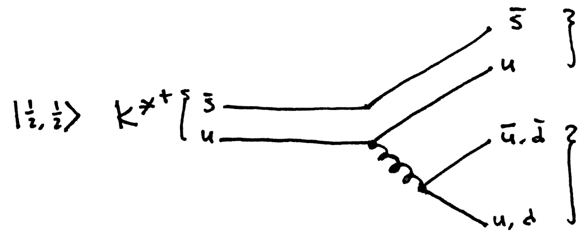
(b) $K^{*+} = |\tfrac{1}{2}, \tfrac{1}{2}\rangle$ $\qquad |I, I^3\rangle$

$K^0 = |\tfrac{1}{2}, -\tfrac{1}{2}\rangle$

$K^+ = |\tfrac{1}{2}, \tfrac{1}{2}\rangle$ $\qquad K^{*0} = |\tfrac{1}{2}, -\tfrac{1}{2}\rangle$

$\pi^0 = |1, 0\rangle$ $\qquad \pi^- = |1, -1\rangle$

$\pi^+ = |1, 1\rangle$

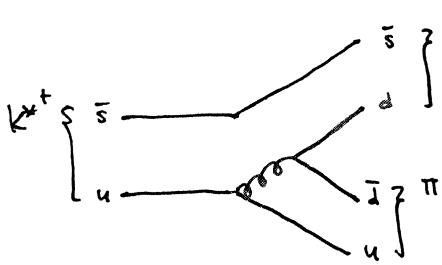


K^+ $|\tfrac{1}{2}, \tfrac{1}{2}\rangle$

π^0 $|1, 0\rangle$

$|\tfrac{1}{2},\tfrac{1}{2}\rangle \otimes |1,0\rangle = \sqrt{\tfrac{1}{3}}\,|\tfrac{1}{2},\tfrac{1}{2}\rangle + \cdots$

LOOK AT CLEBSCH-GORDAN COEFFICIENTS FOR ADDITION OF ISOSPIN, CHECK COEF. FOR STATE MATCHING INITIAL K^{*+}.



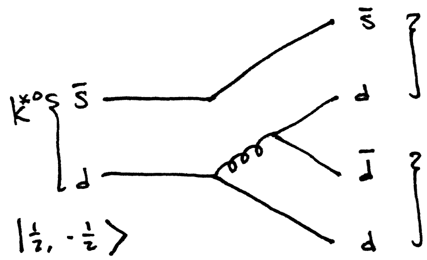
K^0 $|\tfrac{1}{2}, -\tfrac{1}{2}\rangle$

π^+ $|1, 1\rangle$

$|\tfrac{1}{2},-\tfrac{1}{2}\rangle \otimes |1,1\rangle = \sqrt{\tfrac{2}{3}}\,|\tfrac{1}{2},\tfrac{1}{2}\rangle + \cdots$

SINCE THESE ARE THE DOMINANT DECAYS, WE CAN TAKE THE BRANCHING RATIOS RELATIVE TO THESE MODES ONLY. (INDEED, PDG SUGGESTS CORRECTIONS WILL BE ON THE ORDER OF 10^{-3}% !)

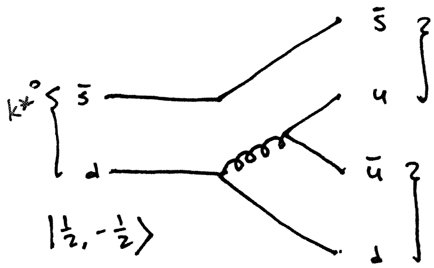
$$\Rightarrow \boxed{\begin{array}{l} \mathcal{B}(K^{*+} \to K^+ \pi^0) = 1/3 \\ \mathcal{B}(K^{*+} \to K^0 \pi^+) = 2/3 \end{array}}$$



$\pi^0 \quad |1,0\rangle$

$K^0 \quad |\tfrac{1}{2},-\tfrac{1}{2}\rangle$

$|\tfrac{1}{2},-\tfrac{1}{2}\rangle \otimes |1,0\rangle = \sqrt{\tfrac{1}{3}}\,|\tfrac{1}{2},-\tfrac{1}{2}\rangle$



$K^+ \quad |\tfrac{1}{2},\tfrac{1}{2}\rangle$

$\pi^- \quad |1,-1\rangle$

$|\tfrac{1}{2},\tfrac{1}{2}\rangle \otimes |1,-1\rangle = -\sqrt{\tfrac{2}{3}}\,|\tfrac{1}{2},-\tfrac{1}{2}\rangle$

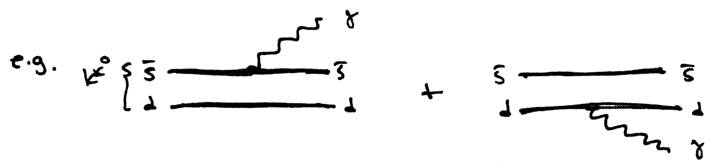
$$\boxed{\begin{array}{l}\mathcal{B}(K^{*0} \to \pi^0 K^0) = 1/3 \\ \mathcal{B}(K^{*0} \to K^+ \pi^-) = 2/3\end{array}}$$

<u>note</u>: ONE DOES NOT NEED TO REFER TO A CLEBSCH-GORDAN TABLE AGAIN FOR THE K^{*0} DECAYS; THEY SHOULD BE THE SAME COEFFICIENTS (UP TO OVERALL SIGN) AS THE K^{*+} DECAYS BECAUSE WE'RE JUST TAKING $|I, I_3\rangle \longleftrightarrow |I,-I_3\rangle$.

(c) WHY DON'T WE CONSIDER THE $K^{*0} \to K^- \pi^+$? WE TOOK THE QUARK CONTENT OF THE K^{*0} TO BE $\bar{s}d$, CONSISTENT WITH THE 2004 PDG, THIS IS IN THE $|I, I_3\rangle$ STATE $|\tfrac{1}{2},-\tfrac{1}{2}\rangle$. THE $K^-\pi^+$ IS IN THE STATE $|\tfrac{1}{2},-\tfrac{1}{2}\rangle \otimes |1,1\rangle = \sqrt{1/3}\,|3/2, 1/2\rangle + \sqrt{2/3}\,|1/2, 1/2\rangle$; THAT IS TO SAY THAT YOU CAN'T HAVE THIS REACTION AND CONSERVE ISOSPIN. HENCE SUCH A PROCESS WOULD BE WEAK. GRIFFITHS LISTS THE $\bar{s}d$ AND THE $s\bar{d}$ AS NEUTRAL K^* MESONS, BUT IT IS CLEAR THAT THESE ARE IN SEPARATE ISOSPIN STATES (WHICH IS WHY K^{*0} IS NOT A LINEAR COMBINATION OF $\bar{s}d$ AND $\bar{d}s$).

4. (d) $K^* \to K \gamma$

e.g.



ONE EM VERTEX IN EA. DIAGRAM

$\mathcal{M} \propto e \quad (\text{or} \propto \sqrt{\alpha})$

SINCE THE DECAYS IN PART (b) OCCUR VIA THE STRONG FORCE, WE KNOW THAT THE TOTAL DECAY WIDTH IS DOMINATED BY THOSE MODES. SO IN OUR ESTIMATION OF $\mathcal{B}(K^* \to K\gamma)$, WE CAN SIMPLY CONSIDER $\dfrac{\Gamma(K^* \to K\gamma)}{\Gamma(\text{STRONG DECAYS})}$.

$\Gamma(K^* \to K\gamma) \propto |\mathcal{M}|^2 \propto \alpha_{EM}$ (ONE VERTEX)

$\Gamma(\text{STRONG}) \propto |\mathcal{M}|^2 \propto \alpha_s^2$ (TWO STRONG VERTICES)

$\alpha_s^2 \simeq 1$;

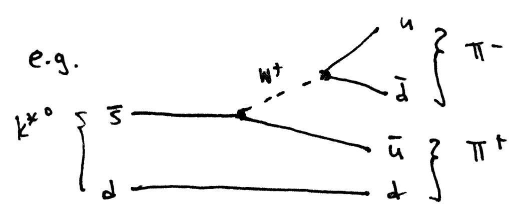
$\mathcal{B}(K^* \to K\gamma) \approx 1/100$

PDG: $\mathcal{B}(K^* \to K^0\gamma) = 2.3 \times 10^{-3}\,\%$

$\mathcal{B}(K^* \to K^\pm\gamma) = 9.9 \times 10^{-4}\,\%$

$K^* \to \pi\pi$

e.g.



STRANGENESS VIOLATING
$\Rightarrow$ WEAK FORCE

$\mathcal{B}(K^* \to \pi\pi) \approx \dfrac{\alpha_w^2}{\alpha_s^2} \approx (10^{-12})^2$ from Griffiths p. 55

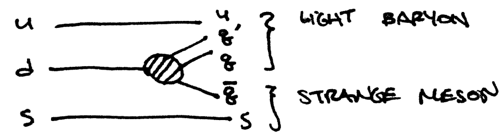
$\approx \boxed{10^{-24}}$

NOT EVEN LISTED IN THE 2004 PDG. ✓

ALTERNATELY, YOU COULD TAKE THE PDG VALUE FOR THE TOTAL WIDTH (& YOU WOULD ONLY BE OFF BY ANGULAR FACTORS), BUT IF YOU HAVE TO LOOK AT THE PDG, YOU'VE DEFEATED THE PURPOSE OF A "BACK-OF-THE ENVELOPE," HAVEN'T YOU?

5 (a) $\Lambda = uds$ HAS MASS $m_\Lambda = 1115.6$ MeV
IT CANNOT DECAY INTO A STRANGE BARYON SINCE IT IS THE LIGHTEST SUCH BARYON.

IT CANNOT DECAY INTO A STRANGE MESON (see figure to right) BECAUSE THE LIGHTEST BARYON HAS MASS $m_p = 938$ MeV



AND THE LIGHTEST STRANGE BARYON HAS MASS $m_K = 493$ MeV.

IT CANNOT DECAY INTO ANYTHING ELSE WITH STRANGE QUARKS BY CONSERVATION OF BARYON NUMBER.

$\Rightarrow$ ANY DECAY MUST VIOLATE STRANGENESS
$\Rightarrow$ —— " —— MUST BE <u>WEAK</u>.

(b) $\Delta I = 1/2$ RULE: $\Delta I = 1/2$ MODE DOMINATES THE $\Delta I = 3/2$ MODE BY ~ 25

IN $|I\ I^3\rangle$ SPACE:

$$\Lambda = |0\ 0\rangle \longrightarrow p\pi^- = |\tfrac12\ \tfrac12\rangle|1\ {-1}\rangle = \sqrt{1/3}\,|\tfrac32\ {-\tfrac12}\rangle - \sqrt{2/3}\,|\tfrac12\ {-\tfrac12}\rangle$$
$$\longrightarrow n\pi^0 = |\tfrac12\ {-\tfrac12}\rangle|1\ 0\rangle = \underbrace{\sqrt{2/3}\,|\tfrac32\ {-\tfrac12}\rangle}_{\text{IRRELEVANT TO THIS ORDER}} + \sqrt{1/3}\,|\tfrac12\ {-\tfrac12}\rangle$$

FEYNMAN DIAGRAMS (see (d)) CARRY SAME PROPAGATORS, COUPLINGS (SOME $\mathcal{O}(1)$ ANGULAR CORRECTION FROM PHASE SPACE, BUT LET US IGNORE THIS.); CALL THIS AMPLITUDE $\mathcal{M}$

THEN
$$\mathcal{M}(\to p\pi^-) \sim -\sqrt{2/3}\,\mathcal{M}$$
$$\mathcal{M}(\to n\pi^0) \sim \sqrt{1/3}\,\mathcal{M}$$

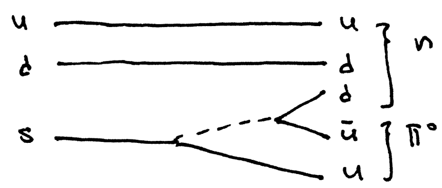
take $\Gamma(\Lambda) = \mathcal{M}^2(p\pi^-) + \mathcal{M}^2(n\pi)$ (dominant decay modes)

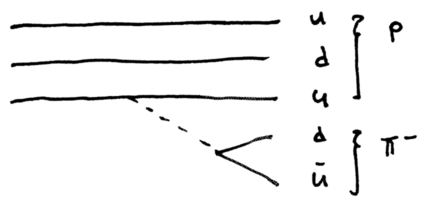
$$\boxed{\begin{aligned}\mathcal{B}(p\pi^-) &= 2/3\\ \mathcal{B}(n\pi^0) &= 1/3\end{aligned}}$$

(c) PDG: $\mathcal{B}(p\pi^-) = 63\ \%$
$\mathcal{B}(n\pi^0) = 36\ \%$

PRETTY GOOD!

(d)





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Selection Rules for the Dematerialization of a Particle into Two Photons

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Selection rules governing the disintegration of a particle into two photons are derived from the general principle of invariance under rotation and inversion. The polarization state of the photons is completely fixed by the selection rules for initial particles with spin less than 2. These results which are independent of any specific assumption about the interactions may possibly offer a method of deciding the symmetry nature of mesons which decay into two photons.

I. INTRODUCTION

IT has been pointed out[1] that a positronium in the ^3S state cannot decay through annihilation with the emission of two photons. Recent calculation[2] shows that also a vector or a pseudovector neutral meson cannot disintegrate into two photons. It is the purpose of the present paper to show that these facts are immediate consequences of certain selection rules which can be derived from the general principle of invariance under space rotation and inversion.

These selection rules also yield information on the polarization state of the two photons emitted. In particular, one concludes that the two photons resulting from the annihilation of slow positrons in matter always have their planes of polarization perpendicular to each other.[3] This has been pointed out by Wheeler who also proposed a possible experimental verification.[4]

An especially interesting consequence of these selection rules is that they could conceivably offer a means of studying the nature of particles which dematerialize into two photons. If, for example, neutral mesons are found which disintegrate into two photons,[5] one would conclude that they cannot be vector or pseudovector mesons. Besides, as will be apparent from the selection rules, the two disintegration photons from a scalar particle always have parallel planes of polarization while those from a pseudoscalar particle always have mutually perpendicular planes of polarization. An experimental determination of the relative orientation of the planes of polarization of the two disintegration photons would therefore decide whether the neutral meson is a scalar or a pseudoscalar meson.

TABLE I. Eigenvalues of the rotations $\mathcal{R}_\varphi$, $\mathcal{R}_\xi$, and the inversion $\mathcal{P}$ for the four polarization states.

	$\Psi^{RR}+\Psi^{LL}$	$\Psi^{RR}-\Psi^{LL}$	Ψ^{RL}	Ψ^{LR}
$\mathcal{R}_\phi$ (rotation around z axis through an angle φ)	1	1	$e^{2i\varphi}$	$e^{-2i\varphi}$
$\mathcal{R}_\xi$ (rotation around x axis through 180°)	1	1		
$\mathcal{P}$ (inversion)	1	-1	1	1

In Section II we shall give a simple but mathematically somewhat incomplete treatment of the symmetry nature of a state of two photons propagating in opposite directions, the detailed mathematical treatment being discussed in Section IV. The selection rules are derived in Section III and are based on the symmetry nature of the two photon states discussed in Section II. In the last section the parity of mesons and of the positronium is discussed.

II. BEHAVIOR OF THE STATE OF TWO PHOTONS UNDER ROTATION AND INVERSION

Consider two photons of equal wave-length λ_0 propagating in opposite directions along the z axis. There are four such states which we shall denote by Ψ^{RR}, Ψ^{RL}, Ψ^{LR} and Ψ^{LL}. The first index refers to the circular polarization state of the photon propagating in the $+z$ direction, the second to that of the other photon. E.g., Ψ^{RL} would represent a state with a right circularly polarized photon propagating along the $+z$ axis and a left circularly polarized photon propagating along the $-z$ axis.

In order to investigate the behavior of these four states under a space rotation or an inversion let us first write down the electric field for a right circularly polarized electromagnetic wave propagating along the z axis,

$$\begin{aligned}(E_x)_+{}^R&=E_0\cos(kz-\omega t+\delta_+{}^R),\\ (E_y)_+{}^R&=E_0\sin(kz-\omega t+\delta_+{}^R).\end{aligned}\tag{1}$$

TABLE II. Circular polarization of disintegration photons.

parity \ J	0	1	2, 4, 6 ⋯	3, 5, 7 ⋯
even	$\Psi^{RR}+\Psi^{LL}$	forbidden	$\Psi^{RR}+\Psi^{LL}$, Ψ^{RL}, Ψ^{LR}	Ψ^{RL}, Ψ^{LR}
odd	$\Psi^{RR}-\Psi^{LL}$	forbidden	$\Psi^{RR}-\Psi^{LL}$	forbidden

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[1] J. A. Wheeler, Ann. N. Y. Acad. Sci. 48, 219 (1946).

[2] S. Sakata and Y. Tanikawa, Phys. Rev. 57, 548 (1948); R. Finkelstein, Phys. Rev. 72, 415 (1947); J. Steinberger, Phys. Rev. 76, 1180 (1949).

[3] They are not individually plane polarized. But if they are analyzed into plane polarized waves their planes of polarizations show the stated correlation.

[4] J. A. Wheeler, reference 1. See also M. H. L. Pryce and J. C. Ward, Nature 160, 435 (1947), and Snyder, Pasternack and Hornbostel, Phys. Rev. 73, 440 (1948). Experimental verification has been reported by E. Bleuer and H. L. Bradt, Phys. Rev. 73, 1398 (1948).

[5] Bjorklund, Moyer, and York, Phys. Rev. 77, 213 (1950).

TABLE III. Correlation of the planes (see reference 3) of polarization of disintegration photons ($\perp$ = planes of polarization perpendicular, $\|$ = planes of polarization parallel).

parity \ J	0	1	2, 4, 6 ⋯	3, 5, 7 ⋯
even	$\|$	forbidden	$\| \geqq 50\%$ $\perp \leqq 50\%$	$\|$ 50% $\perp$ 50%
odd	$\perp$	forbidden	$\perp$	forbidden

For a right circularly polarized wave propagating in the opposite direction,

$$\begin{aligned}(E_x)_-^R &= E_0 \cos(-kz-\omega t+\delta_-^R),\\ (E_y)_-^R &= -E_0 \sin(-kz-\omega t+\delta_-^R).\end{aligned} \tag{2}$$

Under a rotation through an angle φ around the z axis,

$$\begin{aligned}x &= x' \cos\varphi + y' \sin\varphi,\\ y &= -x' \sin\varphi + y' \cos\varphi,\\ z &= z'.\end{aligned} \tag{3}$$

We have

$$\begin{aligned}(E_x)_+^{R'} &= E_0 \cos(kz-\omega t+\delta_+^R+\phi),\\ (E_y)_+^{R'} &= E_0 \sin(kz-\omega t+\delta_+^R+\phi);\end{aligned} \tag{4}$$

$$\begin{aligned}(E_x)_-^{R'} &= E_0 \cos(-kz-\omega t+\delta_-^R-\varphi),\\ (E_y)_-^{R'} &= -E_0 \sin(-kz-\omega t+\delta_-^R-\phi).\end{aligned} \tag{5}$$

Thus the phase of a right circularly polarized wave along the z axis changes by $+\varphi$ while that of a right circularly polarized wave along the $-z$ axis changes by $-\varphi$ under the rotation. For the quantum state Ψ^{RR} the total phase factor is the product of the two phase factors of the two photons. (This will become evident in Section IV.) Hence we conclude that the state Ψ^{RR} is an eigenstate of the rotation (3) with the eigenvalue 1.

Mathematically the states are changed under the rotation (3) by a unitary transformation which we shall call $\mathcal{R}_\phi$. We conclude that:

$$\mathcal{R}_\varphi \Psi^{RR} = \Psi^{RR}. \tag{6}$$

Similar conclusions are reached for a rotation around the x axis through 180° and for an inversion. We summarize the results in Table I.

It is of course evident that the angular momentum along the z axis for the different states is related to the eigenvalue of $\mathcal{R}_\varphi$ in the usual way.

III. SELECTION RULES

The selection rules governing the disintegration of a particle into two photons follow immediately from Table I. We take the center-of-mass reference system and take the z axis along the direction of one of the outgoing photons.

(i) For an odd initial state the only possible mode of disintegration is to go into the state $\Psi^{RR}-\Psi^{LL}$. For an even initial state the three final states Ψ^{RL}, Ψ^{LR}, and $\Psi^{RR}+\Psi^{LL}$ are possible.

(ii) For an initial state with total angular momentum $J=1, 3, 5\cdots$ the only possible final states are Ψ^{RL} and Ψ^{LR}. This is so because $\Psi^{RR}+\Psi^{LL}$ and $\Psi^{RR}-\Psi^{LL}$ are both simultaneous eigenstates of $\mathcal{R}_\varphi$ and $\mathcal{R}_\xi$ with eigenvalues one, while the initial state that is an eigenstate of $\mathcal{R}_\varphi$ with eigenvalue one has the rotation properties of the spherical harmonics Y_{J0} and therefore changes sign under $\mathcal{R}_\xi$ for $J=1, 3, 5\cdots$.

(iii) For an initial state with $J=0, 1$ the only possible final states are $\Psi^{RR}+\Psi^{LL}$ and $\Psi^{RR}-\Psi^{LL}$, because the states Ψ^{RL} and Ψ^{LR} have angular momentum values $\pm 2\hbar$ along the z axis, which is too big for $J=0$ or 1.

These results are summarized in Table II.

It will be shown in the next Section that

(i) $\Psi^{RR}+\Psi^{LL}$ represents two photons with their planes of polarization always[3] parallel;

(ii) $\Psi^{RR}-\Psi^{LL}$ represents two photons with their planes of polarization always perpendicular; and

(iii) Ψ^{RL} and Ψ^{LR} both represent two photons with their planes of polarization 50 percent of the time parallel and 50 percent of the time perpendicular.

These facts combined with Table II lead to the conclusions summarized in Table III concerning the correlation of the planes of polarization of the disintegration photons.

IV. SPACE ROTATION AND INVERSION IN QUANTUM ELECTRODYNAMICS

In the electromagnetic field and the meson field, the number of particles is not a constant of motion. A complete formulation of the principle of invariance can only be made with the quantized field theory. Let us first consider the electromagnetic field described by the vector potential $\mathbf{A}(xyz)$. These are operators operating on state vectors Ψ which are usually represented as functions of occupation numbers. Under a space rotation ρ defined by

$$x_i = \sum_{j=1}^{3} \rho_{ij} x_j' \quad (x_1=x,\, x_2=y,\, x_3=z), \tag{7}$$

the operators $\mathbf{A}(xyz)$ and the wave function Ψ undergo a unitary transformation $\mathcal{R}_\rho$ and the invariance under rotation requires that

$$\mathcal{R}_\rho A_i(xyz)\mathcal{R}_\rho^{-1} = \sum_{j=1}^{3} \rho_{ij} A_j(x'y'z'). \tag{8}$$

It is of course further required that the $\mathcal{R}$'s form a group isomorphic to the group of rotations.

To see in detail what this means let us expand the vector potential $\mathbf{A}$ into plane waves as usual:

$$\mathbf{A}(xyz) = \sum_{\mathbf{k}} \sum_{\lambda=1}^{2} (2\pi\hbar c/vk)^{\frac{1}{2}} \mathbf{e}_{k\lambda}(a_{k\lambda}e^{i\mathbf{k}\cdot\mathbf{r}} + a_{k\lambda}^* e^{-i\mathbf{k}\cdot\mathbf{r}}), \tag{9}$$

where $\mathbf{e}_{k1}$ and $\mathbf{e}_{k2}$ are two unit vectors forming with $\mathbf{k}/k$ a right-handed orthogonal system of unit vectors. It will be more convenient to use circular polarization for the study of rotation and we define

$$(\mathbf{e}_{k1}+i\mathbf{e}_{k2})/\sqrt{2} = \mathbf{e}_k{}^L, \quad (\mathbf{e}_{k1}-i\mathbf{e}_{k2})/\sqrt{2} = \mathbf{e}_k{}^R; \tag{10}$$

$$(a_{k1}-ia_{k2})/\sqrt{2} = a_k{}^L, \quad (a_{k1}+ia_{k2})/\sqrt{2} = a_k{}^R. \tag{11}$$

TABLE IV. The parity of particles at rest.

Scalar meson	Vector meson	Pseudovector meson	Pseudoscalar meson	Positronium in ${}^1S$ and ${}^3S$ states
even	odd	even	odd	odd

Evidently

$$\mathbf{A}(xyz)=\sum_k(2\pi\hbar c/vk)^{\frac{1}{2}}(\mathbf{e}_k{}^L a_k{}^L e^{i\mathbf{k}\cdot\mathbf{r}}+\mathbf{e}_k{}^{L*}a_k{}^{L*}e^{-i\mathbf{k}\cdot\mathbf{r}})$$
$$+\sum_k(2\pi\hbar c/vk)^{\frac{1}{2}}(\mathbf{e}_k{}^R a_k{}^R e^{i\mathbf{k}\cdot\mathbf{r}}+\mathbf{e}_k{}^{R*}a_k{}^{R*}e^{-i\mathbf{k}\cdot\mathbf{r}}). \quad (12)$$

The operators $a_{k\lambda}$, $a_k{}^R$ and $a_k{}^L$ all satisfy the usual commutation relations

$$a_k{}^R a_k{}^{R*}-a_k{}^{R*}a_k{}^R=1, \quad \text{etc.}$$

We are particularly interested in those modes of electromagnetic waves propagating along the $+z$ or the $-z$ direction with a definite wave-length, There are four such modes and we shall write their a operators as $a_+{}^R$, $a_+{}^L$, $a_-{}^R$ and $a_-{}^L$; $+$ and $-$ meaning the direction $+z$ or $-z$ of propagation. For definiteness we choose the phases of those modes such that the $\mathbf{e}_{k1}$, $\mathbf{e}_{k2}$ vectors in Eqs. (10) have as their xyz components:

$$\mathbf{e}_{+,1}=\mathbf{e}_{-,1}=(1,0,0), \quad \mathbf{e}_{+2}=-\mathbf{e}_{-,2}=(0,1,0). \quad (13)$$

With the operators a one can express in a very convenient form the states Ψ^{RR}, Ψ^{RL}, Ψ^{LR} and Ψ^{LL} defined in Section II.

$$\begin{array}{ll}\Psi^{RR}=a_+{}^{R*}a_-{}^{R*}\Psi_{00\cdots}, & \Psi^{RL}=a_+{}^{R*}a_-{}^{L*}\Psi_{00\cdots},\\ \Psi^{LR}=a_-{}^{R*}a_+{}^{L*}\Psi_{00\cdots}, & \Psi^{LL}=a_+{}^{L*}a_-{}^{L*}\Psi_{00\cdots},\end{array} \quad (14)$$

where $\Psi_{00\cdots}$ is defined to be the state with no photons.

We make a digression here to prove the statement at the end of the last section about the correlation in the planes of polarization of the two photons for the states $\Psi^{RR}+\Psi^{LL}$, $\Psi^{RR}-\Psi^{LL}$, Ψ^{RL} and Ψ^{LR}. By (14) and (11),

$$\begin{aligned}\Psi^{RR}+\Psi^{LL}&=(a_+{}^{R*}a_-{}^{R*}+a_+{}^{L*}a_-{}^{L*})\Psi_{00\cdots}\\ &=(a_{+,1}{}^*a_{-,1}{}^*-a_{+,2}{}^*a_{-,2}{}^*)\Psi_{00\cdots},\\ \Psi^{RR}-\Psi^{LL}&=-i(a_{+,1}{}^*a_{-,2}{}^*+a_{+,2}{}^*a_{-,1}{}^*)\Psi_{00\cdots},\\ \Psi^{RL}&=\tfrac{1}{2}(a_{+,1}{}^*a_{-,1}{}^*+a_{+,2}{}^*a_{-,2}{}^*\\ &\quad+ia_{+,1}{}^*a_{-,2}{}^*-ia_{+,2}{}^*a_{-,1}{}^*)\Psi_{00\cdots},\\ \Psi^{LR}&=\tfrac{1}{2}(a_{+,1}{}^*a_{-,1}{}^*+a_{+,2}{}^*a_{-,2}{}^*\\ &\quad-ia_{+,1}{}^*a_{-,2}{}^*+ia_{+,2}{}^*a_{-,1}{}^*)\Psi_{00\cdots}. \quad (15)\end{aligned}$$

Noticing that e.g., $a_{+,1}{}^*a_{-,1}{}^*\Psi_{00\cdots}$ represents a state with two photons with parallel planes of polarization one completes the proof with no difficulty.

Returning to the investigation of the behavior of the states Ψ^{RR}, Ψ^{RL}, etc., under rotation let us consider the rotation around the z axis through an angle φ, as defined by (3). Substitution of (13) and (12) into (8) shows[6] that

$$\begin{array}{ll}\mathcal{R}_\varphi a_+{}^R\mathcal{R}_\varphi{}^{-1}=e^{-i\varphi}a_+{}^R, & \mathcal{R}_\varphi a_+{}^L\mathcal{R}_\varphi{}^{-1}=e^{+i\varphi}a_+{}^L,\\ \mathcal{R}_\varphi a_-{}^R\mathcal{R}_\varphi{}^{-1}=e^{+i\varphi}a_-{}^R, & \mathcal{R}_\varphi a_-{}^L\mathcal{R}_\varphi{}^{-1}=e^{-i\varphi}a_-{}^L.\end{array} \quad (16)$$

These and similar equations for the other annihilation operators and for a general rotation ρ determine the operators $\mathcal{R}_\rho$ if the additional condition is imposed that $\mathcal{R}_\rho$ form a group. It is not difficult to prove that

$$\mathcal{R}_\rho\Psi_{00\cdots}=\Psi_{00\cdots}. \quad (17)$$

We can now prove Eq. (6) which asserts that Ψ^{RR} is an eigenstate of $\mathcal{R}_\varphi$ with eigenvalue 1. Take the Hermitian conjugate of (16) and multiply from the right by $\mathcal{R}_\varphi$:

$$\mathcal{R}_\varphi a_+{}^{R*}=e^{i\varphi}a_+{}^{R*}\mathcal{R}_\varphi, \quad \mathcal{R}_\varphi a_-{}^{R*}=e^{-i\varphi}a_-{}^{R*}\mathcal{R}_\varphi.$$

Hence,

$$\mathcal{R}_\varphi a_+{}^{R*}a_-{}^{R*}=a_+{}^{R*}a_-{}^{R*}\mathcal{R}_\varphi.$$

Operating on $\Psi_{00\cdots}$ with this last equation and making use of (17) and (14) one proves (6).

The other conclusions tabulated in the first row of Table I can be obtained in similar ways. For the rotation ξ:

$$x=x', \quad y=-y', \quad z=-z',$$

we have

$$\begin{array}{ll}\mathcal{R}_\xi a_+{}^R\mathcal{R}_\xi{}^{-1}=a_-{}^R, & \mathcal{R}_\xi a_+{}^L\mathcal{R}_\xi{}^{-1}=a_-{}^L,\\ \mathcal{R}_\xi a_-{}^R\mathcal{R}_\xi{}^{-1}=a_+{}^R, & \mathcal{R}_\xi a_-{}^L\mathcal{R}_\xi{}^{-1}=a_+{}^L.\end{array} \quad (18)$$

These lead to the results in the second row of Table I. Under an inversion the states are transformed by a unitary transformation $\mathcal{P}$ satisfying

$$\mathcal{P}\mathbf{A}(xyz)\mathcal{P}^{-1}=-\mathbf{A}(-x,-y,-z). \quad (19)$$

It is further required that

$$\mathcal{P}^2=1. \quad (20)$$

Expanding (19) into Fourier components we obtain

$$\begin{array}{ll}\mathcal{P}a_+{}^R\mathcal{P}^{-1}=-a_-{}^L, & \mathcal{P}a_+{}^L\mathcal{P}^{-1}=-a_-{}^R,\\ \mathcal{P}a_-{}^R\mathcal{P}^{-1}=-a_+{}^L, & \mathcal{P}a_-{}^L\mathcal{P}^{-1}=-a_+{}^R.\end{array} \quad (21)$$

It is to be noticed that (19) and (20) together do not completely determine the operator $\mathcal{P}$, as a change of sign of $\mathcal{P}$ does not affect either equation. However, changing the sign of $\mathcal{P}$ merely means a change in the name-calling of the even and odd states and is of no physical consequence. For definiteness we shall fix the sign by calling the vacuum an even state:

$$\mathcal{P}\Psi_{00\cdots}=\Psi_{00\cdots}. \quad (22)$$

Equations (21) and (22) lead to the third row of Table I.

[6] Actually since the Maxwell's equations are of the second order in $\partial/\partial t$, one should write together with (8)

$$\mathcal{R}_\rho\frac{\partial A_i(xyz)}{\partial t}\mathcal{R}_\rho{}^{-1}=\sum_j\rho_{ij}\frac{\partial A_j(x'y'z')}{\partial t}. \quad (A)$$

The operators $\partial A_i/\partial t$ are expanded into Fourier series similar to (12). Equation (A) together with (8) give Eq. (16).

V. PARITY OF MESONS AND THE POSITRONIUM

The above method for obtaining the symmetry nature of photon states can be easily extended to the meson field. In particular, if a_0 is the annihilation operator for a scalar meson at rest it is easy to see in analogy with Eq. (21) that

$$\mathcal{P} a_0 \mathcal{P}^{-1} = a_0. \tag{23}$$

There is no change of sign of a_0 (see Eq. (21)) because the scalar meson field, unlike the vector potential of the electromagnetic field, retains its sign under an inversion. If we again call the state of no meson an even state, it is evident that a state with one scalar meson at rest is also even. This and similar conclusions concerning the parity of the vector, the pseudoscalar and pseudovector mesons are summarized in Table IV.

With the electron-positron field the situation is quite similar. The behavior of the field $\psi_i(xyz)$ under rotation and inversion is evidently given by

$$\mathcal{R}_\rho \psi_i(xyz) \mathcal{R}_\rho^{-1} = \sum_{j=1}^{4} S_{ij}(\rho) \psi_j(x'y'z'), \tag{24}$$

$$\mathcal{P} \psi_i(xyz) \mathcal{P}^{-1} = \sum_{j=1}^{4} \beta_{ij} \psi_j(x'y', z'), \tag{25}$$

where $S_{ij}(\rho)$ represents the spinor transformation corresponding to the rotation ρ and β_{ij} are the elements of Dirac's β-matrix.

If we expand ψ into plane waves and consider the particular mode representing an electron at rest in a positive or negative energy state it is evident that (25) shows

$$\mathcal{P} b_{0+} \mathcal{P}^{-1} = b_{0+}, \quad \mathcal{P} b_{0-} \mathcal{P}^{-1} = -b_{0-}, \tag{26}$$

where b_{0+} and b_{0-} are the annihilation operators for an electron at rest with positive and negative energy values, respectively. The negative sign in (26) comes from the operation on ψ with the β-matrix. It is therefore evident that an electron-positron pair, both at rest, has an odd parity. Here, as before, we adopt the convention that the state of vacuum is to be called even.

Extension of the above argument to the ^1S and ^3S states of the positronium is evident. One gets for both states an odd parity. As mentioned in the introduction, Wheeler has pointed out[1] that the annihilation photons from the ^1S state of the positronium always have mutually perpendicular[3] planes of polarization. We see from Table III that the assignment of an odd parity to the ^1S state of positronium leads directly to the same conclusion.

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