

1. (a) GRIFFITHS 5.22

$$\text{(eq. 5.99)} \quad M_{\text{MESON}} = m_1 + m_2 + A \frac{\vec{S}_1 \cdot \vec{S}_2}{m_1 m_2}$$

$$m_u = 310 \text{ MeV}$$

$$m_d = 310 \text{ MeV}$$

$$m_s = 483 \text{ MeV}$$

$$A = (2m_u)^2 160 \text{ MeV}$$

<u>MESON</u>	<u>CALCULATED</u>	<u>OBSERVED</u>	(MeV)
$\pi$	140	138	
K	484	496	
$\eta$	559	549	
$\rho$	780	776	
$\omega$	780	783	
$K^*$	896	892	
$\phi$	1032	1020	
$\eta'$	349	956	

(b) SEE GRIFFITHS' REFERENCE:

C. Quigg, *Gauge Theories of the Strong, Weak, & EM Interactions*  
 New York: Benjamin 1983 (STANFORD LIBRARIES: QCT93.3.F5 Q53 1983)  
 PAGE 252

ALSO, Quigg's REFERENCE:

G. 't Hooft, *Phys Rev. Lett.* 37, 8 (1976)

THE IDEA: THE CHIRAL SYMMETRY THAT LED US TO BELIEVE IN EQUATION 5.99 IS BROKEN BY AN ANOMALOUS AXIAL CURRENT. (an "anomaly" is a case where quantum fluctuations break classical symmetries)

↳ e.g. VIRTUAL GYON STATES

2 Griffiths 5.23

	<u>MESON</u>	<u>CALCULATED</u>	<u>OBSERVED</u>
	$\eta_c$	2979	2981
	D	1711	1865
$D_s^+ \rightarrow$	F	1919	1968
	J/ψ	3007	3097
	D*	1843	2007
$D_s^{+*} \rightarrow$	F*	2004	2112
	B (bū)	4978	5279
	B <sub>s</sub> (b $\bar{s}$ )	5163	5370
	B <sub>c</sub> (b $\bar{c}$ )	6193	6400
	Υ (b $\bar{b}$ )	9401	9460

BY THE WAY, GRIFFITHS USES THE J/ψ PARTICLE ONLY AS "ψ," PERHAPS THAT IS BECAUSE HE WROTE THE BOOK WHILE ON SABBATICAL AT SLAC! (BURTON RICHTER OF SLAC NAMED THE c $\bar{c}$  BOUND STATE THE ψ, WHILE SAMUEL TING OF MIT NAMED IT THE J. BOTH WERE AWARDED THE 1976 NOBEL PRIZE AND HALF OF THE PARTICLE'S "OFFICIAL" NAME.)

3. Griffiths 5.28

WE WOULD LIKE TO CONSTRUCT A TOTALLY ANTISYMMETRIC SPIN/FLAVOR BARYON OCTET. WE HAVE THE PARTIALLY ANTISYMMETRIC SPIN 1/2 WAVE FUNCTIONS  $\psi_{12}^{(s)}$  AND  $\psi_{23}^{(s)}$  FROM EGS. (5.102) AND (5.103). FURTHER, WE HAVE THE PARTIALLY ANTISYMMETRIC FLAVOR WAVE FUNCTIONS  $\psi_{12}^{(f)}$  AND  $\psi_{23}^{(f)}$  (P. 177). FROM THESE WE CAN CONSTRUCT  $\psi_{13}^{(s)}$  [eg. (5.104)] AND  $\psi_{13}^{(f)}$  (P. 178).

LET ME USE MORE ACCESSIBLE NOTATION AND WRITE

$s_{ij} = \psi_{ij}^{(s)}$  PARTIALLY ANTISYM. SPIN STATE  
 $f_{ij} = \psi_{ij}^{(f)}$  " " FLAVOR STATE

I WILL ALSO USE THE CONVENIENT FLAVOR STATES  
 $F_{ij} = f_{ik} + f_{jk}$  PARTIALLY SYMMETRIC FLAVOR STATE

THANKS TO ONE OF YOUR COLLEAGUES, ARIEL SOMMER, FOR USING THIS CLEVER DEFINITION

NOW CONSIDER THE WAVE FUNCTION (SPIN ⊗ FLAVOR) ← "ANTISYM" × "SYMM" = "ANTISYM!"

$\psi = \eta (s_{12} F_{12} + s_{23} F_{23} + s_{31} F_{31})$

THIS IS TOTALLY ANTISYMMETRIC!  
 (NORMALIZATION,  $\eta$  FROM  $\langle \psi | \psi \rangle = 1 \Rightarrow \eta = 1/\sqrt{2}$ )

4. Griffiths 5.30

eq. (5.117)  $M_B = \langle B \uparrow | (M_1 + M_2 + M_3)_z | B \uparrow \rangle$

$$\begin{aligned} \Psi &= \eta (S_{12} F_{12} + S_{23} F_{23} + S_{31} F_{31}) \\ &= \eta (S_{12} f_{13} + S_{23} f_{21} + S_{31} f_{32} \\ &\quad + S_{12} f_{23} + S_{23} f_{31} + S_{31} f_{12}) \\ &= \eta (-S_{12} f_{31} - S_{23} f_{12} - S_{31} f_{23} \\ &\quad + S_{12} f_{23} + S_{23} f_{31} + S_{31} f_{12}) \end{aligned}$$

$f_{ij} = \Psi_{ij}$  in eqs. (5.102) - (5.104)  
 $S_{ij} = \Psi_{ij}$  on p. 177 - 178

**PROTON**

ABSORB NORMALIZ. OF S, & INTO  $\eta$

$$\begin{aligned} &= \eta [ (\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow) \{ (uud - duu) + (udu - duu) \} \\ &\quad + (\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow) \{ -(udu - duu) + (udu - duu) \} \\ &\quad - (\uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow) \{ -(uud - udu) + (udu - duu) \} \end{aligned}$$

$f_{12} = -S_{12} f_{31} + S_{12} f_{23}$   
 $f_{21} = -S_{23} f_{12} + S_{23} f_{13}$   
 $f_{23} = -S_{31} f_{23} + S_{31} f_{12}$

$$\Psi_p = \eta [ \downarrow\uparrow\uparrow (3udu - 3uud) + \uparrow\downarrow\uparrow (3uud - 3duu) + \uparrow\uparrow\downarrow (3duu - 3udu) ]$$

} notice that each term is a permutation of  $d(\uparrow), u(\uparrow), u(\downarrow)$

DISTINCT TERMS ARE ORTHOGONAL

FOR EACH TERM,  $\langle \text{term} | (M_1 + M_2 + M_3)_z | \text{term} \rangle = M_d$   
 (UP TO A COMMON NORMALIZATION THAT WE ABSORB INTO  $\eta$ )

$\Rightarrow \langle \Psi_{p\uparrow} | (M_1 + M_2 + M_3)_z | \Psi_{p\uparrow} \rangle = 6 \eta^2 M_d$

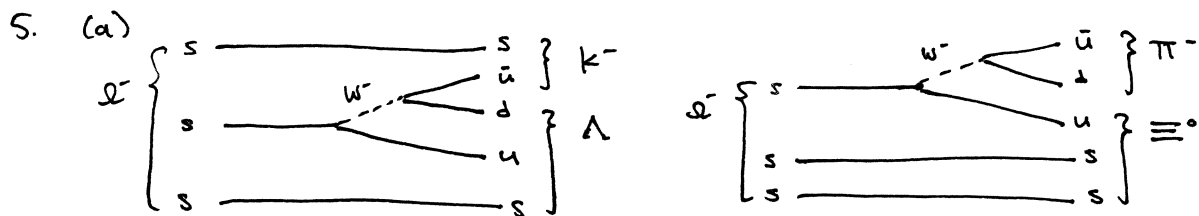
FOR THE NEUTRON, WE REPEAT ALL OF THE ABOVE WITH  $u \leftrightarrow d$   
 ALL STEPS FOLLOW ACCORDINGLY:

$\Rightarrow \langle \Psi_{n\uparrow} | (M_1 + M_2 + M_3)_z | \Psi_{n\uparrow} \rangle = 6 \eta^2 M_u$

$\Rightarrow \frac{M_n}{M_p} = \frac{M_u}{M_d} = \boxed{-2}$

$M_u = \frac{2}{3} \left( \frac{e}{2m_u} \right)$   
 $M_d = -\frac{1}{3} \left( \frac{e}{2m_d} \right)$   
 eqns. (5.116) } limit  $m_u = m_d$

FROM GRIFFITHS P. 182, EXPERIMENTAL VALUE IS  $\approx -\frac{2}{3}$



(b)  $Z^0$  IS IN THE  $|\frac{3}{2}, \frac{3}{2}\rangle$  STATE  $(|J, J_z\rangle)$   
 THE  $\Lambda$  IS SPIN  $\frac{1}{2}$   
 THE  $K^-$  IS SPIN  $0$  } IN ORDER TO CONSERVE  $J$ ,  $l = 1$

SO FINAL STATE MUST BE IN  $Y_l^m = Y_1^1$

ANGULAR DISTRIBUTION  $\propto |Y_1^1|^2 \propto \boxed{\sin^2 \theta}$   $\checkmark = \frac{3}{8\pi} \sin^2 \theta$

(c)  $|\frac{3}{2}, \frac{1}{2}\rangle$ : CAN HAVE  $(S_N)_z = \pm \frac{1}{2} \Rightarrow |l, m\rangle = |1, 0\rangle, |1, 1\rangle$   
 THE RELATIVE COEFFICIENTS MATTER:  
 REFERRING TO OUR CLEBSCH GORDAN TABLE:

$$|J, m\rangle = |\frac{3}{2}, \frac{1}{2}\rangle = \sqrt{\frac{1}{3}} |1, -\frac{1}{2}\rangle + \sqrt{\frac{2}{3}} |\frac{1}{2}, \frac{1}{2}\rangle$$

$$\Rightarrow \text{DISTRIBUTION} \propto \frac{1}{3} |Y_1^0|^2 + \frac{2}{3} |Y_1^1|^2 = \boxed{\frac{1}{8\pi} \sin^2 \theta + \frac{1}{2\pi} \cos^2 \theta}$$

$|\frac{3}{2}, \frac{1}{2}\rangle$  SAME CG COEFFICIENTS AS  $|\frac{3}{2}, \frac{1}{2}\rangle$  (just taking  $\frac{1}{2} \rightarrow -\frac{1}{2}$ )  
 $\Rightarrow$  SAME DISTRIBUTION

$|\frac{3}{2}, -\frac{3}{2}\rangle$  SAME AS  $|\frac{3}{2}, \frac{3}{2}\rangle$ .  $|Y_1^{-1}|^2 = |Y_1^1|^2$

(d) PHYSICALLY THE  $Z^0$  IS IN AN INHERENT SUPERPOSITION OF STATES W/ DIFFERENT  $J_z$  AND EQUAL PROBABILITY. THUS THE ANGULAR DISTRIBUTION SHOULD BE UNIFORM!

AND, INDEED:

$$2 |Y_1^0|^2 + 2 \left( \frac{1}{3} |Y_1^0|^2 + \frac{2}{3} |Y_1^1|^2 \right) = \frac{3}{4\pi} \sin^2 \theta + \frac{1}{4\pi} \sin^2 \theta + \frac{1}{\pi} \cos^2 \theta = \frac{1}{\pi} \checkmark$$

THAT IS TO SAY, WE CANNOT MAKE ANY MEASUREMENT OF  $Z^0$  SPIN!