2.2. MEAN FREE PATH IS GIVEN BY $l = 1/\sigma n$
CROSS SECTION, $\sigma = \pi r^2$

$$l = \frac{1}{\pi r^2 n}$$

$$l_\star = \frac{1}{\pi r_\star^2 n_\star} = \boxed{6.2 \times 10^{17} \text{ Mpc}}$$

$$l_g = \frac{1}{\pi r_g^2 n_g} = \boxed{8.0 \times 10^{4} \text{ Mpc}}$$

2.4 WE KNOW MASSES ARE NON-NEGATIVE
$\Rightarrow$ SET LIGHTEST NEUTRINO TO $\boxed{m_{\nu_e} = 0}$

then $m_{\nu_\mu} = \sqrt{5 \times 10^{-5}\ eV^2/c^4} = \boxed{.0071\ eV/c^2}$

$m_{\nu_\tau} = \sqrt{3 \times 10^{-3}\ eV^2/c^4 + m_{\nu_\mu}^2} = \boxed{.055\ eV/c^2}$

2.5 $$\frac{dE}{E} = -K\,dr$$

$$\ln E = -Kr + C$$

$$E = Ce^{-Kr} \longleftarrow E = h\nu = h\left(\frac{c}{\lambda}\right)$$

$$\frac{1}{\lambda} = Ce^{-Kr} \quad \text{(RESCALING OVERALL CONSTANT)}$$

$$z = \frac{\lambda_{OB} - \lambda_{EM}}{\lambda_{em}}$$

$r_{em} = 0$

$$= Ce^{-Kr_{em}}\left(C^{-1}e^{Kr_{OB}} - C^{-1}e^{Kr_{em}}\right)$$

$$= e^{Kr_{OB}} - 1$$

for $z \ll 1$ WE CAN TAYLOR EXPAND RHS

$\boxed{z = Kr + \mathcal{O}((Kr_{OB})^2)}$ where $Kr_{OB} \ll 1$

IF $z = \frac{H_0}{c} r$, $\boxed{K = \frac{H_0}{c}}$

3.2 (3.9) $ds^2 = \cancel{dr}^2 + R^2 \sin^2 r/R \; d\theta^2$

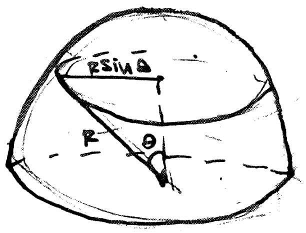
$\qquad\qquad 0$

$$\Rightarrow \boxed{d\theta = \frac{ds}{R \sin r/R}}$$

AS $r \to \pi R$, $d\theta \to \infty$

THIS CORRESPONDS TO ANTIPODAL POINTS ON THE SPHERE WHERE θ IS NOT WELL DEFINED

3.3



$\theta = r/R$

$\Rightarrow C = 2\pi R \sin r/R$

$|C_E - C| > 1$ m

$|2\pi r - 2\pi R \sin r/R| > 1$ m

$R \sin r/R < r - \frac{1}{2\pi}$

$\boxed{r > 33 \text{ KM}}$

3.4 a) for $K = +1$, THE SURFACE IS COMPACT $\Rightarrow \exists$ MAXIMUM AREA

A = (AREA OF SURFACE) = $\boxed{4\pi R^2}$

(IMAGINE AN ARBITRARILY SMALL TRIANGLE ON THE SURFACE, NOTE THAT THE COMPLIMENT OF THIS TRIANGLE IS ALSO A TRIANGLE!)

b) for $K = 0$ $\boxed{A = \infty}$

c) (3.10) $3\gamma = \pi - A/R^2$ $\qquad \gamma$ = ANGLE OF TRIANGLE (EQUILATERAL)

$\gamma \geq 0 \Rightarrow A = \boxed{\pi R^2}$

3.5

$$dx = \sin\theta\cos\phi\, dr + r\cos\theta\cos\phi\, d\theta + r\sin\theta(-\sin\phi)\, d\phi$$

$$dx^2 = \sin^2\theta\cos^2\phi\, dr^2 + r\sin\theta\cos\theta\cos^2\phi\, dr\, d\theta - r\sin^2\theta\cos\phi\sin\phi\, dr\, d\phi$$
$$r\sin\theta\cos\theta\cos^2\phi\, dr\, d\theta + r^2\cos^2\theta\cos^2\phi\, d\theta^2 - r^2\cos\theta\sin\theta\cos\phi\sin\phi\, d\theta\, d\phi$$
$$-r\sin^2\theta\cos\phi\cos\phi\, dr\, d\phi - r^2\cos\theta\sin\theta\cos\phi\sin\phi\, d\theta\, d\phi + r^2\sin^2\theta\sin^2\phi\, d\phi^2$$

$$= \sin^2\theta\cos^2\phi\, dr^2 + 2r\sin\theta\cos\theta\cos^2\phi\, dr\, d\theta - 2r\sin^2\theta\cos\phi\sin\phi\, dr\, d\phi$$
$$+ r^2\cos^2\theta\cos^2\phi\, d\theta^2 - 2r^2\cos\theta\sin\theta\cos\phi\sin\phi\, d\theta\, d\phi$$
$$+ r^2\sin^2\theta\sin^2\phi\, d\phi^2$$

$$dy = \sin\theta\sin\phi\, dr + r\cos\theta\sin\phi\, d\theta + r\sin\theta\cos\phi\, d\phi$$

$$dy^2 = \sin^2\theta\sin^2\phi\, dr^2 + r\sin\theta\cos\theta\sin^2\phi\, dr\, d\theta + r\sin^2\theta\cos\phi\sin\phi\, dr\, d\phi$$
$$(\text{SYMMETRIC}) + r^2\cos^2\theta\sin^2\phi\, d\theta^2 + r^2\cos\theta\sin\phi\cos\phi\sin\theta\, d\theta\, d\phi$$
$$(\text{SYMMETRIC}) + (\text{SYMMETRIC}) + r^2\sin^2\theta\cos^2\phi\, d\phi^2$$

$$dz = \cos\theta\, dr - r\sin\theta\, d\theta$$
$$dz^2 = \cos^2\theta\, dr^2 - 2r\cos\theta\sin\theta\, dr\, d\theta + r^2\sin^2\theta\, d\theta^2$$

$$ds^2 = \sin^2\theta(\cos^2\phi + \sin^2\phi)\, dr^2 + 2r\sin\theta\cos\theta(\cos^2\phi + \sin^2\phi)\, dr\, d\theta$$
$$+ r^2\cos^2\theta(\sin^2\phi + \cos^2\phi)\, d\theta^2 + r^2\sin^2\theta(\sin^2\phi + \cos^2\phi)\, d\phi^2 + dz^2$$

$$= \sin^2\theta\, dr^2 + 2r\sin\theta\cos\theta\, dr\, d\theta + r^2\cos^2\theta\, d\theta^2$$
$$+ r^2\sin^2\theta\, d\phi^2 + \cos^2 dr^2 - 2r\cos\theta\sin\theta\, dr\, d\theta + r^2\sin^2\theta\, d\theta^2$$

$$= dr^2 + r^2 d\theta^2 + r^2\sin^2\theta\, d\phi^2$$
$$= dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2) \quad \checkmark$$

4.1 $E_\Lambda = \frac{4}{3}\pi r^3 \varepsilon_\Lambda$ $\left\{ \begin{array}{ll} \varepsilon_\Lambda = 5200 & \text{MeV m}^{-3} \\ r = 1 \text{ AU} & = 1.5 \times 10^{11} \text{ m} \end{array} \right.$

$= \boxed{7.3 \times 10^{37} \text{ MeV}}$

$E_\odot = \boxed{1.1 \times 10^{60} \text{ MeV}}$ (GOOGLE CALCULATOR)

$E_\Lambda \ll E_\odot$, SHOULD NOT HAVE A SIGNIFICANT EFFECT.

4.2 IF SOME MATTER IS CONVERTED INTO RADIATION THEN THE PRESSURE OF THE UNIVERSE INCREASES SINCE $w_{matter} \ll w_{radiation}$ & ε is CONSTANT.

BY THE ACCELERATION EQUATION THIS CAUSES $\dot{a}$ TO DECREASE w/ TIME, hence THE UNIVERSE CONTRACTS.

$\Rightarrow$ this is slightly unintuitive based on how we usually think about pressure.

4.3 (4.69) $R_0 = \frac{c}{2\sqrt{\pi G \rho}} = (6.3 \times 10^{17}\,s)\,c$

IT WOULD TAKE A PHOTON $2\pi R_0 \sim 10^{18}$ sec TO CIRCUMNAVIGATE $\sim 10^{11}$ YEARS!

4.5 $E = \sqrt{m^2c^4 + h^2c^2/\lambda^2}$ E FOR ONE PARTICLE

NONRELATIVISTIC : $a \to \infty$ $\Rightarrow \lambda \to \infty$, $E = mc^2$

$\boxed{P = -\frac{\partial E}{\partial V} = 0}$ ✓

RELATIVISTIC : $a \to 0$ $\Rightarrow \lambda \to 0$

$\Rightarrow m^2c^4 \ll h^2c^2/\lambda^2 \Rightarrow E \approx \frac{hc}{\lambda}$

let $\lambda = \lambda_0 a = \lambda_0 V^{1/3}$

$E = \frac{hc}{\lambda_0} V^{-1/3}$

$P = -\frac{\partial E}{\partial V} = \frac{1}{3}\frac{hc}{\lambda_0} V^{-4/3} = \frac{1}{3}\frac{E}{V} = \boxed{\frac{1}{3}\varepsilon}$ ENERGY DENSITY

EXTRA CREDIT

a) THE EASIEST WAY TO DO THIS IS TO PERFORM A LORENTZ TRANSFORMATION ON THE ENERGY:

$$E' = \gamma E - \beta \gamma c P_x$$
$$h\nu' = \gamma h\nu - \beta \gamma c (E/c \cos\theta)$$

$$\nu' = \gamma \nu (1 - \beta \cos\theta)$$

$$\boxed{\frac{\nu'}{\nu} = \frac{1 - \frac{v}{c}\cos\theta}{\sqrt{1 - v^2/c^2}}}$$

b) SET $\frac{\nu'}{\nu} = 1$

$$1 - \frac{v}{c}\cos\theta = \sqrt{1 - v^2/c^2}$$
$$\boxed{\cos\theta = \frac{c}{v}\left(1 - \sqrt{1 - v^2/c^2}\right)}$$