318 Section 5: Prelim review

22 Feb

Broad overview
→ You ask questions & we'll discuss!

Phys 318 thus far

1. Newton ↔ Lagrange
   - Degrees of freedom: how to count
     - how to specify configuration space
     - UNAMBIGUOUS ANSWER: how many init. conditions are required?
   - Generalized coordinates: q, $\dot{q}$ INDEP.
   - deriving EOM from KE & PE
     - ↳ where did Newton's laws come in? Conservative force assumption?
   - defining L

2. Properties of Euler-Lagrange
   - Non uniqueness of L
   - Hamiltonian (def, when conserved, when = E?)
   - Conserved 'momenta'
   - using 1st integrals
     - ↑ potential pitfalls, eg. signs in V_{eff}

3. Constraints

- usually/often included in our choice of generalized coordinates

  → but: sometimes hard
  sometimes obfuscating
  (eg picking a gauge in E&M

- Lagrange multipliers
  - few ways of doing -- all the same
    eg: HW: $\Delta L = \lambda |\vec{r}_1 - \vec{r}_2|$ ↑ use $|\vec{r}_1 - \vec{r}_2| = d$
    vs: $\Delta L = \lambda(|\vec{r}_1 - \vec{r}_2| - d)$ ↑ λ = AUX DOF
  - λ ~ constraint force
    GIVES A CONTRIBUTION THAT FORCES GENERALIZED COORDINATES TO VARY ALONG CONSTRAINT.

  eg. Pdynar chain: time dep of λ would have been really hard to do using gen. coords alone.

- Holonomic vs Non-holonomic
  - we mostly deal w/ holonomic
  - understand definition

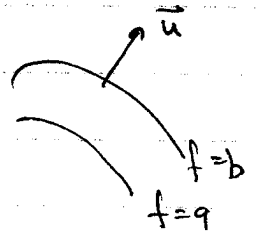
In gen: Non Holonomic

FIND EXTREMA OF $f(x,y,\ldots)$ s.t. $\delta\vec{F}$ IS PERPENDICULAR TO SOME # OF CONSTRAINT VECTOR FIELDS, $\vec{u}_1, \vec{u}_2, \ldots$

$\Updownarrow$

IN TERMS OF λ MULTIPLIERS

SOLVE: $\vec{\nabla} f = \lambda_1 \vec{u}_1 + \lambda_2 \vec{u}_2 + \ldots$



eg for just 1 constraint

Holonomic: if $u = \nabla g$
then: $f = g$.

- See how we circumvented nonholonomic constraints for the penny-inclined-plane prob.

↑ REMARK: eg. DRIVING OUT OF A TIGHT PARKING SPOT.

Calculus of variations

- Variations w/r/t paths (∞ # of variables)
  - ↑ $\dot{q}(t)^2$ terms cost a lot for very wildly varying paths. PHYSICALLY: WANT A STRAIGHT LINE PATH IN ABS OF FORCE.

- it's really just ordinary calculus
- yields EULER–LAGRANGE
- what if multiple dep. vars?
- other types of "L" (eg surface area of a bubble)

2 body : not on exam

relation to QM :

- QM is democratic, try all paths
- classical path gives largest contrib to AMPLITUDE — why? slowest rotation of phasor.