Section 8 or something

Announcements:

- Mon strut held by chivalry

Some remarks about HwJ

HwJ 4.2 $B(t) = B - t \mathbf{A}$ "Assume axial sym"

Call $\omega$ "Assuming $\phi = -$"

This is implicitly a gauge choice!

Axial sym $\Rightarrow$ want you to pick circular current!

Path s1. $\mathbf{A} \sim \mathbf{A}$ choice of $\mathbf{A}$ could have

had uncomb in $\phi$ s dip. But $\phi$ comb

Not compatible w1 $\phi = 0$.

In general (for $\mathbf{A} = A_0 \mathbf{A} + A_0 \mathbf{E}$), would need

to use Faraday's law $\mathbf{E} = -\nabla \phi - \frac{1}{c} \frac{d\mathbf{A}}{dt}$

to determine $\phi$.

But then the question is stupid B/c it asks

to find $\mathbf{E}$ then confirm Faraday.

But you need Faraday to (in general gauge) find $\mathbf{E}$. 
New ("macroscopic") Maxwell Eqs

\[ \nabla \cdot D = 4\pi j \]

\[ \nabla \cdot B = 0 \]

\[ \nabla \times E = -\frac{1}{c} \frac{\partial B}{\partial t} \]

\[ \nabla \times H = \frac{\partial D}{\partial t} + \frac{1}{c} \frac{\partial E}{\partial t} \]

Note: \[ \nabla \cdot H = -4\pi J \]

\[ \text{not "analogous to B"} \]

Divergence Laws \rightarrow (4s) Continuity of Material Comp.

All Laws \rightarrow (4s) Continuity of Tangential...

Eq. in absence of Free Charge,

\[ D_{1} \text{ is continuous} \rightarrow E \cdot \text{grad} D \text{ is not.} \]

Hjm 1.13

\[ \mathbf{P} \]

Brainstorm:

\[ S_{\rho} = n \cdot \mathbf{P} = \mathbf{P} \cos \theta \]

\[ E(r) = \int_{S_{\rho}} \frac{P(s)}{|r - s|^{2}} \, d^2 s \rightarrow \int_{a}^{r} \frac{P_{2}(r)}{a^2(r)} \, (2\pi) \, a^2 \, d(\cos \theta) \]

Go Ahead + Project on \[ E_{2} \text{, only Non Zero Comp} \]

\[ E_{2}(r) = \int_{a}^{r} \frac{P \cos \theta}{a \cos \theta} \cdot (\frac{r}{a}) \, (2\pi) \, a^2 \, d(\cos \theta) \]

\[ -\cos \theta \]
\[ E_2(0) = -2 \pi P \int_0^1 \cos^2 \theta \, d(\cos \theta) = -\frac{4\pi P}{3} \]

\[ E(0) = -\frac{4\pi}{3} P \]

b) DOES THIS LOOK REASONABLE? WHAT ABOUT OTHER \( r < a \)?

CLAIM: COMPARE TO OVERLAP OF OPP. CHARGED UNIF. SHOES

\[ \begin{align*}
&\text{Choose } P = \delta \text{ s.t. overlap config} \\
&\text{matches part (a)} \\
&\text{Note: Part a has total dipole moment } \begin{bmatrix} P \frac{4\pi a^3}{3} \end{bmatrix} = P \delta
\end{align*} \]

\[ \begin{align*}
&\text{S. superimposed config has dipole moment} \\
&P = 0 \delta \\
&w \delta = \frac{4\pi a^3}{3} \\
\text{The dipole moments match when } P = P \delta
\end{align*} \]
CLAIM: IN A UNIFORM SPHERE $E(r) = \frac{4\pi P}{3}$.

\[
E = \frac{4\pi}{3} \left[ (r - \frac{a}{2}) - (r + \frac{a}{2}) \right] = -\frac{4\pi P a}{3}
\]

\[
E = -\frac{4\pi P a}{3}
\]

What we found at $r = 0$

In fact, $E$ is constant everywhere inside.

Alternate derivation: SPHERICAL HARMONICS

\[
P_0 = P_0 P_0 (\cos \theta)
\]

(c) Now consider a polarized medium w/ a sphere cut out. What is $E$ in the cavity?

What $E$ in the medium? (E is external)

Same trick: superimpose a polarized sphere

\[
E_{\text{ext}} + E_{\text{sup}} = E - \frac{4\pi}{3} P_{\sup} = E + \frac{4\pi}{3} P
\]
Note: we are ignoring dipole contribution!

Example: midterm w/ dielectric

\[ \phi(r) = (C_0 r^0 + D_1 \frac{1}{r^{2+1}}) P_e \]
\[ = A^2 \frac{1}{r^2} P_e \]

BC: Continuity of \( E_z \): \( \phi \) is continuous

Discontinuity of \( D_z \):
\[ \varepsilon \frac{\partial \phi}{\partial r} \big| \frac{1}{r} - \varepsilon \frac{\partial \phi}{\partial r} = \quad \text{at} \quad a, b \]

Start @ Boundary \( r = b \), simplest:

\[ \phi(b) = 0 \Rightarrow \left[ D_e = -C_e b^{2+1} \right] \]
\[ \phi_e = \frac{b^{2+1}}{c^{2+1}} P_e \]

Next simplest: Continuity @ \( r = a \)

\[ \phi'(a) = \phi(a) \Rightarrow Al a^4 = C_e (a^5 - b^{2+1}/a^{2+1}) \]
\[ A l = C_e \left( 1 - \left( \frac{a}{b} \right)^{2+1} \right) \]

So far: same!
Hard BC

\[ e^{-r_0} = e^{-r_0}, \quad e^{2r_0} \Phi(a) - 2r_0 \Phi'(a) = -4\pi \delta(0) \]

\[ e^{C e \left[ (l +1) - (l+1) \right]} \frac{b^{2l+1}}{a^{l+2}} \left[ P_0 - \frac{2}{3} C e - l \left( \frac{1}{2} \right)^{2l+1} \right] l a^l P_0 \]

\[ = -4\pi \sigma_\infty \left( \frac{1}{3} P_0 + P_0 + \frac{2}{3} P_2 \right) \]

\[ Q^2 : \quad e^{C e \left[ l +1 \right]} \frac{b^{2l+1}}{a^{l+2}} - C e l a^{-l} + C e l a^{l+2} = \cdots \]

\[ C e \left[ (l+1) +1 \right] \frac{b^{2l+1}}{a^{l+2}} + (l-1) C e l a^{-l} = \cdots \]

So matching coefficients:

\[ C_0 = \left( -4\pi \sigma_\infty \right) \frac{a^2}{b^2} \]

\[ C_1 \left[ (l+1) \right] \frac{b^3}{a^3} + (l-1) C_1 \left[ \left( (l+1) \right) \right] = -4\pi \sigma_\infty \]

\[ C_1 = -4\pi \sigma_\infty \left( (l+2) \frac{b^3}{a^3} + (l-1) \right)^{-1} \]

\[ C_2 \left[ (l+1) +1 \right] \frac{b^5}{a^5} + (l-1) C_2 a \]

\[ C_2 = -4\pi \sigma_\infty \left( \frac{2}{3} \right) \left( \left( (l+3) \right) \frac{b^5}{a^5} + (l-1) a \right)^{-1} \]
$- \Theta \phi (b) = - 4 \pi \sigma \rightarrow \sigma = \frac{\Theta \phi (b)}{4 \pi}$

$2r \phi (r) = 2 C e^{l} l r^{l-1} + (l+1) \frac{b^{2l+1}}{r^{l+2}} \int P_{l}$

$\frac{\Theta \phi (b)}{4 \pi} = - \sigma \left[ \frac{4}{3} \frac{a^{2}}{b} \left( \alpha + \frac{1}{b} \right) P_{0} \right.$

$+ \frac{2}{3} \left( \frac{(3+2)}{a^{3}} \right) \int (3-1) \int^{l+2} P_{l} \left. \right]$

$+ \frac{2}{3} \left( \frac{(3+2)}{a^{3}} \right) \int (3-1) \int^{l+2} P_{l} \left. \right]$

"WUUGuy! But can see: $\sigma$ affects different multipoles differently."