**Remark:** These are rough notes which may or may not correspond to what we actually did in section! - Fuj

**Announcements**

- Encouraged: Read §3.9 §3.10
  - for culture
  - for later

- HW Extensions

  We've been fairly lax - but just because you get an extension, it doesn't mean that the class is slowing down!

  → Each extra day spent on old HW is one less day for current HW

  → Please soon!

  We're giving you wiggle room because you're grown ups, but make sure you don't end up screwing yourself.

- Repeat: Work with other people!!
  - it's a matter of efficiency.

- HW#9 Hint - to be posted

  Use KelvinBar KelvinBar

- HW: Hand writing, be honest with?
WARM UP

Getting used to $\varepsilon$. (Polarizability)

Q: is $\varepsilon > 1$
   or $\leq 1$?

\[ D = \varepsilon E \]

What is $D$? This is the "electric field" that is sensitive to (sourced by) only free charge, not bound charge. (PBZ of the medium)

So: Which is bigger, $D$ or $E$?

$D$ is! Bound dipoles align according to $E$, create a contribution that wants to cancel $E$.

\[ E \rightarrow \begin{array}{c} + \\downarrow \\uparrow \\
\end{array} \]

\[ \rightarrow \begin{array}{c} - \\uparrow \\downarrow \\
\end{array} \]

\[ \overset{\text{Stirrer}}{\varepsilon > 1, D > E} \]

\[ \overset{\text{(do not confine \mu) \rightarrow p/\varepsilon}}{\text{field \& long distances}} \]

\[ \overset{\text{\mu} \rightarrow 0 \text{ doesn't make sense.}}{\text{}} \]
Remarks on $\mu$

$D = \varepsilon E \quad \text{but} \quad H = \frac{1}{c} \mathbf{B}$

\[ \uparrow \quad \uparrow \]

$= E + \mu\mathbf{P}$

$= B - \mu\mathbf{M}$

\[ \mu \varepsilon \mathbf{E} = \mu_0 (\mathbf{B} + \mathbf{P_e}) \]

\[ = -\nabla \mathbf{E} \]

\[ \nabla \mathbf{B} = \frac{\mu_0}{c} (\mathbf{j}_f + \mathbf{j}_b) \]

\[ = \mathbf{c} \nabla \times \mathbf{M} \]

In fact:

$\mu \gg 1$ diamagnetic

$\mu \ll 1$ paramagnetic

($\mu \gg 1$ ferromagnetic)

But for typical materials, $\mu \approx 1$ so for now we stick to this regime.
**Dielectric (ε) Media**

\[ f \approx 1 \]
\[ \sigma = 0 \quad \text{Why?} \quad \sigma \neq 0 \Rightarrow \text{J nons} \]

This will be all about this.

\[ \nabla \times \mathbf{B} - \frac{\varepsilon_0}{c} \frac{d}{dt} \mathbf{E} = \frac{4\pi}{c^2} \mathbf{J} \quad \text{(Faraday's Law)} \]

**Punchline:** Light travels slower in media.

index of refraction \( n \)

\[ n = \frac{c}{v} \]

**Easy to see,**

\[ \nabla \times \mathbf{B} = \frac{1}{c} \frac{d}{dt} \mathbf{E} \quad \Rightarrow \quad \nabla \times \mathbf{B} = \left( \frac{\varepsilon_0}{c} \right) \mathbf{E} \]

\[ \text{Then} \quad \nabla \times \mathbf{E} = \mathbf{D} = \varepsilon_0 \mathbf{E} \]

\[ \nabla \times \mathbf{D} = \nabla (\mathbf{D} \cdot \mathbf{B}) - \nabla B = \frac{8\pi}{c} \mathbf{G} \]

\[ = -\frac{8\pi}{c} \mathbf{G} \left( \frac{\varepsilon_0}{c} \right) \]

**But why?** (Microscopically)

**Problem:** Show, by supposition, that \( v = \frac{1}{c} \). Assuming the incident wave travels in \( \varepsilon = 0 \). Show \( v \neq 0 \).

In conclusion, the induced charge...
Superposition

\[ \text{IDEA: } \pm \frac{1}{2} ( \hat{z} - i \hat{y}) \]

WAVE CAUSES OSCILLATIONS IN DIPOLAR MEDIUM

SUPERPOSITION OF INITIAL WAVE + INDUCED WAVES GIVES NEW MONOCHROMATIC WAVE

\[ V = \frac{\lambda}{2N} = \frac{\lambda}{v} \]

Two PART Problem

1. ("nonperturbative")

\[ E_1 = E_0 e^{i k' x - i \omega t} \]

\[ B_1 = \frac{E_0}{c} e^{i k' x - i \omega t} \]

\[ E_r = E_0 e^{i k x - i \omega t} \]

\[ B_r = \frac{E_0}{c} e^{i k x - i \omega t} \]

\[ k' = \frac{n \lambda}{c} = \frac{n \lambda}{v} \]

\[ k = \frac{\omega}{v} \]

Assume \[ V = \frac{\lambda}{v} \] here (well prove later)

we will study this in ch. 6 (Basis of optics)

FIND \[ E_T \neq \text{EXPAND in } h = 4\pi \hbar e \]

\[ E_T = E_0 e^{i k' x - i \omega t} \]

\[ B_T = \frac{E_0}{c} e^{i k' x - i \omega t} \]

\[ \hat{y} \]

\[ \hat{y} \]

Show that superposition above gives same expansion in \( h \).
BC (from Maxwell @ interface)

\[ \mathbf{D} = 0 \Rightarrow \mathbf{e}_1 \mathbf{E}_1^\perp = \mathbf{e}_2 \mathbf{E}_2^\perp \]
\[ \mathbf{D} \times \mathbf{E} = 0 \Rightarrow \mathbf{E}_1^\perp = \mathbf{E}_2^\perp \]
\[ \mathbf{D} \cdot \mathbf{B} = 0 \Rightarrow \mathbf{B}_1^\perp = \mathbf{B}_2^\perp \]
\[ \mathbf{D} \times \mathbf{H} = 0 \Rightarrow \mathbf{H}_1^\perp = \mathbf{H}_2^\perp \]

\[ \mathbf{E} - \text{field} : \quad \mathbf{E}_0 + \mathbf{E}_n = \mathbf{E}_T \]
\[ \mathbf{B} - \text{field} : \quad \mathbf{E}_0 - \mathbf{E}_n = \mathbf{n} \mathbf{E}_T \]

\[ 2 \mathbf{E}_0 = (n+1) \mathbf{E}_T \Rightarrow \mathbf{E}_T = \frac{2}{n+1} \mathbf{E}_0 \]

Similarly: \[ \mathbf{E}_n = -\left( \frac{n-1}{n+1} \right) \mathbf{E}_0 \]
but we don't care here.

Now: Assume \( n^2 = 1 \Rightarrow n = \sqrt{n^2} = \sqrt{1+\alpha^2 x} = \sqrt{1+h} \)

\[ \mathbf{E}_T = \frac{2}{\sqrt{1+h}+1} \mathbf{E}_0 e^{i\frac{\sqrt{1+h} kx - iwt}{\sqrt{1+h}}} \]

\[ \sqrt{1+h} = 1 + \frac{1}{2}h - \frac{1}{8}h^2 \]

\[ \mathbf{E}_T = \frac{2}{2+\frac{1}{2}h - \frac{1}{8}h^2\cdots} \mathbf{E}_0 e^{i\left(\frac{1}{2}h - \frac{1}{8}h^2 + \cdots\right) kx - iwt} \]

\[ \mathbf{E}_0 \]
For simplicity, take only the $0$ in $h$
so you can check the rest using Mathematica
(use series function)

$$E_{1} = (1 - \frac{1}{4}h(1 - 2ikx) + ...)E_{0}e^{ikx}e^{jyt}$$

**Part II: Iterative Solution from Superposition**

**Incident Plane Wave** $\boxed{E_{i}}$ as before

**Induced** A Polarization

$$\mathbf{P} = \chi_{e}E_{i} = \chi_{e}E_{0}e^{ikx}e^{-jyt}$$

Time Varying = **Induced Current**

$$J_{P}^{(1)} = \mathbf{\dot{P}} = -j\omega \chi_{e}E_{0}e^{ikx}e^{-jyt}$$
**Lemma:** Given a neutral plane surface current \( k(t) \) is

\[
E = -\frac{2\pi}{c} \int \frac{k(t-s/c)}{\sqrt{s^2 + \frac{r^2}{c^2}}} ds
\]

Finite time effects

More on this later!

**Proof**

\[
A(t) = \frac{1}{c} \int \frac{k(t-r/c)}{\sqrt{s^2 + \frac{r^2}{c^2}}} ds
\]

\( t_r = t - \frac{s^2 + r^2}{c} \)

Retarded time

\[
= \frac{2\pi}{c} \int_0^{t_r} k(t-r/c) \frac{s}{s^2 + \frac{r^2}{c^2}} ds
\]

\[
u = \frac{1}{c} \left( \frac{s^2 + r^2}{c} - \frac{s^2}{c^2} \right)
\]

\[
du = \frac{1}{c} \frac{d}{s^2 + r^2} ds
\]

\[
t - \frac{s^2 + r^2}{c} = t - \frac{\nu}{c} - u
\]

\[
= \frac{2\pi}{c} \int_0^{t_r} k(t-r/c-u) du
\]

\[
E = -\frac{1}{c} \frac{2K}{2\pi} = -\frac{2\pi}{c} \int_0^{t_r} \frac{2K}{2\pi} k(t-r/c-u) du
\]

\[
+ \frac{2\pi}{c} \int_0^{t_r} \frac{2K}{2\pi} k(t-r/c-u) du
\]

\[
= -\frac{2\pi}{c} K(t-r/c) - 0 = -\text{assumed!}
\]
\[ E = -\frac{\eta}{c} k (t - \frac{x}{c}) \]

So: INCIDENT \to \text{POLT} \to \text{CURRENT} \to E

\[ E_0 = \left( -\frac{2\pi}{\alpha} \right)^2 (\int_{x}^{\infty} e^{-iwx} E_0 e^{\frac{x-x'}{2\sigma}} dx') \left( \int_{-\infty}^{x} e^{-iwx} E_0 e^{\frac{x-x'}{2\sigma}} dx' \right) \]

\[ k = \frac{\omega}{c} \frac{\hbar}{2} E_0 \frac{\hat{z}}{z} \left( e^{i(kx-\omega t)} \int_{0}^{\infty} e^{-i\tilde{\omega}t} \tilde{\omega} \tilde{M} \tilde{M}^{\dagger} \tilde{M} \tilde{M}^{\dagger} \right. \]

\[ = i \frac{\hbar}{c} \hat{z} E_0 \left( e^{i(kx-\omega t)} \int_{0}^{\infty} e^{-2ik \tilde{\omega} t} \tilde{M} \tilde{M}^{\dagger} \tilde{M} \tilde{M}^{\dagger} \right) \]

WAVE REFLECTED FROM FAR SIDE OF DIELECTRIC.

ARTIFACT OF PURE PLANAR WAVE.

So it vanishes.
\[
E_1^{(1)} = E_i \frac{\hbar}{4} i k (2x - \frac{i}{ik})
\]
\[
= E_i \frac{\hbar}{4} (2ikx - 1)
\]
\[
= -E_i \frac{\hbar}{4} (1 - 2ikx)
\]

\[\text{and} \quad E_\downarrow \]

\[\text{WOW! EXPLAINS TRANSPARENCY}\]