REMARKS: HW2 – OVERALL GOOD JOB
MY FAULT FOR NOT INCLUDING INTEGRAL RELATION

- WORK W/ OTHER PEOPLE!
- DIMENSIONAL ANALYSIS
- SPEAKING OF HINTS, USUALLY A GOOD IDEA TO FOLLOW ANY HINTS GIVEN IN THE PROBLEM.

Q. CONTOUR INTEGRALS?
- SHOULD YOU REVIEW THEM
- SHOULD I GIVE A TUTORIAL ← no! ☺

THESE ARE JUST TOOLS; WON'T HAVE TO WORRY IF YOU'VE MADE IT THIS FAR.
↳ ALSO, THE LEVEL OF THIS COURSE IS SUCH THAT YOU SHOULD BE ABLE TO REVIEW THE RELEVANT BG ON YOUR OWN OR W/ FRIENDS.
↑ this is what research is like!

ANYWAY, NEXT HW HAS MORE PHYSICS.

ANY FURTHER QUESTIONS?

THE FEYNMAN PROPAGATOR

PHYSICAL INTUITION:

$$\langle 0|\Phi(x)\,\Phi(y)|0\rangle$$

- $\langle 0|\Phi(x)$: OBSERVE (ANNIHILATE) HERE, @ x
- $\Phi(y)|0\rangle$: CREATE A PARTICLE HERE @ POINT y

SO THIS IS A MEASUREMENT OF THE LIKELIHOOD THAT A PARTICLE AT $y \longrightarrow x$.
↳ this is motion; no surprise DEPENDS ON KINETIC TERM (DERIVATIVES).

ROLE OF MASS TERM: EXPONENTIAL DAMPING ← of $K_1(mr)$
eg IF THIS WERE A FORCE PARTICLE (eg W BOSON), MASS TERM TELLS US THAT FORCE IS NOT LONG RANGED.

BIG PIC: LAST WK: EVERYTHING MUST BE LOCAL IN $\mathcal{L}$ TO GIVE CAUSALITY. NOW THAT WE HAVE A CAUSAL QFT, CALCULATE NONLOCAL (BUT CAUSAL?) EFFECTS.

WE HAVEN'T SAID ANYTHING ABOUT HOW THE PARTICLE GOT TO y TO BEGIN WITH, OR WHETHER IT MAKES SENSE TO TALK ABOUT POSITION SPACE VS MOMENTUM SPACE INIT/FINAL STATES.

↳ MORE ON THIS SOON IN LECTURE!

BUT WE DID INTRODUCE THE FEYNMAN PROPAGATOR

$$\langle 0 | T\, \phi(x)\phi(y) | 0 \rangle \sim \int d^4k \; e^{-ik\cdot(x-y)} \frac{i}{k^2 - M^2 + i\varepsilon}$$

↑ TIME ORDERING
⇒ EXPLICITLY IMPOSE CAUSALITY

↑ FUNNY ε THING

vs. $\int d^3p \, \frac{1}{2E_p} e^{-ip\cdot(x-y)}$

NO $i\varepsilon$ PERSCRIPT.

WHAT TO MAKE OF THIS?
WHAT WE SAW IN THE HOMEWORK IS THAT THIS IS EQUIVALENT TO A PERSCRIPTION TO PERFORM THIS CONTOUR INTEGRAL.

COULD CHOOSE OTHER PERSCRIPTIONS, BUT WOULDN'T GET THE CAUSAL PROPERTIES WE WANT.

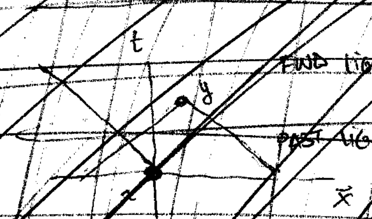
↳ $+i\varepsilon$ IS THE "<u>RIGHT</u>" PROPAGATOR

NOTE THAT RHS HAS NO TIME ORDERING OP! TIME ORDERING OCCURED AUTOMATICALLY

↳ IN FACT, PATH INTEGRAL FORMALISM DOES THIS AUTOMATICALLY (SEE ZEE 1.8. APP 1)

~~THIS = $\theta(x^0-y^0)\langle 0|\phi(x)\phi(y)|0\rangle + \theta(y^0-x^0)\langle 0|\phi(y)\phi(x)|0\rangle$~~

~~SUPPOSE~~



~~GIVES CORRECT PROPAGATOR~~

~~WHAT IS THIS?~~

~~CAN THINK OF PARTICLES W/ NEGATIVE ENERGY GOING BACKWARD IN TIME.~~

~~→ CANCELS PARTICLE CONTRIBUTION OUTSIDE LIGHTCONE.~~

FOR THE REAL SCALAR FIELD (PESKIN §2.4)

CAN A MEASUREMENT @ x AFFECT A MEASUREMENT @ y?

↳ NOT IF $[\phi(x), \phi(y)] = 0$ ← PROBLEM 1!

$$[\phi(x), \phi(y)] = \int d^3p \frac{1}{2E_p} e^{-ip\cdot(x-y)} - ((x-y) \leftrightarrow (y-x))$$

USING $[a, a^\dagger]$ RELATIONS.

BUT FOR SPACELIKE SEPARATION, CAN TAKE $(x-y) \to (y-x)$ S.T. THESE CANCEL, B/C INTEGRAL OVER $(-p)$ IS THE SAME

REMARK: IF ϕ WERE A FERMION, THEN $[\cdot,\cdot] \to \{\cdot,\cdot\}$ & THIS WOULDN'T WORK!

↳ SPIN-STATISTICS!

MENTIONED IN CLASS

THIS CANCELATION CAN BE INTERPRETED IN TERMS OF AN ANTIPARTICLE MOVING BACKWARDS IN TIME (neg freq.)

↳ an interesting complementary discussion: Weinberg Vol 1 § 5.2

YOU SHOULD START THINKING ABOUT EVERYTHING YOU KNOW ABOUT ANTIPARTICLES
- IS THERE ANTI MASS?
- ANTI PHOTON?
- WHAT IS CHARGE?
- CP VIOLATION

⇒ WE'LL FORMALIZE OUR UNDERSTANDING

I REALLY DON'T WANT TO GO INTO DETAILS ... IT'S FRIDAY, WE'RE ALL TIRED.

BUT WE SHOULD AT LEAST REVIEW THE IDEA.

⇒ **SYMMETRY**

2 PAIRS OF CREATION/ANN OPS, CAN READ THEM EITHER WAY.

IMAGINE 2 $\mathbb{R}$ SCALAR FIELDS. (IDENTICAL ACTIONS) ϕ_1, ϕ_2

IDEA: PACKAGE AS ONE $\mathbb{C}$ SCALAR FIELD. $\phi = \phi_1 + i\phi_2$

↳ 2 DoF OF $\mathbb{R}$ FIELD = ϕ & $\phi^\dagger$ IN $\mathbb{C}$ FIELD

LINEARLY INDEPENDENT.

WHY? ∃ SYMMETRY ROTATING ϕ_1 & ϕ_2 INTO ONE ANOTHER,

→ ∃ CONSERVED CURRENT

→ ∃ CONSERVED CHARGE → $[Q, H] = 0$

⇒ EIGENSTATES OF SYSTEM MUST BE EIGENSTATES OF Q.

→ ϕ & $\phi^\dagger$; WHICH CONTAIN eg CREATION FOR PARTICLE OR ANN OF ANTIPARTICLE.

SYMMETRY & NOETHER'S THM (for continuous symmetries)

SYMMETRY → $\delta S = 0$ ⇒ $\delta \mathcal{L} = \partial_\mu f^\mu$ for some f^μ

REALLY: $\int d^4x\, \mathcal{L}$ ⇒ $\delta(d^4x\, \mathcal{L}) = d^4x\, \partial_\mu f^\mu$ (↑ includes Jacobian)

USUALLY (INTERNAL SYM):

$$\boxed{\delta \mathcal{L} = \frac{\delta \mathcal{L}}{\delta \phi}\delta\phi + \frac{\delta \mathcal{L}}{\delta \partial \phi}\delta \partial \phi} \quad *$$

$$= \partial_\mu \left(\frac{\delta \mathcal{L}}{\delta \partial_\mu \phi} \delta\phi \right)$$

$=0$! (INTERNAL) $\qquad \equiv j^\mu$

$\mathcal{L} = |\partial\phi|^2 + \cdots = (\partial_\mu\phi)(\partial^\mu\phi^*)$

for ℂ SCALAR FIELD $\quad \phi \to e^{i\alpha}\phi = (1+i\alpha)\phi$
$\qquad \phi^* \to e^{-i\alpha}\phi^* = (1-i\alpha)\phi^*$

$$j^\mu = (\partial^\mu\phi^*)(i\alpha\phi) + (\partial^\mu\phi)(-i\alpha\phi^*)$$

$$= i\phi\partial^\mu\phi^* - i\phi^*\partial^\mu\phi \qquad \text{(DROP PARAMETER } \alpha)$$

↳ THIS SHOULD BE IDENTIFIED W/ THE USUAL CURRENT IN, SAY, ELECTROMAGNETISM

SPACETIME SYMMETRIES : HAVE TO BE MORE CAREFUL

$$* \to \partial_\mu f^\mu = \partial_\mu \left(\frac{\delta \mathcal{L}}{\delta \partial_\mu \phi} \delta\phi \right)$$

$$\Rightarrow j^\mu = \frac{\delta \mathcal{L}}{\delta \partial_\mu \phi}\delta\phi - f^\mu$$

eg: $\mathcal{L}(x) \to \mathcal{L}(x) + \varepsilon^\nu \partial_\nu \mathcal{L}(x)$ ← ALREADY IN FORM $\partial_\mu f^\mu$ (4 PARAM)

$$(j^\mu)_\nu = \frac{\delta \mathcal{L}}{\delta \partial_\mu \phi}\underbrace{(\partial_\nu \phi)}_{\delta\phi} - \delta^\mu_\nu \mathcal{L} \equiv T^\mu{}_\nu$$

CANONICAL (vs symm.)

ON NEXT HW

A NEW TWIST : SCALE TRANSFORMATION

$\Rightarrow x \to e^{\alpha} x$

SPACETIME SYMMETRY $\Rightarrow$ SHOULD HAVE TO DO W/ $T^{\mu}{}_{\nu}$

BUT: UNLIKE TRANSLATIONS + LORENTZ, d^4x NOT INVT!

POINCARE!

DEPENDS ON x

$$d^4x\, \mathcal{L} \to d^4x \left|\frac{\partial x'}{\partial x}\right| \mathcal{L}[\phi', \partial'\phi', x'] = (\delta\mathcal{L} + (\ )^{\nu}\partial_{\nu}\mathcal{L})$$

WANT THIS TO BE TOTAL DIV.

HAVE TO BE CAREFUL!

SCALE INVARIANCE IS SPECIAL!
INTERESTING FEATURES WHICH WILL APPEAR LATER IN QFT

$\hookrightarrow$ BROKEN QUANTUM MECHANICALLY
$\to$ RG FLOW

QUANTUM NOETHER

SO FAR EVERYTHING WE'VE DONE W/ NOETHER'S THM IS ESSENTIALLY CLASSICAL FIELD THEORY.

$\hookrightarrow$ E&M, GR

LET'S REMIND OURSELVES THAT IN QM, THE CONSERVED CHARGE IS ALSO THE GENERATOR OF THE SYM!

CONSIDER INTERNAL SYM (SPACETIME $\to$ SAME)

$$Q = \int d^3x\, j^0 = \int d^3x \frac{\delta\mathcal{L}}{\delta\partial_0\phi}\delta\phi$$

$$= \int d^3x\, \pi(x)\,\delta\phi(x) \quad \leftarrow \text{fune of } \phi(x)$$

$$[Q, \phi] = \int d^3x\, [\pi, \phi]\,\delta\phi + \cancel{\ldots}^{\,0}$$

$$= \delta\phi \quad \checkmark$$

THINGS TO LOOK FORWARD TO

- SYMMETRIES
  ↳ WE'VE LOOKED @ CONTINUOUS SYM
  -- KNOW THIS WELL!
  NEXT: DISCRETE SYMMETRIES
  ↑ C, P, T
  ↑ SUBGROUPS of POINCARE
  CHARGE (antimatter)

SYM. IS REALLY IMPORTANT
eg. PROBLEM 2 of HW
↳ CONSTRAINS form OF $\mathcal{L}$
STATISTICS of FIELDS
CLASSIFICATION OF PARTICLES

MUCH LATER: ORIGIN of FORCE
↳ GAUGE FIELDS.
cf CURRENT ASSOC. w/ φ FIELD
WHAT DOES IT COUPLE TO?

$$A_\mu j^\mu$$

(YOU CAN BELIEVE THAT THIS GENERATES A 3-POINT INTERACTION, RIGHT?)

- REVIEW: PROBLEM 3 & 4 of HW

BIG PICTURE AGAIN: (Q)FT of A SCALAR
→ BUT NONRELATIVISTIC
↳ SCHRÖDINGER FIELD THEORY.