LEPP-EX UNDERGRAD TALK : CMS students — t.o.c.

3 LECTURES COVERING : feynman RULES , SM , EWSB

the plan :

THE FEYNMAN DIAGRAM GAME

↓

MEANING → picture of quantum mechanics
→ spacetime diagrams

QED & ITS RELATIVES
→ QED + μ
QED + Z → VIRTUAL PARTICLES
WEAK FORCE
QCD → **STRONG COUPLING** → loops
→ breakdown of P.T.

SM SUMMARY : the "Brian Greene" version

ELECTROWEAK → THE HIGGS VEV & ITS DIAGRAMS
→ MEANING of MASS (mass + spin) → **FERMIONS** → helicity & chirality
→ CP
→ GAUGE BOSONS → GOLDSTONES
↓
ELECTROWEAK THY & EWSB

HIERARCHY PROBLEM → WHY GO BSM
→ WW SCATTERING & THE HIGGS
→ e SELF ENERGY AS ANALOGY TO HIERARCHY.

SUMMER 2012

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LEPP-ex UNDERGRADS, LEC 1a: FEYNMAN RULES

goal: CRASH COURSE in THE LANGUAGE of PARTICLE PHYSICS

- (CRASH COURSE) ↑ no detailed derivations from first principles (BUT SOME MOTIVATION)
- (LANGUAGE) ↑ @ this level: GOAL IS TO BE ABLE TO CONVERSE FLUENTLY IN THE STANDARD MODEL
  - → COMMUNICATE IDEAS SUCCINCTLY
  - → UNDERSTAND THE PHYSICS UNDERLYING THEM W/O GETTING LOST IN TECHNICAL DETAIL

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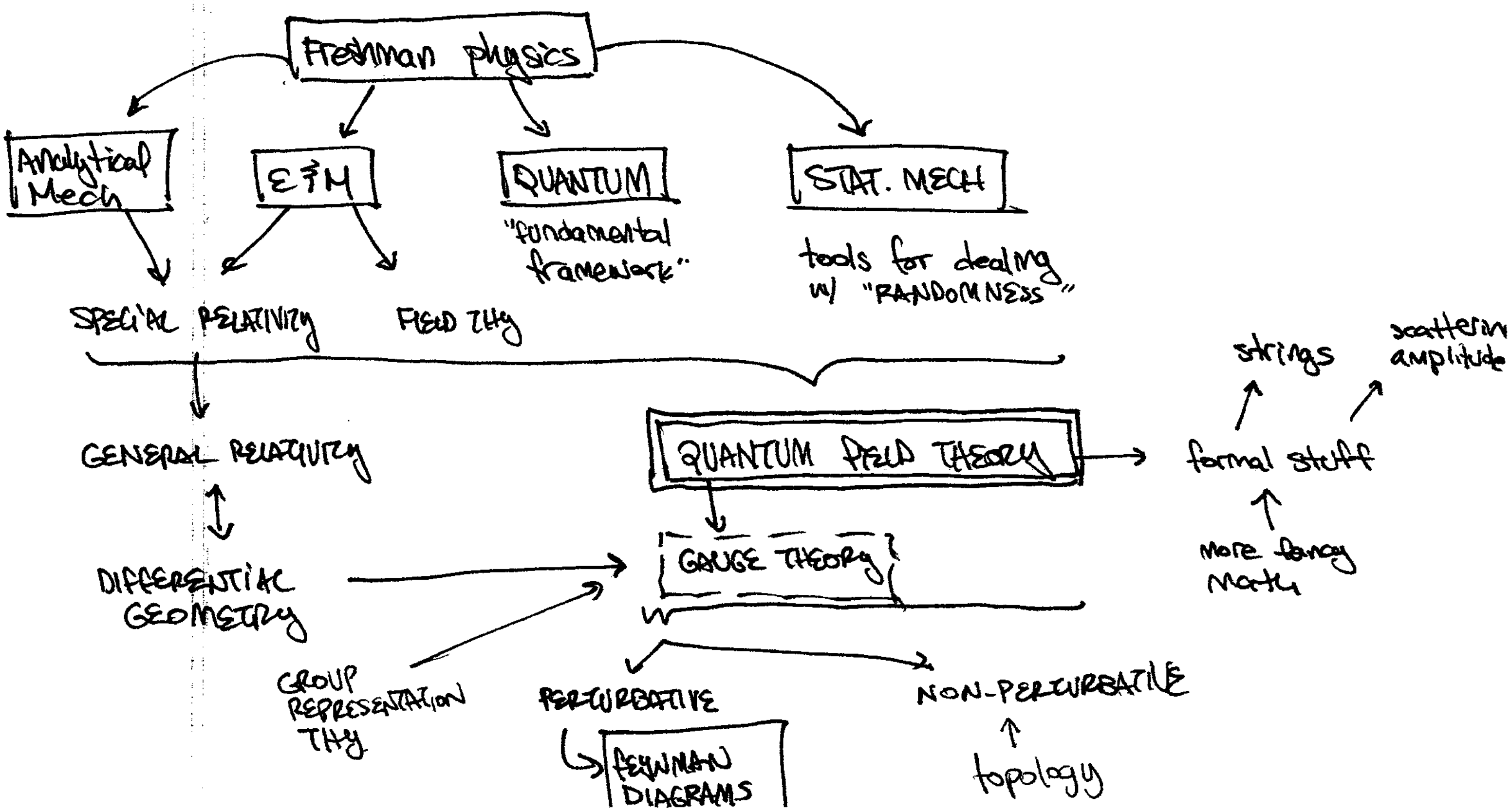
the big picture - theory side



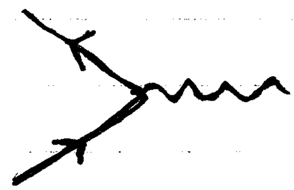
WARM UP: QED quantum electrodynamics

IMAGINE A GAME. HERE ARE THE RULES

① there are two kinds of lines

YOU ARE FREE TO ROTATE THEM AROUND

② there is only one way to connect lines:



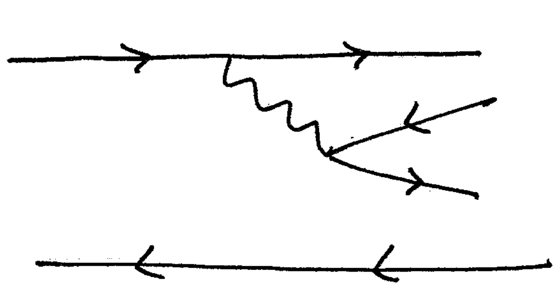
arrow orientation is important!

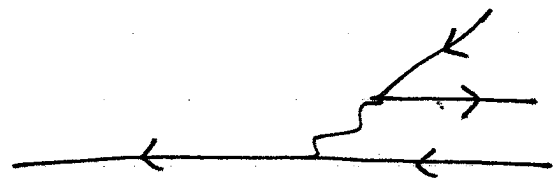
③ the objective of the game:

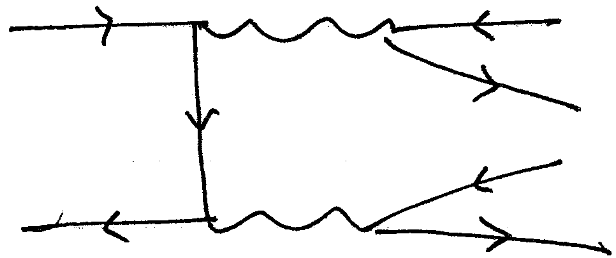
GIVEN A SET OF LINES ON THE LEFT, CAN YOU DRAW GRAPHS (USING AS MANY LINES & VERTICES AS YOU NEED) TO CREATE A SET OF LINES ON THE RIGHT.

eg:

~~DEF: A GRAPH OR DIAGRAM~~







<u>OBSERVE</u>: • CAN MAKE THESE ARBITRARILY COMPLICATED
(WE WILL PREFER <u>SIMPLER</u> GRAPHS)

• SOME LINES ARE DISCONNECTED
(WE WILL PREFER CONNECTED DIAGRAMS)

YOU ALREADY KNOW THE INTERPRETATIONS ...

also see:
- FEYNMAN VEGA LECTURES
- QED by FEYNMAN

Where can I learn more?

- CHECK OUT FLIP'S BLOG ON QUANTUM DIARIES
- POPULAR SCIENCE ARTICLES (SciAm, ...)
- TALK TO PEOPLE! ← eg. GRAD STUDENTS
- GO TO COLLOQUIA

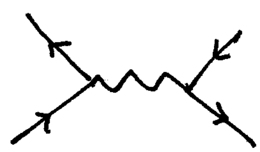
also: STRASSLER, MINUTE PHYSICS

- notes available online

FEYNMAN DIAGRAMS

SIMPLE, PICTORIAL REPRESENTATIONS OF PHYSICAL INTERACTIONS BETWEEN PARTICLES.

eg.



our convention: READ LEFT TO RIGHT

→ WHAT KIND of PHYSICS? QUANTUM FIELD THEORY

just means "RELATIVISTIC"

ie QM + SR

SO WE NEED A REFRESHER (pre-fresher?) ON

QUANTUM PHYSICS

↑ meaning: discrete

NEED-to-KNOW QUANTUM MECHANICS

↳ I'll skip the formalism, take these as facts

YOU PROBABLY HAVE A VAGUE SENSE OF "QUANTUM UNCERTAINTY"

- wave-particle duality
- Heisenberg UNCERTAINTY
  $\Delta x \Delta p \gtrsim \hbar$
  $\boxed{\Delta E \Delta t \gtrsim \hbar}$ ← remember this
- schrödinger's cat

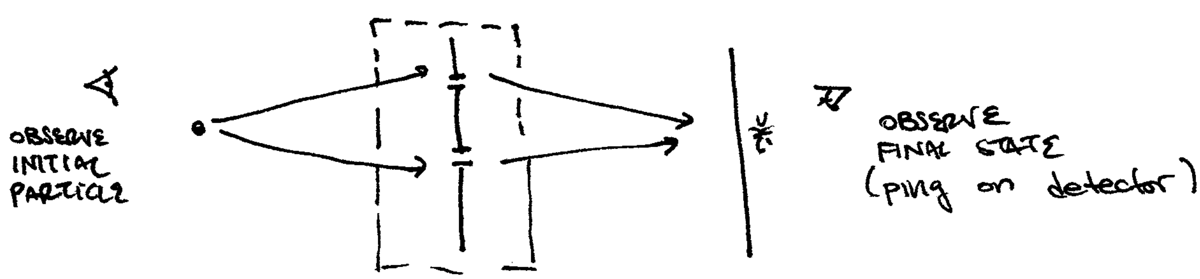
A MORE USEFUL NOTION @ THIS STAGE IS THE

SUM OVER HISTORIES

↳ OR PATH INTEGRAL

- → USUALLY SEEN FIRST IN GRAD QM!
- → SAME MATHEMATICAL OBJECT AS THE PARTITION FUNCTION IN STATISTICAL MECHANICS
- ↓ DEVELOPED BY FEYNMAN @ CORNELL

MAIN IDEA: DOUBLE SLIT EXPERIMENT



* ALL YOU OBSERVE (know) IS THE IN & OUT STATES

QUANTUM:

1. not a well defined question to ask which slit
2. in fact, in a very technially precise sense, it goes through both slits & interferes with itself!

↓

WHAT DOES THIS MEAN

- EACH PATH IS ASSIGNED A COMPLEX NUMBER z_i

  ↑ don't worry about how, BUT IF YOU WANT LOOK UP THE PRINCIPLE OF LEAST ACTION IN THE FEYNMAN LECTURES.

- SUM TOGETHER THE COMPLEX NUMBERS ASSOCIATED WITH EACH PATH THAT CONNECTS THE IN & OUT STATES

$$\sum_i z_i = z_{total}$$

- THE PROBABILITY OF REACHING THE SPECIFIED OUT STATE GIVEN THE PARTICULAR IN STATE IS GIVEN BY THE SQUARED MODULUS OF THIS SUM:

$$\boxed{\text{Prob}(\text{out} \mid \text{in}) = |z_{tot}|^2} \neq |z_1|^2 + |z_2|^2 + \cdots$$

this is the main result of QM

can get interference.

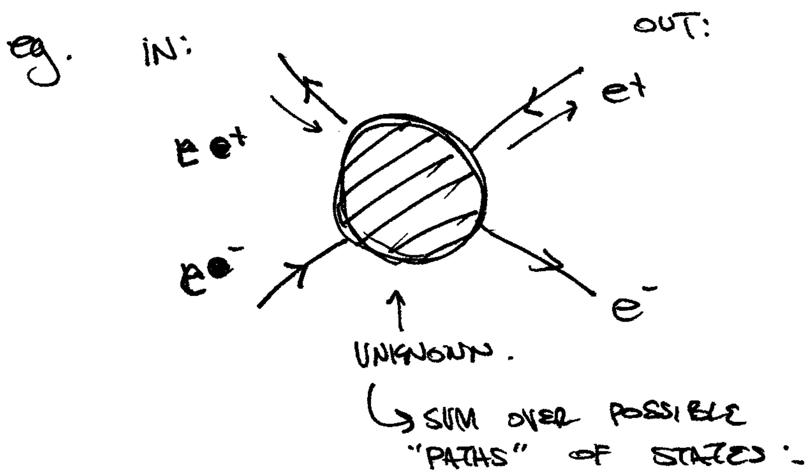
FEYNMAN DIAGRAMS ARE A MANIFESTATION OF THIS PRINCIPLE, THOUGH WE ABSTRACT (generalize) THE IDEA OF AN "IN" & "OUT" STATE.

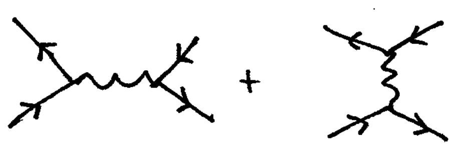
SET of SOME # of PARTICLES W/ SPECIFIED QUANTUM NUMBERS

everything there is to know about a particle

eg: MOMENTUM, MASS, SPIN, COLOR, CHARGE, ...

eg.



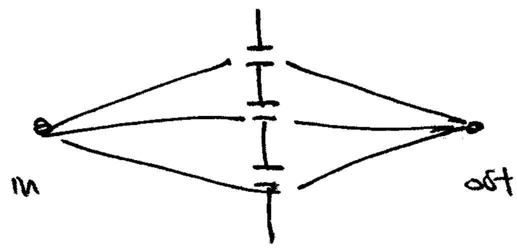


annihilation & pair production

virtual photon exchange

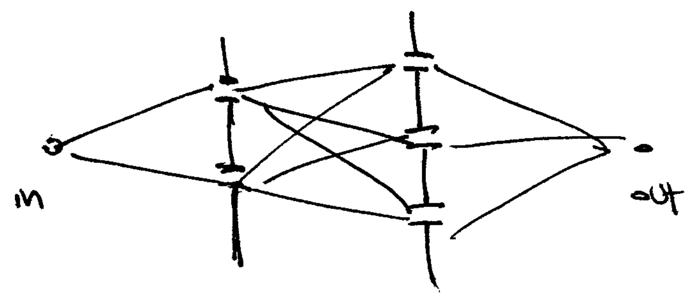
REMARK: from particle to field

IMAGINE NOT A DOUBLE SLIT EXPT, BUT TRIPLE SLIT:



SUM OVER <u>three</u> "paths", EACH IS A $\mathbb{C}$ NUMBER

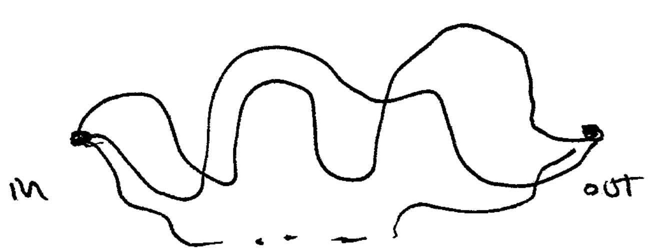
IMAGINE DOUBLE + TRIPLE SLIT EXPT:



SUM over: $2 \times 3 = 6$ paths Σ 6 $\mathbb{C}$ #'s

IMAGINE <u>infinite</u> # of barriers, each w/ <u>infinite</u> slits
$\Leftrightarrow$ no barriers at all.

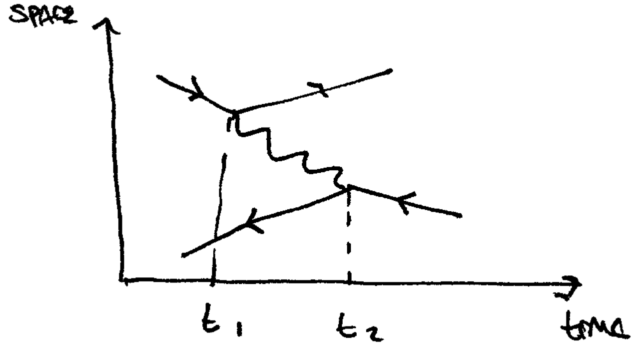
<u>SUM OVER A CONTINUUM</u> (∞) OF PATHS
INTEGRAL OVER A FUNCTION ($\mathbb{C}$ VALUED) $\rightarrow$ "ACTION"



} this is what we're dealing with in quantum <u>field</u> theory!

Feynman diagrams as spacetime diagrams

YOU CAN THINK OF FEYNMAN DIAGRAMS AS SPACETIME PLOTS:



<u>Interpretation</u>: @ time t_1 an electron emits a virtual photon

@ time t_2 this virtual photon is absorbed by a nearby positron.

the result is that the momenta of both the e^+ & e^- are now different

BUT: WE USUALLY LEAVE THE TIMES & POSITIONS IMPLICIT

⇒ WE SUM OVER ALL POSSIBLE TIMES & POSITIONS

SO THE SPACE & TIME AXES ARE IRRELEVANT

↳ WE JUST DRAW THE <u>TOPOLOGY</u>:

SIDE REMARK

THIS IDEA of ASSIGNING A NUMBER TO EACH POSSIBLE PATH ALREADY EXISTS IN CLASSICAL MECHANICS. THE CLASSICAL PATH MINIMIZES THE NUMBER ("principle of least action").

SEE: FEYNMAN LECTURES

IN QUANTUM MECH, NEARBY PATHS INTERFERE DESTRUCTIVELY SO THAT THE CLASSICAL PATH IS MOST PROBABLE. (gives main contribution to the probability)

SEE: FEYNMAN'S POPULAR BOOK, QED;

IN FACT: THE REFORMULATION of CLASSICAL MECHANICS IN TERMS of THE "ACTION" GAVE A NATURAL WAY TO FURTHER GENERALIZE THE FORMALISM TO QUANTUM MECHANICS.

A SIMILAR PROGRAM IS ONE of THE CURRENT FRONTIERS of THEORETICAL PHYSICS — THE GOAL IS TO REPLACE THE FORMALISM of FEYNMAN DIAGRAMS w/ SOMETHING WHICH CAN GENERALIZE TO QUANTUM GRAVITY.

↳ SEE NIMA ARKANI-HAMED'S MESSENGER LECTURES @ CORNELL (2010)

SO LET'S GO BACK TO **QED**

RULES : 

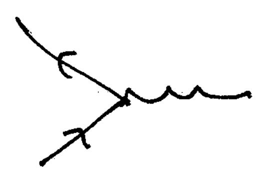
⇕ DEFINE A THEORY.

~~###~~ POINTY = electron

WIGGLY = PHOTON (the 'quantum' of ELECTROMAGNETISM)

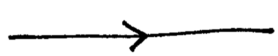
Matter vs. force PARTICLES
BUT for NOW WE WON'T DISTINGUISH BETWEEN THEM — THAT'S ONE of THE BEAUTIFUL THINGS ABOUT THIS.

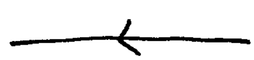
CLASSICAL E&M : PARTICLES + POTENTIAL THEY GENERATE
QED : JUST PARTICLES.

QED INTERACTION : 

- way to turn 1 particle → 2, etc.

INTERPRETATION

 ELECTRON MOVING forward in TIME

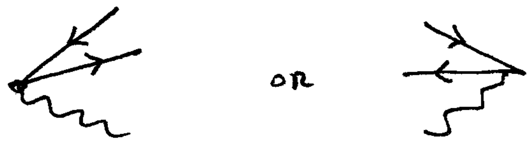
 ELECTRON MOVING BACKWARD IN TIME = ANTI-ELECTRON / POSITRON MOVING FORWARD.

why Antimatter? PART of SPACETIME SYMMETRY.

SO, MANY WAYS TO INTERPRET

e⁻ EMITS γ | e⁺ EMITS γ | PAIR PRODUCTION | ANNIHILATION

WHAT ABOUT:



?

not allowed: topologies okay, but there's an additional rule that we impose when we want to be more technical

RULE: THE IN & OUT STATES MUST OBEY CONSERVATION of ENERGY, MOMENTUM, AND ANY OTHER CONSERVED QUANTITIES IN YOUR THEORY.

→ this is actually built into the feynman rules!

further: EACH VERTEX CONSERVES E & $\vec{p}$

HOMEWORK: 1. SHOW THAT NONE of THE $1\to2$ OR $2\to1$ DIAGRAMS IN QED SATISFY THIS RULE.

2. CONVINCE YOURSELF THAT ELECTRIC CHARGE IS ALWAYS CONSERVED IN ANY ALLOWED DIAGRAM.

SOME E & $\vec{p}$ CONSERVATION COMMENTS

EINSTEIN: $E^2 = |\vec{p}|^2 + m^2$

TOTAL ENERGY — KINETIC ENERGY — MASS ENERGY

HOMEWORK: this isn't dimensionally correct. append factors of c to each term as necessary.

HOMEWORK: TAYLOR EXPAND w/rt $|\vec{p}|/m \ll 1$

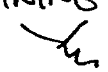
SHOW THAT TO LEADING ORDER YOU RECOVER THE FAMOUS APPROXIMATION $E = mc^2$.

ON SHELL: SATISFIES EINSTEIN RELATION

→ m IS FIXED, SO RELATES E & $|\vec{p}|$

VIRTUAL PARTICLES: MAY BE <u>off shell</u>: E & $|\vec{p}|$

<u>NOT</u> RELATED BY EINSTEIN EQ.

↳ EACH VERTEX (EVEN w/ VIRTUAL PARTICLES) MUST OBEY E & $|\vec{p}|$ CONS. THIS IS CONSTRAINING FOR ON-SHELL PARTICLES (eg. ), BUT NO PROBLEM FOR off-SHELL.

eg:  GIVEN 2 OF THE EXT STATES, ∃ ANY VALUE FOR THIRD $(E, \vec{p})$ s.t. ALLOWED? 4 UNKNOWNS

ONSHELL → 3 UNKNOWNS. CONS of $E, \vec{p} \Rightarrow$ 4 CONSTR.

LINES WHICH EXIT THE DIAGRAM

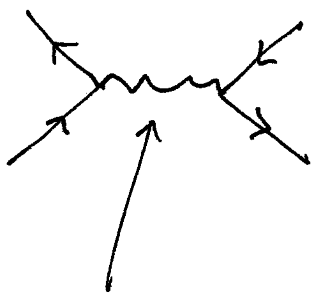
INTERESTING OBSERVATION:

ONLY IN & OUT STATES MUST SATISFY ENERGY & MOMENTUM CONSERVATION! ("ON SHELL")

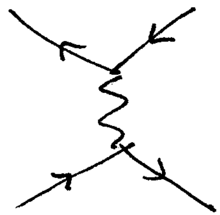
INTERMEDIATE STATES CAN BE "OFF SHELL"

eg. $e^+e^- \to e^+e^-$

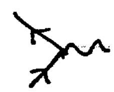
"s-channel"

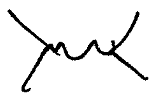


"t-channel"



THIS PHOTON NEEDN'T BE ON SHELL

↳ even though  &  not allowed,

 is allowed.

INTERMEDIATE STATES ARE NEVER DIRECTLY OBSERVED
→ not "physical" [c.f. arbitrary path in Σ over histories doesn't obey momentum conservation]

CALL THEM VIRTUAL PARTICLES.

IN THESE LECTURES WE WON'T WORRY TOO MUCH ABOUT CHECKING MOMENTUM CONSERVATION EXCEPT IN TRIVIAL CASES.

CONSERVATION LAWS

IF YOU HAVENT DONE IT ALREADY, CONVINCE YOURSELF THAT ELECTRIC CHARGE IS ALWAYS CONSERVED. ($Q[e^-] = -1$, $Q[e^+] = +1$, $Q[\gamma] = 0$)

WHAT ABOUT PARTICLE NUMBER?

CAN YOU HAVE $e^- \rightarrow 3e^-$?

$e^- \rightarrow e^- e^+ e^-$?

$\Rightarrow \exists$ CONSERVATION of ELECTRON #

(in this case it's trivial b/c it is identical to electric charge - but this is not always the case)

<u>HOMEWORK</u> (EASY): IS THERE A CONSERVATION LAW FOR PHOTON #?

<u>REMARK</u>: CONSERVATION LAWS ARE A BIG PART OF PHYSICS.

$\Leftrightarrow$ SYMMETRIES of THE THEORY.

↳ you will see this in Analytical Mechanics & then over & over again

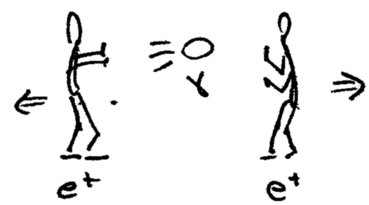
THE EMERGENCE of FORCE

QED IS THE MICROSCOPIC THEORY OF [CLASSICAL] E & M

BUT IN CLASSICAL E & M I HAVE THE NOTION of AN ATTRACTIVE FORCE COMING FROM A POTENTIAL

↑ γ IS ACTUALLY A 'QUANTUM' of THE CLASSICAL POTENTIAL

there is the analogy of two ice skaters tossing a ball



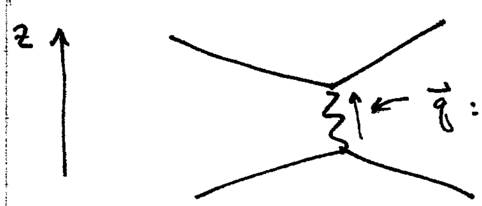
as a toy picture of a repulsive force.

HOW TO GET AN ATTRACTIVE FORCE?

THE POINT: this analogy is misleading!

γ IS VIRTUAL. IT CAN HAVE ANY MOMENTUM

CAN JUST AS EASILY PICK THIS s.t. THE $e^\pm$ ATTRACT vs. REPEL!



← $\vec{q}$: IF $q_z < 0$, then attractive

SO JUST GIVE UP ON THIS ANALOGY

LETS TALK ABOUT MATTER & FORCE PARTICLES

MATTER → "FERMIONS"

FORCE → "BOSONS"

SPIN: QUANTUM MECHANICAL "INTRINSIC" ANGULAR MOMENTUM ASSOCIATED WITH A PARTICLE. COMES IN 1/2 INTEGER UNITS → WHY? PROPERTY OF QM + SPACETIME SYMMETRY. (based on the topology of Lorentz group)

SPIN-STATISTICS THM: MATTER PARTICLES → 1/2 INTEGER SPIN
FORCE PARTICLES → INTEGER SPIN

Further: MATTER PARTICLES: OBEY PAULI EXCLUSION PRINCIPLE can't have two particles in the same state (eg electrons in an atom in chem.)

FORCE PARTICLES: DO NOT OBEY PAULI, CAN BE "STACKED" → eg. LASERS!

REMARK: MATTER PARTICLES ARE ALL SPIN 1/2 (maybe 3/2 in SUPERGRAVITY)
FORCE PARTICLES: SPIN 0: YUKAWA (NUCLEAR INTER.) ATTRACTIVE
fundamental { SPIN 1: ELECTRIC ATTRACTIVE/REPULSIVE
SPIN 2: GRAVITY ATTRACTIVE

REMARK: WHERE DO FUNDAMENTAL FORCES COME FROM? GAUGE SYM (LATER LECTURE)

REMARK: SUPERSYMMETRY RELATES FORCE + MATTER PARTICLES

RHETORICAL Q: Why don't we talk about QCD ever being repulsive?

LEPP-EX UNDERGRADS: Lec 16: MOVING ON FROM QED

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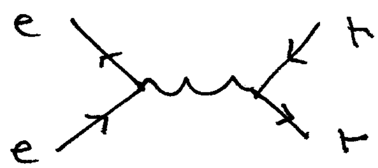
QED :

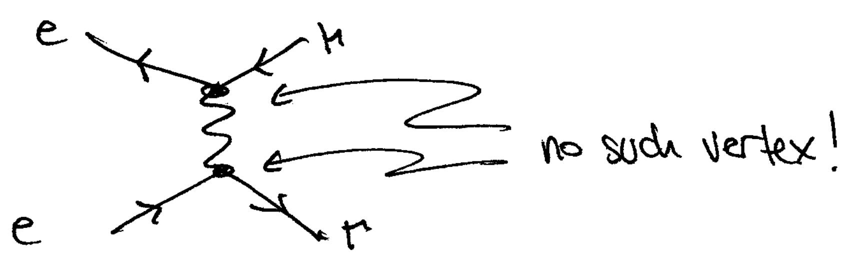
ADD A MUON :

NOW WHAT? CAN HAVE NEW PROCESSES: μ PRODUCTION @ AN e^+e^- COLLIDER :



SAME AS $e^+e^- \to \mu^+\mu^-$?

No: No t-CHANNEL DIAGRAM :



RECALL EACH DIAGRAM IS A ℂ NUMBER
SO THE ℂ # FOR $ee \to \mu\mu$ IS DIFFERENT
FROM THAT FOR $ee \to ee$!

FEYNMAN RULES ENCODE THE ENTIRE THEORY
↳ in QFT, we learn how to use the rules to assign the $\mathbb{C}$ number to each diagram.

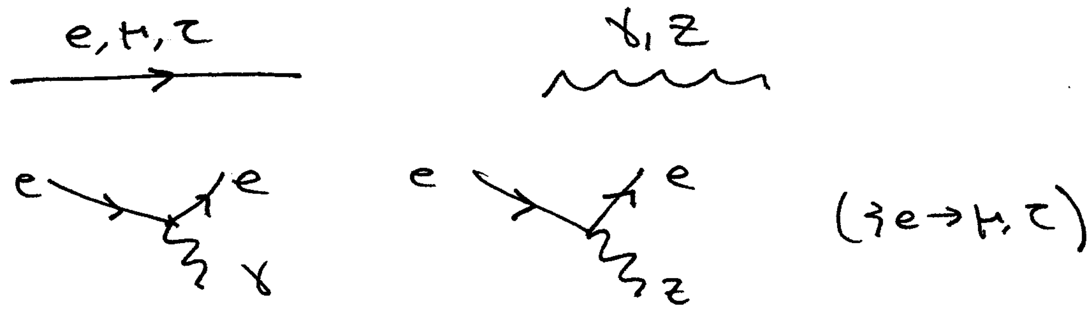
WHAT CAN WE LEARN FROM <u>QED + μ</u> ?

e # STILL CONSERVED
→ NOW ALSO μ # !
} precisely because no e–μ mixing in vertex

THIS IS KIND OF A TRIVIAL EXTENSION.
ALSO TRIVIAL: QED + μ + τ.
as you know, the e, μ, τ differ by their masses.
FLAVOR SYMMETRY

ANOTHER TRIVIAL EXTENSION: ADD A HEAVY PHOTON: Z

RULES



CONVINCE YOURSELF THAT THIS IS BASICALLY SEVERAL COPIES OF (QED + μ).

VIRTUAL PARTICLES & HEISENBERG

RECALL THE HEISSENBERG UNCERTAINTY RELATION

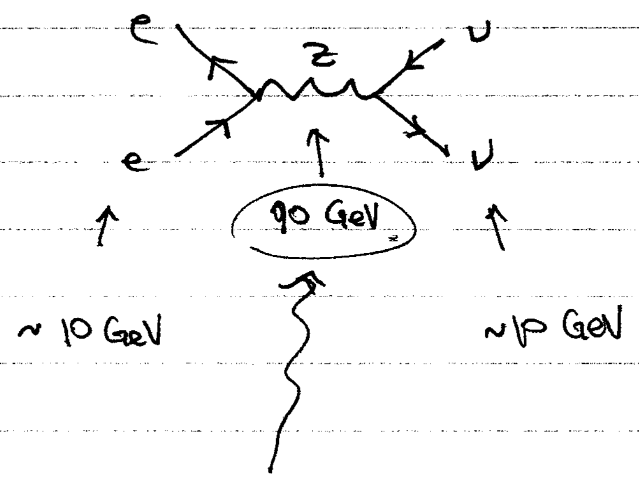
$$\Delta E \Delta t \gtrsim \hbar$$

"I can violate energy conservation, but only for a small amount of time"

⇒ the more I violate, the smaller the amount of time

<u>CONSIDER</u>: e^+e^- COLLIDING @ 10 GeV

& look @ $e^+e^- \rightarrow$ ~~[illegible]~~ $\nu\bar{\nu}$



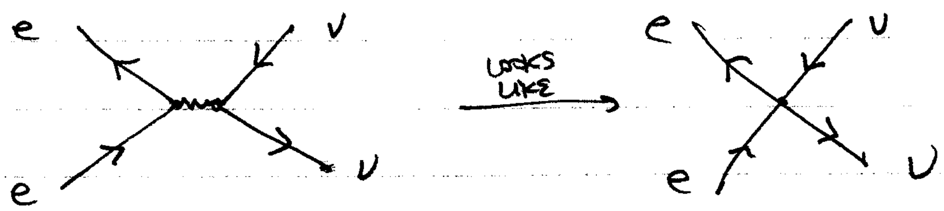
the Z MASS is 90 GeV!

IT IS VERY FAR FROM "ON SHELL"

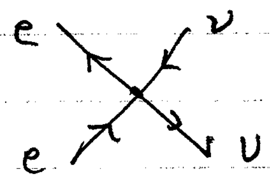
IT MUST HAVE <u>negative</u> kinetic energy to satisfy E-CONSERVATION @ EACH VERTEX. → Z IS VERY <u>VIRTUAL</u>

<u>HEISENBERG</u>: CAN ONLY EXIST FOR A SHORT AMOUNT OF TIME

SO IN REAL SPACE:



IF I ONLY HAD A 10 GeV COLLIDER, I WOULD HAVE OBSERVED $ee \to \nu\nu$ AS A POINT INTERACTION. I'D NEVER SEE A Z (too far off shell), AND I'D MAKE UP A THEORY w/ A "4-fermi" INTERACTION



THIS THY WOULD GIVE GOOD AGREEMENT w/ OTHER DATA FROM MY 10 GeV COLLIDER ...

BUT @ HIGHER ENERGIES WE'D NOTICE THAT ACTUALLY, THERE IS A Z BOSON.

"HOMEWORK": ~~[illegible]~~ MEDITATE ON THIS w/RT THE CURRENT STATE OF HIGH ENERGY PHYSICS & THE LHC.

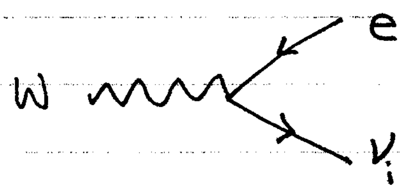
Something new: ν & $W^\pm$: ELECTROWEAK THY

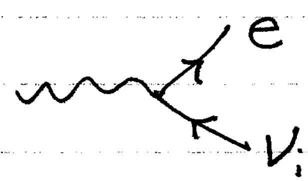
NEUTRINOS:
- MASSLESS-ish (more on this soon)
- UNCHARGED (electrically, at least) → no coupling to γ (but still couples to Z)
- MATTER PARTICLES

3 flavors: ν_e, ν_μ, ν_τ

$W^\pm$ BOSON:
- MASSIVE FORCE PARTICLE
- UNLIKE γ, Z: CARRIES ± CHARGE
- (W^+ is ANTIPARTICLE of W^-)

RULES:

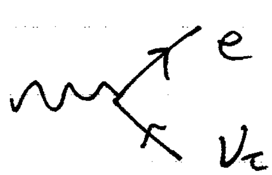


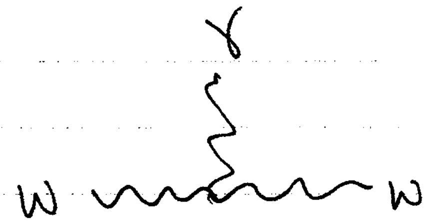


where $i = e, \mu, \tau$

↑

FLAVOR MIXING:





↑ Meaning of "W is charged"

QUESTION: is electron # still conserved? what <u>is</u> conserved?

HOMEWORK: ONE OF MY 'FAVORITE' PROCESSES IS
$\mu \to e\gamma$

1. PREVIOUSLY WE ARGUED THAT $e \to e\gamma$ IS
NOT KINEMATICALLY ALLOWED (E & $\vec{p}$ CONSERVATION)
... WHY IS $\mu \to e\gamma$ OKAY?

2. DRAW A DIAGRAM FOR $\mu \to e\gamma$
USING ELECTROWEAK RULES.

↳ hint: need to go beyond "tree level"

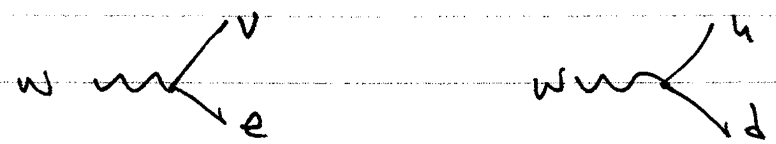
ANSWER: SEE arXiv: 1004.2037 :)

ILLUSTRATES A GENERAL PRINCIPLE IN THE SM:
"there are no flavor-changing neutral
currents (FCNC) at tree level"

↑ HW: PROVE THIS USING FEYNMAN RULES.

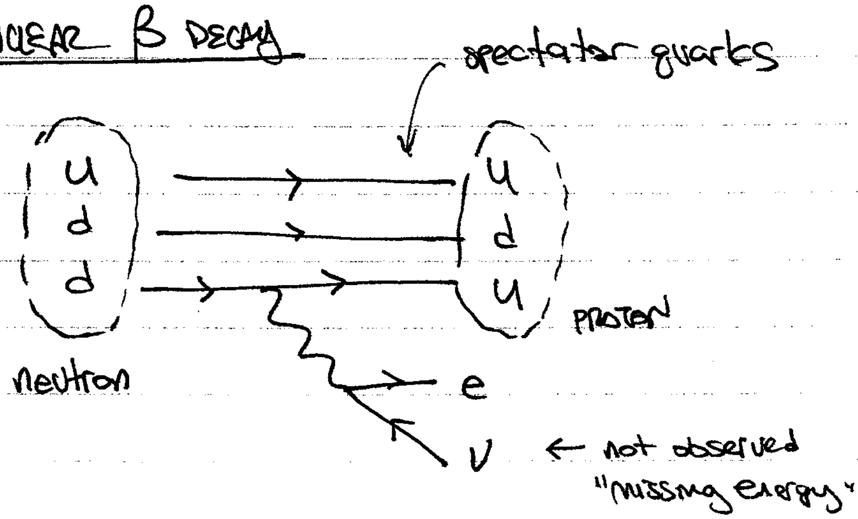
<u>example</u>: WE HAVEN'T INTRODUCED QUARKS, BUT THEIR STRUCTURE IS ANALOGOUS TO THE LEPTONS. LET'S JUST SKIP AHEAD TO SHOW WHERE THE W IS RELEVANT

$$\begin{pmatrix} \nu \\ e \end{pmatrix} \xleftrightarrow{\text{analog}} \begin{pmatrix} u \\ d \end{pmatrix}$$



[this relation is actually much more formal ... we'll see this later.]

<u>NUCLEAR β DECAY</u>



led to Pauli POSTULATING ν

HOMEWORK: WHY DOESN'T THIS "BLOW UP" THE NEUTRON? (something to think about)

→ e-ness, μ-ness, ...

IN THE SM, FLAVOR IS CONSERVED ... EXCEPT BY THE W
SO ALL FLAVOR CHANGING (IN THE SM!) MUST HAVE
A W BOSON INVOLVED.

OBSERVE: THE W is PRETTY HEAVY : 80 GeV
(only fermion heavier is the top)

SO @ LOW ENERGIES (@ WHICH MOST
FLAVOR EXPERIMENTS OCCUR), FLAVOR
CHANGING EFFECTS ARE SUPPRESSED BY
THE VIRTUALITY OF AN INTERMEDIATE Z

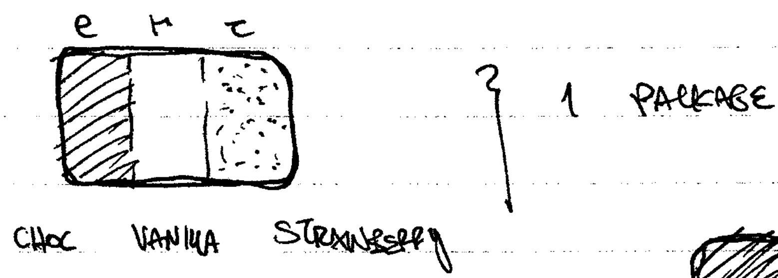
↑ recall $\Delta E \Delta t \gtrsim \hbar$

REMARKS: W^+ & W^- ARE ANTIPARTICLES of ONE ANOTHER

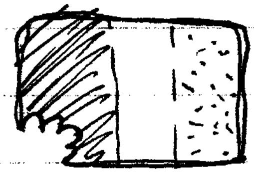
↳ COMPARE TO γ, Z WHICH ARE
THEIR OWN ANTIPARTICLES

W, Z, γ ARE ALL COUSINS
(related by the higgs) } WE'LL GET TO THIS LATER

Neapolitan Astronaut Ice Cream

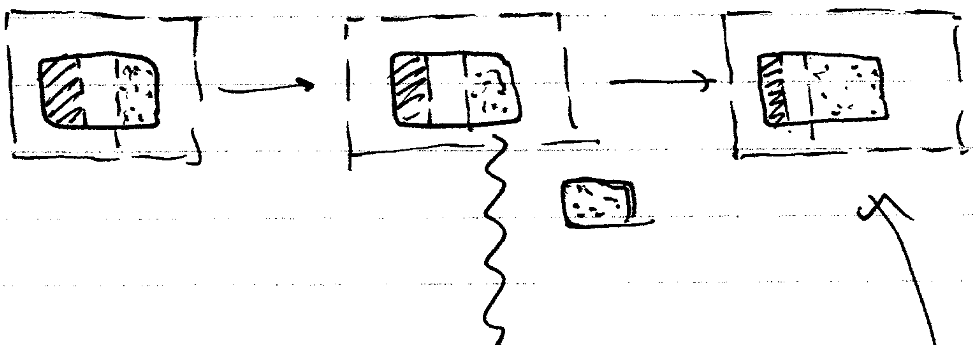


INTERACTIONS occur w/ SPECIFIC FLAVORS



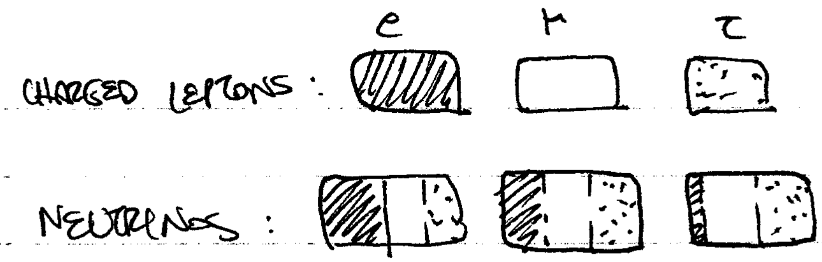
(no fair biting 2 flavors @ one time, you animal)

BUT THE PACKAGE IS THE THING WHICH GETS SHIPPED.



INTERACTION: THIS IS A STRAWBERRY.
@ THIS MOMENT THE ICECREAM IS COMPLETELY STRAWBERRY

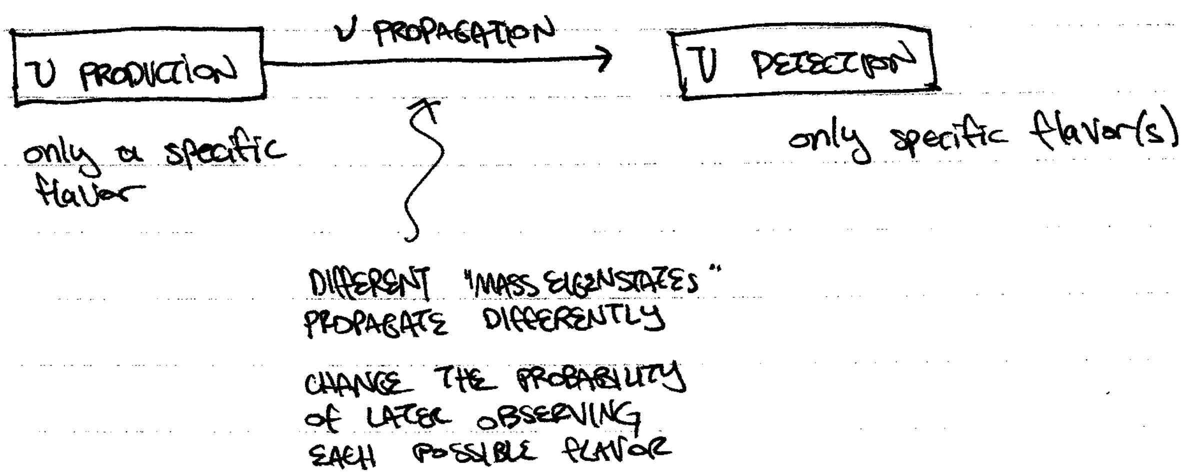
~~[illegible]~~ AFTER THE INTERACTION: AGAIN A NEAPOLITAN BAR, BUT PERHAPS A DIFFERENT RATIO of EACH FLAVOR



WE ONLY OBSERVE THE ν FLAVOR EIGENSTATES
↳ INTERACTION STATES

BUT THE FLAVOR STATES DON'T STAY PUT —
THE "MASS EIGENSTATES" ARE THE PARTICULAR ADMIXTURES THAT STAY PUT.

ν EXPERIMENTS : just broad picture



<u>WHAT ABOUT QUARKS?</u> $\begin{pmatrix} \nu \\ e \end{pmatrix} \longleftrightarrow \begin{pmatrix} u \\ d \end{pmatrix}$

- BUT u, d DO NOT OSCILLATE (for the same reason that the e do not oscillate)

- <u>BUT</u>: <u>MESONS</u> (bound states of $q\bar{q}$) <u>DO</u> OSCILLATE

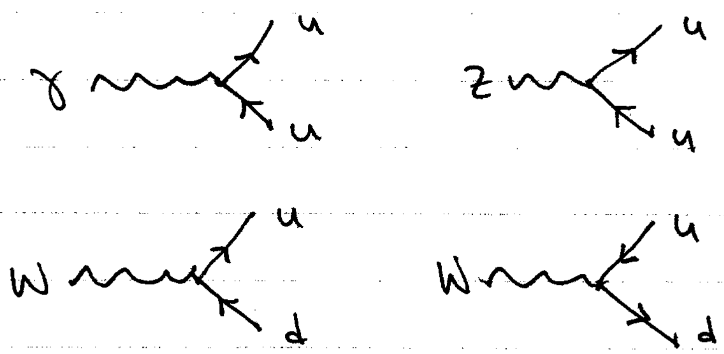
QCD : LEPP EX UNDERGRADS : LEC 2a

LECTURES BY FLIP TANEDO
pt267@cornell.edu

PARTICLE CONTENT MIRRORS LEPTONS

$$\begin{pmatrix} \nu \\ e \end{pmatrix} \longrightarrow \begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix} \quad \begin{matrix} \text{CHARGE} \\ +2/3 \\ -1/3 \end{matrix}$$

So:

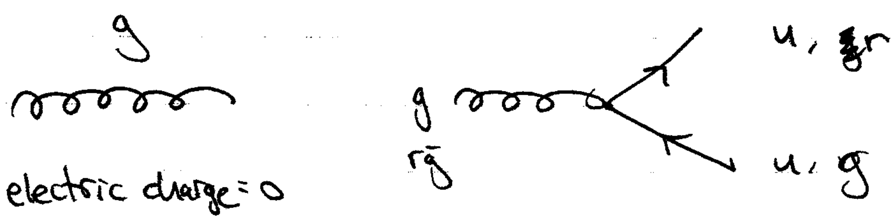


"NON-ABELIAN"

WHATS NEW: QUANTUM CHROMODYNAMICS
↳ LIKE QED, BUT WITH THREE CHARGES (rgb)

NEUTRAL : $(r\bar{r})$, (rgb)

WHAT MEDIATES THE COLOR FORCE? GLUON



electric charge = 0
color charge: $r\bar{g}$

MASSLESS
DOESN'T CHANGE CHARGE, FLAVOR ... A LOT LIKE PHOTON
CHANGES COLOR, BUT WE NEVER OBSERVE COLOR

HW : is BARYON # CONSERVED? WHY?
CAN YOU HAVE $e^+e^+ \to p^+p^+$?

Non-Abelian Vertices

↳ for forces w/ 2 or more "charges"

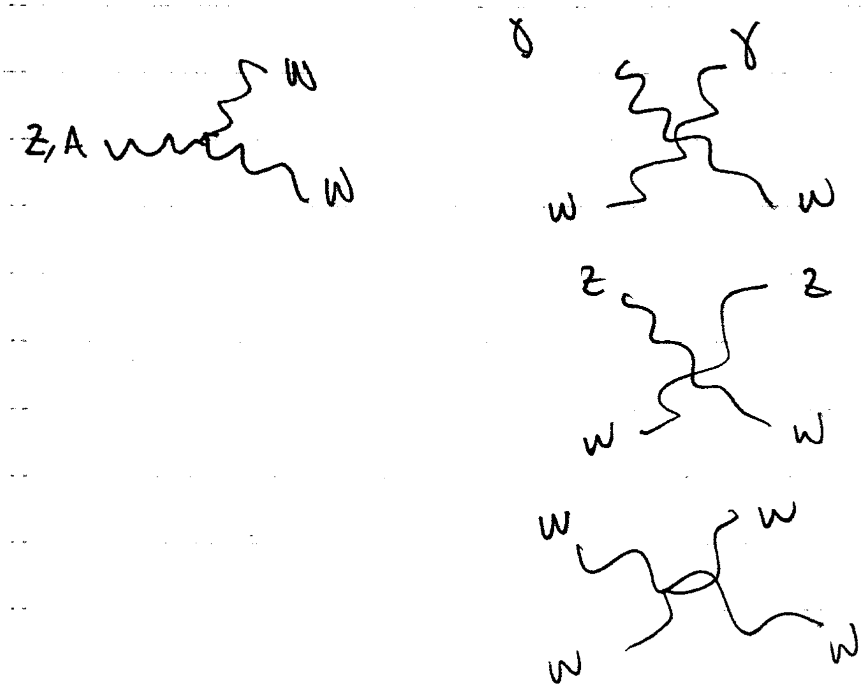
cf: QED: only one electric charge

QCD: three types of charge (rgb)

GLUONS:

IN FACT, W & Z ALSO HAVE SIMILAR VERTICES!!

↳ A HINT THAT THEY ARE NOT JUST COPIES OF QED BUT ACTUALLY COME FROM A THEORY WITH ≥ 2 CHARGES. ⤳ ELECTROWEAK THEORY RELATED TO HIGGS MECHANISM [WE'LL GET TO THAT]

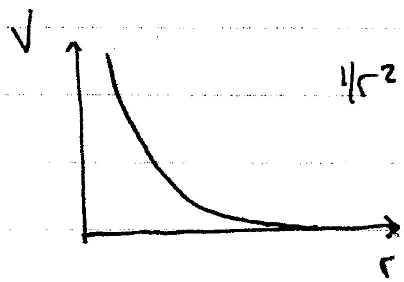


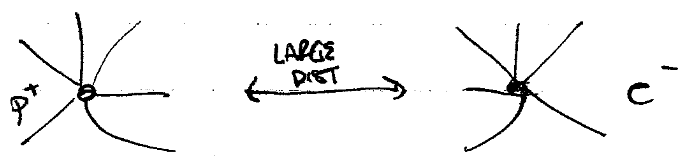
Strong Coupling

BUT QCD IS VERY DIFFERENT FROM QED, AND NOT JUST BECAUSE IT HAS THREE CHARGES.

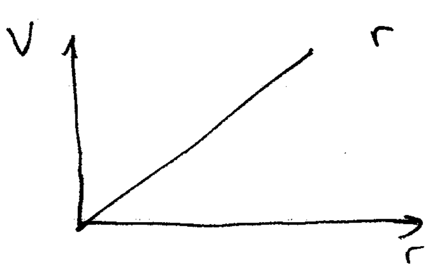
WHY? WE NEVER SEE THESE CHARGES BECAUSE QCD IS CONFINING. THE FORCE IS SO STRONG THAT COLOR CHARGES IMMEDIATELY WANT TO BECOME NEUTRAL — ENERGY COST TO STAY CHARGED IS TOO GREAT. ⤳ eg $p^+ e^- \to H$

eg PULL APART e^- FROM p^+



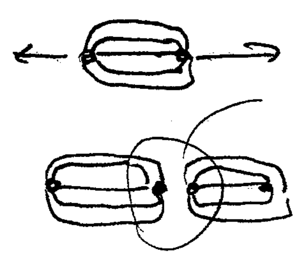


eg. PULL APART q FROM $\bar{q}$



ENERGY KEEPS INCREASING!

EVENTUALLY $V \gtrsim 2Mc^2$



Mesons & Baryons

~~2 ways to obtain color neutral states~~

the strong force is so strong that it hates having free charges: the potential energy of such a charge is much greater than the energy of pair producing quarks out of the quantum mechanical vacuum!

so bare quarks quickly HADRONIZE into color-neutral bound states. TWO WAYS TO DO THIS:

(QUARK)(ANTI-QUARK) = MESON

DIRECT ANALOG of HYDROGEN ATOM (you'll spend a lot of time studying the Hydrogen atom in your QM course... just pretend that it's a meson!)

$u \; \bar{u}$ (or $d\bar{d}$)

π

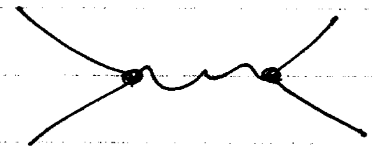
ρ (SPINNING)

(RED q)(BLUE q)(GREEN q) = BARYON, like p^+, n

HW: the lightest mesons are the $\pi^0, \pi^\pm$
the lightest baryons are the p^+, n
THESE ARE ALL MADE UP OF u & d QUARKS.
WHAT IS THE QUARK CONTENT of EACH of THESE HADRONS?

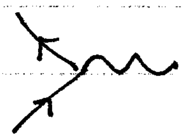
WHAT QUANTIFIES THIS STRONG COUPLING?

RECALL :



EACH DIAGRAM IS SHORTHAND FOR A COMPLEX NUMBER. (THE SUM OF THESE NUMBERS IS THE ~~AMPLITUDE~~ PROBABILITY AMPLITUDE, s.t. $|\sum_i z_i|^2 \sim$ PROBABILITY)

FEYNMAN RULES GIVE A PRESCRIPTION FOR WRITING MATHEMATICAL EXPRESSIONS FOR THESE $\mathbb{C}$ NUMBERS. IN PARTICULAR, THERE IS AN OVERALL PREFACTOR CALLED THE COUPLING CONSTANTS :

 $\sim e$

where $\frac{e^2}{4\pi^2} = \alpha \sim 1/137$

[you know this number]

THE SIZE OF THIS NUMBER $\longleftrightarrow$ STRENGTH OF THE COUPLING.

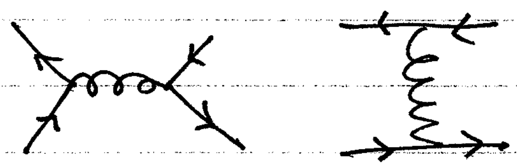
$\alpha_{QCD} \sim \mathcal{O}(1)$ "ORDER ONE"

WHEN FEYNMAN DIAGRAMS **Fail!**

<u>Strong coupling</u> is problematic.

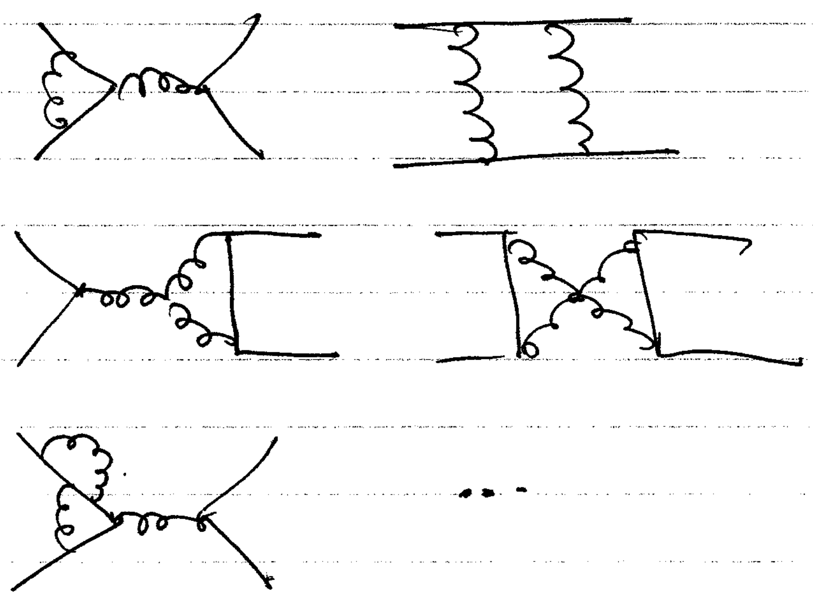
TO SEE WHY, WE HAVE TO UNDERSTAND A LITTLE MORE ABOUT WHAT THE SUM OF FEYNMAN DIAGRAMS CORRESPONDS TO.

<u>QUESTION</u>: WHAT DIAGRAMS CONTRIBUTE TO $u\bar{u} \to u\bar{u}$?



} the 'obvious' answers

BUT ALSO:



...

<u>ARBITRARILY MANY</u> !!

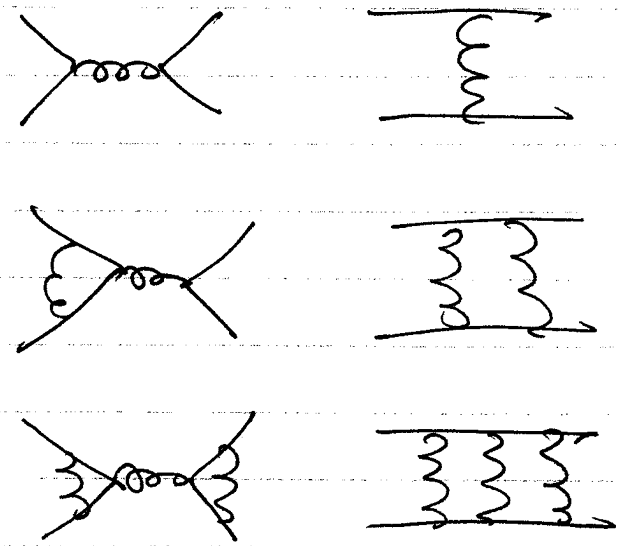
[maybe you've noticed this already!]

WE HAVE WORDS FOR THESE DIAGRAMS:

tree

loop

2-loop



etc.

WE CAN ALWAYS ADD MORE VIRTUAL PARTICLES TO MAKE HIGHER LOOP DIAGRAMS.

↳ in principle, there are an infinite # of diagrams contributing

THIS is OKAY! USUALLY ~~THE~~ ONLY THE SIMPLEST DIAGRAMS CONTRIBUTE.

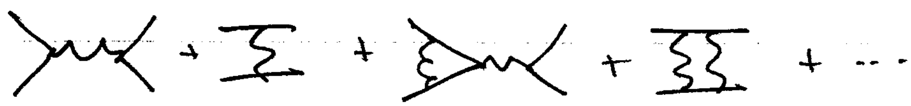
MORE COMPLICATED DIAGRAMS ARE SMALLER

- IN QED, EACH VERTEX IS SUPPRESSED BY A FACTOR OF THE CHARGE: e $\left(e^2/4\pi \sim 1/137 \right)$

- EACH CLOSED LOOP SUPPRESSED BY $1/16\pi^2 \sim 1/160$

⇒ EACH LOOP SUPPRESSES THE DIAGRAM BY $e^2/16\pi^2 \sim \frac{1}{4}\,\frac{1}{137}$ RELATIVE TO LESSER-LOOP DIAGRAMS.

SO WHILE THERE IS AN INFINITE SERIES OF DIAGRAMS,



EACH SUCCESSIVE TERM IN THE SERIES IS SMALLER.
IT IS USUALLY SUFFICIENT TO CONSIDER ONLY THE FIRST FEW TERMS (tree diagrams)

does this sound familiar?

it is a [fancy version of a] TAYLOR EXPANSION.

ALMOST ALL OF PHYSICS REDUCES TO A TAYLOR EXPANSION

↑ know how to make good approximations!

recall: $f(x) = f(0) + x f'(0) + \frac{1}{2}x^2 f''(0) + \cdots$

or

$$f(x) = f(a) + (x-a) f'(a) + \frac{1}{2}(x-a)^2 f''(a)$$

DEPENDS ON $(x-a) \ll 1$

$(x-a)$ is the analog of $\frac{g^2}{16\pi^2}$, or "loopiness"

BUT WE KNOW THAT THE TAYLOR EXPANSION FAILS WHEN $(x-a) \gtrsim 1$. FOR THE STRONG FORCE, THE COUPLING CONSTANT IS NOT SMALL.

$$\hookrightarrow \quad \alpha_s = \frac{g_s^2}{4\pi} \sim 1$$

IN THIS CASE HIGHER LOOP DIAGRAMS ARE NOT MUCH SUPPRESSED RELATIVE TO LOWER LOOP DIAGRAMS.

↳ ALL DIAGRAMS SEEM TO CONTRIBUTE EQUALLY! IMPOSSIBLE TO CALCULATE USING OUR NORMAL TOOLS

some ways around this

① LATTICE QCD : USE COMPUTERS TO CALCULATE THESE THINGS ON A DISCRETE SPACETIME LATTICE.

② EFFECTIVE THEORY : WHEN THE COUPLING IS STRONG, USUALLY THE "PARTICLES" THAT YOU SEE AT LOW ENERGIES ARE DIFFERENT (composites: mesons & baryons) → these can be weakly coupled

③ HOLOGRAPHY (fairly recent) : SEE YUHSIN'S LECTURE
↳ CALCULATE IN STRONGLY COUPLED 4D THEORY BY USING A DUAL 5D THEORY !!

(important!)

HOMEWORK: NUCLEON EFFECTIVE THEORY

YOU NOW KNOW 'EVERYTHING' ABOUT QCD.
IN THE 60's, NOBODY KNEW ABOUT QUARKS OR GLUONS.
THEY ONLY HAD NUCLEONS (PROTONS & NEUTRONS) & PIONS ($\pi^0, \pi^\pm$). AS FAR AS THEY WERE CONCERNED, NUCLEONS & PIONS INTERACTED AS FUNDAMENTAL PARTICLES.
(old QUANTUM FIELD THEORY BOOKS EVEN TALK ABOUT THIS.)

BASED ON WHAT YOU KNOW ABOUT THE QCD FEYNMAN RULES, WRITE OUT ALL THE FEYNMAN RULES FOR THIS NUCLEON / PION EFFECTIVE THEORY.

CONSIDER A FEW SIMPLE PROCESSES (eg $p^+ n \to p^+ n$) & draw the feynman diagrams for both the NUCLEON EFFECTIVE THEORY AND THE FULL QCD THEORY.

COMMENT ON CONSERVATION LAWS.
eg. IS PION # CONSERVED?
NUCLEON #?
ELECTRIC CHARGE?
PROTON-NESS / NEUTRON-NESS? (called ISOSPIN)

Some remarks

I. WE SAID THAT FEYNMAN DIAGRAMS ARE A KIND OF TAYLOR EXPANSION. WHAT ARE WE TAYLOR EXPANDING? (WHAT IS THE ANALOG OF $f(x)$?)

these diagrams are actually a generalization of a Taylor expansion. instead of expanding a function, we are expanding a function-of-functions ("FUNCTIONAL").

the rough analog of $f(x)$ is something called the PARTITION FUNCTION.

$$Z = e^{iS/\hbar}$$

$\hbar$ ← PLANK'S CONSTANT

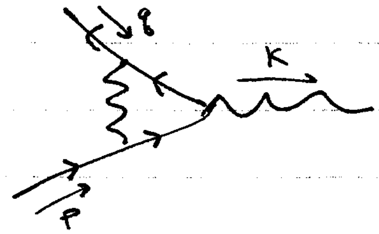
S: THE ACTION (you'll meet this in analytical mechanics)

Z: this is a kind of weighting that gives the 'importance' of different diagrams / paths.

YOU'LL MEET Z IN STATISTICAL MECHANICS.

II. WHAT'S UP WITH LOOP DIAGRAMS?

<u>HOMEWORK</u>: CONSIDER ANY LOOP DIAGRAM ... eg.



CORRECTION TO ELECTRIC CHARGE

YOU KNOW THAT EACH VERTEX CONSERVES MOMENTUM & ENERGY. FIND THE MOMENTUM & ENERGY of THE INTERNAL (VIRTUAL) LOOP PARTICLES.

↳ ANSWER: UNDETERMINED!
WHAT DOES THIS MEAN?
HAVE TO SUM OVER DIAGRAMS WHERE THE VIRTUAL PARTICLES HAVE DIFFERENT MOMENTA! ⟶ INTEGRAL.

IN FACT, THIS is A 4D INTEGRAL (E, P_x, P_y, P_z). FURTHER, THIS COMES WITH A FACTOR of $1/(2\pi)^4$ because we've <u>FOURIER TRANSFORMED</u> INTO MOMENTUM SPACE. (ie the cost of summing momentum states vs. positions.)

$$\int \frac{d^4k}{(2\pi)^4} \longrightarrow \sim \underbrace{\frac{1}{16\pi^2}}\int k^3\,dk \cdots$$

(↰ do angular part)

↳ where this comes from

LOOP DIAGRAMS ARE NOTORIOUSLY MORE DIFFICULT TO CALCULATE THAN TREE DIAGRAMS.

→ I used to be terrified of them.
THEN I SPENT 2 YEARS CALCULATING LOOP DIAGRAMS IN 5D IN A CURVED SPACETIME ...

BUT THEY ELUCIDATE THE DEEP STRUCTURE of QUANTUM THY
LOOPS REPRESENT "VERY QUANTUM" (more virtual) CONTRIBUTIONS. THESE CONTRIBUTIONS CAN DO VERY INTERESTING THINGS TO YOUR THEORY!

REMARK: 1 LOOP IS HARD
2 LOOP IS REALLY HARD (mostly done only by german physicists!)
3 LOOP IS CRAZY HARD (mostly done only by large groups of german physicists!)

⋮

TOM KINOSHITA, ONE OF OUR EMERITUS PROFESSORS, IS DOING FIVE LOOP CALCULATIONS IN QED. THIS IS NOTHING SHORT OF HEROIC.

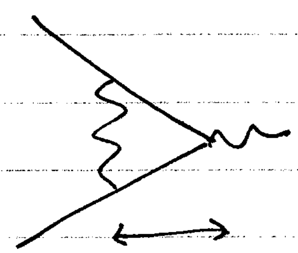
III. A HINT of RENORMALIZATION

one of the weird quantum effects associated w/ loop diagrams is that PHYSICS CHANGES DEPENDING ON HOW YOU LOOK AT IT.

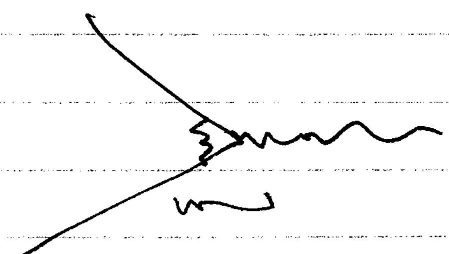
↳ eg: BIO → CHEM → ATOMIC PHYS → QUANTUM } BUT A "CHEF DOES NOT NEED TO KNOW FIELD THEORY"

[reiterates idea of EFFECTIVE THEORY]

CONSIDER FEYNMAN DIAGRAM:

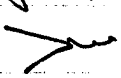


SOME LENGTH SCALE



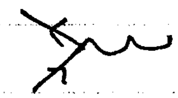
CAN YOU RESOLVE THIS?
DO YOU KNOW THAT IT'S A LOOP DIAGRAM?

LOOKS LIKE A POINT INTERACTION

IF MY MICROSCOPE CAN ONLY ~~SEE~~ RESOLVE A CERTAIN DISTANCE, THEN EVERYTHING SMALLER THAN THAT IS EFFECTIVELY A POINT INTERACTION (~~correction~~ CONTRIBUTION TO )

THIS MEANS: OUR OBSERVED CHARGES DEPEND ON <u>HOW CLOSELY</u> WE'RE LOOKING

WHAT WE MEAN BY

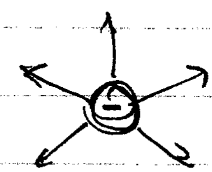


LENGTH ~ 1/MOMENTUM

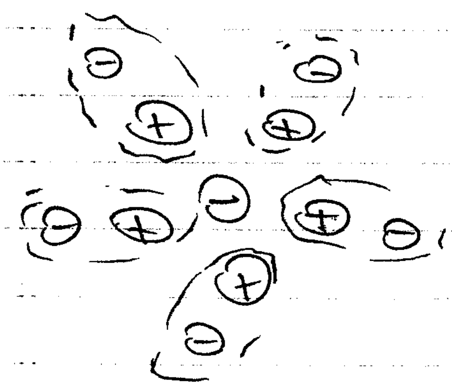
(HIGH ENERGIES PROBE SHORT LENGTH SCALES)

↳ LHC IS THE WORLD'S MOST POWERFUL <u>MICROSCOPE</u>.

eg SCREENING:



@ LONG DISTANCES: $E \sim 1/r$ (classical)



@ SHORT DISTANCES: VIRTUAL e^+e^- PAIRS APPEAR AND DILUTE THE ELECTRIC CHARGE.

<u>WE OBSERVE THIS!</u> @ $E \sim M_Z$, $\alpha_{EM} \sim 1/128$

@ Hi ENERGIES, α_{STRONG} IS SMALLER!

III. SOME PROCESSES DO NOT OCCUR @ TREE LEVEL & ONLY APPEAR AT LOOP LEVEL! THESE OFTEN HAVE TO DO WITH (APPROXIMATE) SYMMETRIES OF YOUR THEORY, eg. <u>FLAVOR</u>.

SEE MONIKA'S TALK ON FLAVOR PHYSICS.

A NICE EXAMPLE: $\mu \to e\gamma$

Review: the "low energy" Standard Model

↑ sm that we've discovered

$\begin{pmatrix} \nu \\ \ell^- \end{pmatrix}$ × 3 FLAVORS (e, μ, τ) + ANTIPARTNERS

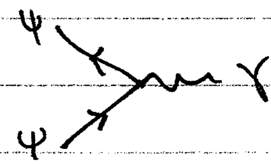
$\begin{pmatrix} u \\ d \end{pmatrix}$ × 3 flavors $\begin{pmatrix} u & c & t \\ d & s & b \end{pmatrix}$

× 3 colors

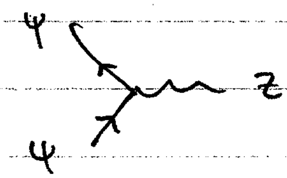
$\gamma \quad Z \quad W^\pm \quad g$

↑ × 8 color/anticolor combinations

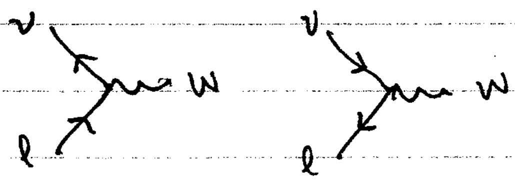
RULES:



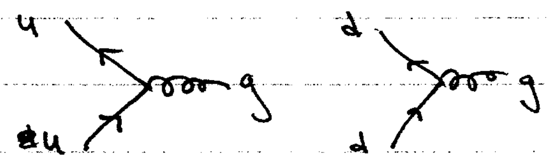
FOR $\psi = u, d, \ell$



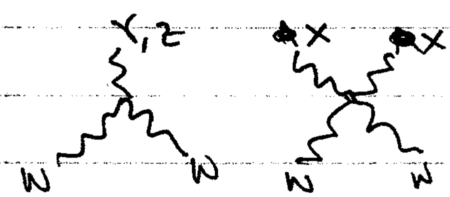
FOR $\psi = u, d, \ell, \nu$



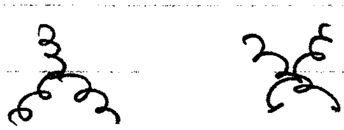
ALSO $\begin{pmatrix} \nu \\ \ell \end{pmatrix} \to \begin{pmatrix} u \\ d \end{pmatrix}$ for all flavor combinations



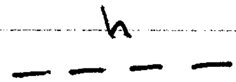
for all color combinations

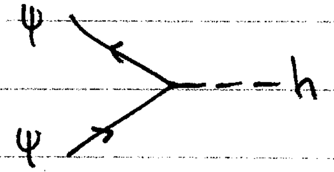


for $X = \gamma, Z, W$

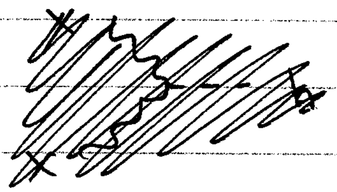


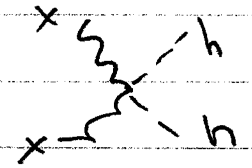
A PREVIEW of THE HIGGS



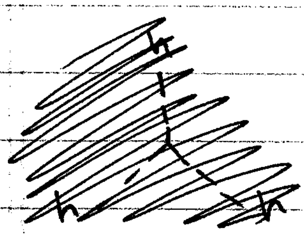


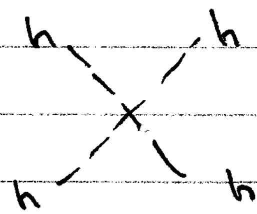
for $\psi = u, d, \ell, (\nu)$



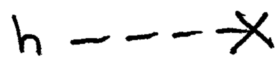


for $X = Z, W$





SOMETHING TOTALLY WEIRD



↳ terminates.

RULE: A non-trivial DIAGRAM ~~WITH ONLY 2~~ CONNECTING A PARTICLE & ANTIPARTICLE IS A **MASS**.

eg.