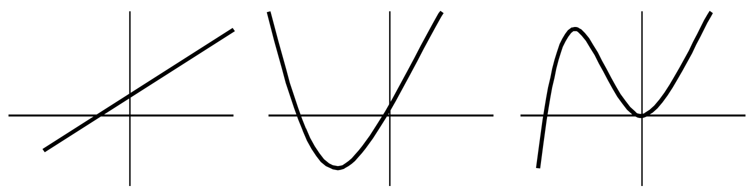
Physics 214, Fall 2002
Assignment #1, due Thurday, Sept. 5 before 5:00 pm

1. (a) Each of the following graphs represents U, the potential energy of a particle, as a function of x. For each:
   (i) sketch a graph of the force $F_x = -\partial U/\partial x$ as a function of x,
   (ii) identify any equilibrium points, and
   (iii) for each equilibrium point, determine whether the particle would oscillate if displaced slightly from equilibrium.



   (b) Each of the graphs above now represents F_x, the force on a particle, as a function of x. For each:
   (i) sketch a graph of the potential energy U as a function of x,
   (ii) identify any equilibrium points, and
   (iii) for each equilibrium point, determine whether the particle would oscillate if displaced slightly from equilibrium.

2. (a) Write Taylor series expansions about $x = 0$ for the functions e^x, $\cos x$, and $\sin x$. Use the expansions to prove the Euler identity: $e^{i\theta} = \cos \theta + i \sin \theta$.
   (b) Use the Euler identity to show that $\cos \theta = (e^{i\theta} + e^{-i\theta})/2$ and $\sin \theta = (e^{i\theta} - e^{-i\theta})/2i$.
   (c) Use complex exponentials to prove the trig identities
   (i) $\cos a \cos b = [\cos (a + b) + \cos (a - b)]/2$ and
   (ii) $\sin^2 a + \cos^2 a = 1$.

3. A farmer named Albert is getting old and all the ruts in the field make his back hurt when he drives his tractor. He gets the idea of supporting the tractor seat with a spring instead of having it bolted to the tractor. His mass is $m = 75$ kg and the spring compresses 1 cm when he sits in the seat.

   (a) What is the spring constant k of the spring?

   With his new invention, Albert finds that when he hits a bump in the field, he oscillates up and down in the seat.

   (b) What is the frequency of oscillation?

   In order to understand what's happening, Albert goes back to the barn to do some experiments. He sits in the tractor seat, stretches the spring away from equilibrium, and lets go. Let y be the vertical position of Albert while oscillating in the seat; $y = 0$ is his equilibrium position. Mathematically, what he does is specify the initial condition $y(0) \neq 0$. After that, his motion is determined by the force due to the spring and gravity.

(c) Use Newton's second law to derive the equation of motion (the differential equation for y). [Note that you don't have to include the gravitational force as a separate term; the force on Albert due to the spring *and gravity* is $F = -ky$. (Why?)] Show that $y(t) = A \cos □t$ is a solution of the equation of motion.

In order to reduce the amount of bouncing up and down, Albert adds a shock absorber to damp out the oscillations. The shock absorber exerts a force $F = -b\, dy/dt$ on the seat. Unfortunately, the shock absorber provides way too much damping. The constant b is so large that Albert's acceleration is negligibly small.

(d) Derive the equation of motion, neglecting the acceleration term. Find a solution to the equation of motion. What does it tell you?

Albert replaces the shock absorber with another one with a much smaller b.

(e) Derive the equation of motion. Show that $y = Ae^{-□t+i□t}$ is a solution to the equation of motion for certain values of □ and □. Find those values of □ and □ in terms of m, k, and b.

Albert's position y can't be a complex number; what we really mean is $y = \mathrm{Re}\,\{Ae^{-□t+i□t}\}$, where Re {} mean *the real part of.*

(f) Assuming □ is a real number, write down the real solution for y. What does the solution tell you—what does Albert's motion look like? What values of b ensure that □ is a real number? What would be a good choice of b to let Albert have a smooth ride?