Physics 214

Assignment #2, due Thursday, Sept. 12 before 5:00 pm

Reading for coming lectures

Y&F, review Chapter 13 as necessary (especially Section 13.9)
Y&F, Sections 19.1-19.5 and 20.1-20.4.
E&H, Chapter 1

Skills to be mastered

- know how to verify a solution to an equation of motion
- find a particular solution to the equation of motion given initial conditions
- be able to determine the complex amplitude from the initial conditions for a simple harmonic oscillator
- be able to get the (real) amplitude and the initial phase from the complex amplitude
- be able to use complex representation to solve differential equations

Problems

1. An ideal mass-spring system of $k = 1250$ N/m and $m = 0.50$ kg has initial position $x_0 = x_{eq} + 2.0$ cm and initial velocity $v_{0x} = +5.0$ cm/s.

(a) Find the complex amplitude $\underline{A}$ of the motion.

(b) Find the real amplitude A and the initial phase φ_0.

2. (a) Show, starting from either of the forms

$$x(t) = x\mathrm{eq} + B\cos\omega t + C\sin\omega t$$

or

$$x(t) = x\mathrm{eq} + A\cos(\omega t + \varphi_0)$$

that the general solution to the equation of motion of a simple harmonic oscillator (SHO) can be written also as:

$$x(t) = x\mathrm{eq} + \frac{1}{2}\left(\underline{A}e^{i\omega t} + \underline{A}^{*}e^{-i\omega t}\right)$$

where $\underline{A}$ is the complex amplitude and $\underline{A}^*$ is its complex conjugate. Hint: For any complex number $\underline{z}$,

$$\mathrm{Re}\{\underline{z}\} = \frac{1}{2}(\underline{z} + \underline{z}^*)$$

where Re {} means *the real part of.*

(b) Confirm (by explicit differentiation) that

$$x(t) = x_{eq} + \frac{1}{2}\left(\underline{A}e^{i\omega t} + \underline{A}^* e^{-i\omega t}\right)$$

satisfies the equation of motion of a SHO.

(c) Express $\underline{A}$ and $\underline{A}^*$ in terms of x_0, v_0, x_{eq}, and ω.

3. [This problem is a modified version of Problem 3 from Assignment #1. Part (d) was unphysical as stated. The modifications are underlined.] A farmer named Albert is getting old and all the ruts in the field make his back hurt when he drives his tractor. He gets the idea of supporting the tractor seat with a spring instead of having it bolted to the tractor. His mass is m = 75 kg and the spring compresses 1 cm when he sits in the seat.

(a) What is the spring constant k of the spring?

With his new invention, Albert finds that when he hits a bump in the field, he oscillates up and down in the seat.

(b) What is the frequency of oscillation?

In order to understand what's happening, Albert goes back to the barn to do some experiments. He sits in the tractor seat, stretches the spring away from equilibrium, and lets go. Let y be the vertical position of Albert while oscillating in the seat; $y = 0$ is his equilibrium position. Mathematically, what he does is specify the initial condition at $t = 0$: $y(0) \neq 0$. After that, his motion is determined by the force due to the spring and gravity.

(c) Use Newton's second law to derive the equation of motion (the differential equation for y). [Note that you don't have to include the gravitational force as a separate term; the force on Albert due to the spring *and gravity* is $F = -ky$. (Why?)] Show that $y(t) = A \cos \omega t$ is a solution of the equation of motion.

In order to reduce the amount of bouncing up and down, Albert adds a shock absorber to damp out the oscillations. The shock absorber exerts a force $F = -b\, dy/dt$ on the seat. Unfortunately, the shock absorber provides way too much damping. The constant b is so large that the restoring force due to the spring and gravity is negligible except for very short times after $t = 0$.

(d) Derive the equation of motion, neglecting the force due to the spring and gravity. Find a solution to the equation of motion. (This solution is good for all but the shortest times after $t = 0$.) What does it tell you?

Albert replaces the shock absorber with another one with a much smaller b.

(e) Derive the equation of motion. Show that $y = Ae^{-\beta t + i\omega t}$ is a solution to the equation of motion for certain values of β and ω. Find those values of β and ω in terms of m, k, and b.

Albert's position y can't be a complex number; what we really mean is $y = \mathrm{Re}\ \{Ae^{-\beta t + i\omega t}\}$, where Re {} means *the real part of.*

(f) Assuming ω is a real number, write down the real solution for y. What does the solution tell you—what does Albert's motion look like? What values of b ensure that ω is a real number? What would be a good choice of b to let Albert have a smooth ride?

4. (a) Using the equation of motion for a damped harmonic oscillator, with $x_{eq} = 0$ for simplicity, show that the time derivative of the total energy at any time is given by

$$dE/dt = -bv^2$$

where $v = dx/dt$ is the instantaneous velocity of the particle.

(b) Draw sketches of dE/dt and $E(t)$ on the same time scale. Is the behavior of $E(t)$ what you expected? Use physical arguments to justify your answer.