Physics 214 Fall 2002
Assignment #4, due Sept. 26

Reading for coming lectures

Young & Freedman, Sections 21.1—21.3.

Skills to be mastered

- be able to relate the wave speed (v) to the physical parameters of the system in which the waves are propagating: i.e., $v = c \equiv \sqrt{\tau/\mu}$ for a string;
- given the wave solution $y(x,t)$, be able to get and interpret "snapshots" $y(x, t = t_0)$ and "segment histories" $y(x = x_0, t)$;
- understand the boundary conditions for both fixed and free ends of a string;
- be able to sketch the allowed modes for different types of boundary conditions;
- be able to determine the wavelength given a picture of a standing wave mode;
- be able to get the allowed frequencies from the allowed wavelengths;

Problems

1. A transverse wave on a string is described by $y(x,t) = \mathfrak{Re}\left\{\breve{A}e^{i(kx+\omega_0 t)}\right\}$, where $\breve{A} = (-0.3-0.4i)$ mm, $k = 1.57$ m^{-1}, and $\omega_0 = 628$ s^{-1}.

   (a) Sketch a *snapshot* of the string at $t = 5$ ms (*i.e.*, a graph of $y(x)$ at $t = 5$ ms). Start at $x = 0$ and show at least two complete cycles.

   (b) Sketch a *segment history* for the particle at $x = 1.0$ m (*i.e.* a graph of $y(t)$ at $x = 1.0$ m). Start at $t = 0$ and show at least two complete cycles.

2. One end of a horizontal string is attached to an electrically-driven vibrator that vibrates at 120 Hz. The other end passes over a pulley and supports a 1.50-kg mass. The distance between two adjacent nodes on the string is 24.0 cm. What is the mass of a 100-m length of the string?

3. The two waves $y_1 = A\cos(kx + \omega t)$ and $y_2 = A\sin(kx - \omega t + \pi/3)$ travel together on a stretched string. Sketch snapshots of the resulting wave disturbance at $t = 0$ and $t = \pi/2\omega$, showing at least two complete cycles in each snapshot.

4. The E-string of a violin is 32 cm long (between the two fixed points) and has diameter 0.18 mm. It is made of steel ($\rho = 7800$ kg/m^3).

   (a) If its fundamental frequency is 623 Hz, what is the tension τ?

(b) Suppose the string is vibrating in its fundamental mode:

$$y(x,t) = A\sin(kx)\Re[e^{i(\omega t+\phi)}] \tag{1}$$

where $A = 0.2$ mm. Find the *maximum* net force on the segment of string between $x = 3.00$ cm and $x = 3.01$ cm. Do it in two different ways: (1) by analyzing the forces directly and (2) by finding the maximum acceleration and multiplying by the mass of the segment. Make any approximations that are justifiable. Your answer should be a numerical value in newtons.

(c) The violinist can make the string vibrate in a higher frequency mode (without any of the lower frequency modes) by touching the string *lightly* at a node of the desired mode; this is called playing a harmonic. Touching lightly at $x = x_0$ ensures that $y(x_0, t) = 0$, but the string still vibrates on *both sides* of x_0. In order to play a harmonic at 4984 Hz (without any lower frequency modes), how far from the end of the string should the violinist touch it? Sketch the resulting standing wave pattern and indicate where the violinist is touching the string.

(d) If instead the violinist presses *firmly* at this same point, effectively shortening the length of the string, what fundamental frequency results? (Note that string is divided into two unequal lengths; the part of the string that is free to vibrate is the longer of the two.)

5. A string of length L is under tension τ and has linear mass density μ. The string is fixed at one end ($x = L$) while the other end ($x = 0$) is attached to a massless ring that can slide freely (without friction) up and down a rod. Attached to the ring is an ideal spring that exerts a vertical force on the ring; when the ring is at $y = 0$ the spring is relaxed. (See Figure 1.)

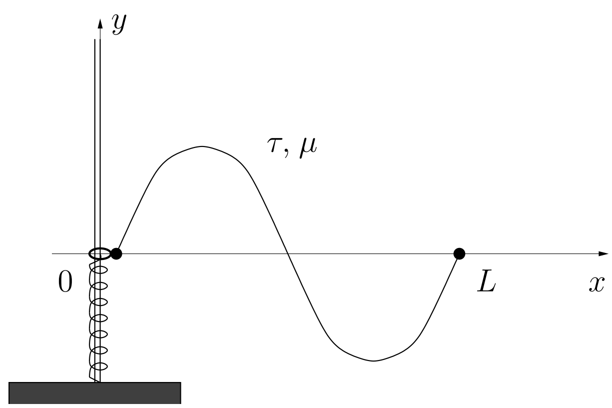

Figure 1: Boundary condition at $x = 0$ due to an ideal spring. $x = L$ is fixed.

In this problem, K_{spr} represents the spring constant to avoid confusion with the wavenumber $k = 2\pi/\lambda$. If the spring is extremely stiff (K_{spr} is very large), $x = 0$ acts like a fixed end. If the spring is extremely flexible (K_{spr} is very small), $x = 0$ acts like a free (slip-ring) end.

(a) Use $\vec{F} = m\vec{a}$ to find a relationship between the slope of the string at $x = 0$ and the displacement $y(x = 0, t)$ of that point (which is the same as the displacement of the ring).

(b) In the limit $K_{\mathrm{spr}} \to \infty$, the point $x = 0$ acts like a *fixed* end. What does your answer to part (a) tell you about the motion and/or the shape of the string at $x = 0$ in that limit? Check that a standing wave on the string consistent with the boundary condition for $K_{\mathrm{spr}} = \infty$ must have the form

$$y(x,t) = A\sin(kx)\cos(\omega t) \tag{2}$$

where $k = 2\pi/\lambda$. (Assume $A \ll \lambda$.) What are the allowed values of λ (the *normal modes*) in terms of the string length L, for a standing wave (2) to be formed on the string?

(c) In the limit $K_{\mathrm{spr}} \to 0$, the point $x = 0$ acts like a *free* end. What does your answer to (a) tell you about the motion and/or the shape of the string at $x = 0$ in that limit? Check that a standing wave on the string consistent with the boundary condition for $K_{\mathrm{spr}} = 0$ must have the form

$$y(x,t) = A\cos(kx)\cos(\omega t) \tag{3}$$

where $k = 2\pi/\lambda$. (Assume $A \ll \lambda$.) What are the allowed values of λ (i.e., the *normal modes*) in terms of the string length L, for a standing wave (3) to be formed on the string?

(d) Now let the spring constant K_{spr} have an *arbitrary* value. Would any of the standing wave solutions (2) or (3) work for the general boundary condition you derived in (a)? Why not?

Argue that a standing wave of the form

$$y(x,t) = A\cos(kx + \phi')\cos(\omega t) \tag{4}$$

would work in this case. Find the condition(s) that the initial phase ϕ' and the wavelength λ should satisfy if a standing wave (4) is to be formed on the string.

(e) Check that the condition(s) you derived in part (d) would give you the same results for the allowed values of λ as parts (b) and (c) in the limits $K_{\mathrm{spr}} \to \infty$ and $K_{\mathrm{spr}} \to 0$. (You are *not* asked to try to solve the conditions for λ in the general case!)