Physics 214 Fall 2002
Problem Set #5, due 10/3

Skills to be mastered

- be able to identify traveling wave solutions $f(x-ct)$ and $g(x+ct)$ and pick out the velocity;
- be able to use the general solution $f(x-ct)+g(x+ct)$ to the wave equation to find particular solutions given the initial conditions (i.e., the string shape and velocity distribution at $t=0$) and boundary conditions.

Problems

1. Figure 1 shows a pulse of height 2.4 mm traveling in the $+x$-direction on a string at $t=0$. The pulse travels at a speed of 200 m/s. Note the differing scales on the x- and y-axes. Sketch the following snapshot graphs (graphs at $t=0$), labeling the axes with numerical values (estimate when necessary). (a) The slope of the string at $t=0$. (b) The segment velocity at $t=0$. (c) $\partial^2 y/\partial x^2$ at $t=0$. (d) The segment acceleration at $t=0$. (e) Is the wave equation satisfied? Explain.

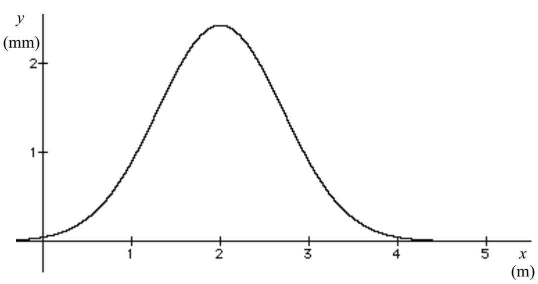

Figure 1: Pulse for Problem 1.

2. (a) A pulse on a string is headed to the left toward a fixed end at $x=0$. The wave speed is 100 m/s. Figure 2 shows the pulse at $t=0$. Sketch graphs of the displacement and segment velocity of the string at $t=0.03$ s and $t=0.05$ s. (b) Repeat part (a), but let the end at $x=0$ be *free* (massless, frictionless slip ring) instead of fixed.

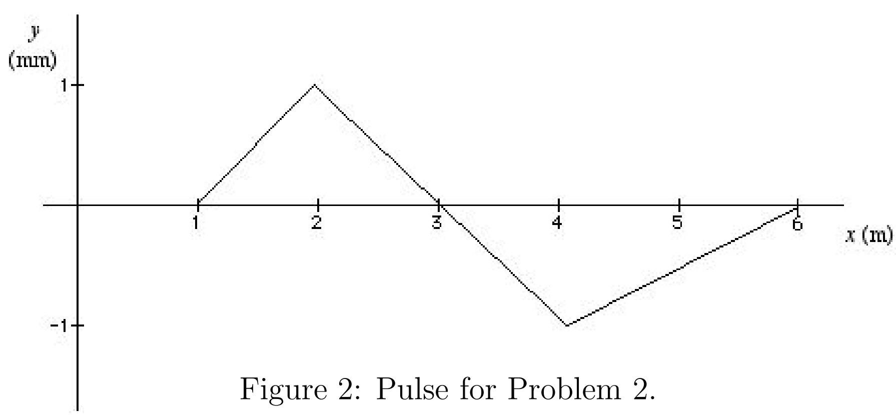

Figure 2: Pulse for Problem 2.

3. In lecture we learned that the general solution of the wave equation in one dimension can be written as $y(x,t) = f(x - ct) + g(x + ct)$. Find an explicit expression for the solution $y(x,t)$ for the particular choice $f(u) = g(u) = A\cos ku$ of the functions f and g (A is a constant). What kind of wave is this? [*Hint:* Rewrite $A\cos ku$ using the complex representation.] What kind of wave is this?

4. In this problem, ignore reflections from the ends of the strings (assume the ends are at $x = \pm\infty$).

(a) Consider the form of the general solution of the wave equation $y(x,t) = f(x - ct) + g(x + ct)$. What must the relationship between the functions f and g be, given the initial condition that the segment velocity $v_y = \partial y/\partial t$ is zero everywhere at $t = 0$? (The string is instantaneously motionless at $t = 0$ but it is *not* flat.)

(b) At t = 0, a very long string is plucked in an unusual way. The points $x = -d$ and $x = d$ are held at $y = 0$ and the point $x = 0$ is pulled up a distance A. Thus the string has an initial shape shown in Figure 3 and is initially motionless ($\partial y/\partial t = 0$ everywhere). At $t = 0$ the string is released. Sketch what the string looks like at $t = d/c$ and $t = 2d/c$. [*Hint*: use your result from part (a).]

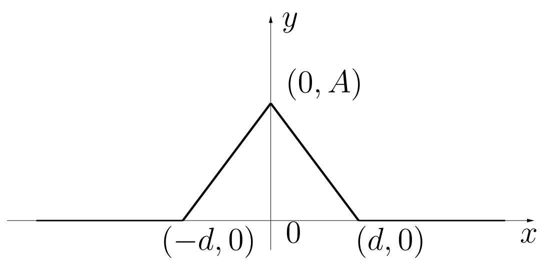

Figure 3: Initial shape of the string for Problem 4(b).

(c) Consider the form of the general solution of the wave equation $y(x,t) = f(x - ct) + g(x + ct)$. What must the relationship between the functions f and g be given the initial condition that the string is initially flat ($y(x,0) = 0$)?

(d) At $t = 0$, the hammer of a piano hits a piano string centered at point $x = 0$. In a simplified model, the string is perfectly flat at $t = 0$, but the hammer has given it an initial velocity distribution $v_y(x,0)$. The wave on the piano string for $t \geq 0$ is described by $y(x,t) = f(x - ct) + g(x + ct)$, where the function $f(u)$ is sketched on Figure 4. Sketch the string's initial velocity distribution $v_y(x,0)$. [*Hint*: use your result from part (c).]

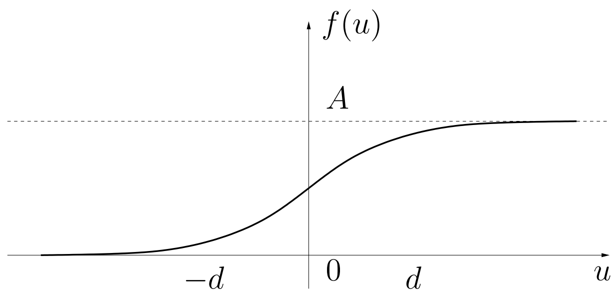

Figure 4: The function $f(u)$ for the piano string in Problem 4(d) and 4(e).

(e) Sketch a snapshot (a graph of y vs. x) of the piano string at $t = d/c$ and at $t = 2d/c$.