Skills to be mastered

- be able to identify traveling wave solutions $f(x - ct)$ and $g(x + ct)$ and pick out the velocity;
- be able to use the general solution $f(x - ct) + g(x + ct)$ to the wave equation to find particular solutions given the initial conditions (i.e., the string shape and velocity distribution at $t = 0$) and boundary conditions.

Problems

1. Figure 1 shows a pulse of height 2.4 mm traveling in the $+x$-direction on a string at $t = 0$. The pulse travels at a speed of 200 m/s. Note the differing scales on the $x$- and $y$-axes. Sketch the following snapshot graphs (graphs at $t = 0$), labeling the axes with numerical values (estimate when necessary). (a) The slope of the string at $t = 0$. (b) The segment velocity at $t = 0$. (c) $\partial^2 y / \partial x^2$ at $t = 0$. (d) The segment acceleration at $t = 0$. (e) Is the wave equation satisfied? Explain.

2. (a) A pulse on a string is headed to the left toward a fixed end at $x = 0$. The wave speed is 100 m/s. Figure 2 shows the pulse at $t = 0$. Sketch graphs of the displacement and segment velocity of the string at $t = 0.03$ s and $t = 0.05$ s. (b) Repeat part (a), but let the end at $x = 0$ be free (massless, frictionless slip ring) instead of fixed.
3. In lecture we learned that the general solution of the wave equation in one dimension can be written as \( y(x, t) = f(x - ct) + g(x + ct) \). Find an explicit expression for the solution \( y(x, t) \) for the particular choice \( f(u) = g(u) = A \cos ku \) of the functions \( f \) and \( g \) (\( A \) is a constant). What kind of wave is this? [Hint: Rewrite \( A \cos ku \) using the complex representation.] What kind of wave is this?

4. In this problem, ignore reflections from the ends of the strings (assume the ends are at \( x = \pm \infty \)).

(a) Consider the form of the general solution of the wave equation \( y(x, t) = f(x - ct) + g(x + ct) \). What must the relationship between the functions \( f \) and \( g \) be, given the initial condition that the segment velocity \( v_y = \partial y / \partial t \) is zero everywhere at \( t = 0 \)? (The string is instantaneously motionless at \( t = 0 \) but it is not flat.)

(b) At \( t = 0 \), a very long string is plucked in an unusual way. The points \( x = -d \) and \( x = d \) are held at \( y = 0 \) and the point \( x = 0 \) is pulled up a distance \( A \). Thus the string has an initial shape shown in Figure 3 and is initially motionless (\( \partial y / \partial t = 0 \) everywhere). At \( t = 0 \) the string is released. Sketch what the string looks like at \( t = d/c \) and \( t = 2d/c \). [Hint: use your result from part (a).]

![Figure 3: Initial shape of the string for Problem 4(b).](image)

(c) Consider the form of the general solution of the wave equation \( y(x, t) = f(x - ct) + g(x + ct) \). What must the relationship between the functions \( f \) and \( g \) be given the initial condition that the string is initially flat \( (y(x, 0) = 0) \)?

(d) At \( t = 0 \), the hammer of a piano hits a piano string centered at point \( x = 0 \). In a simplified model, the string is perfectly flat at \( t = 0 \), but the hammer has given it an initial velocity distribution \( v_y(x, 0) \). The wave on the piano string for \( t \geq 0 \) is described by \( y(x, t) = f(x - ct) + g(x + ct) \), where the function \( f(u) \) is sketched on Figure 4. Sketch the string’s initial velocity distribution \( v_y(x, 0) \). [Hint: use your result from part (c).]

![Figure 4: The function \( f(u) \) for the piano string in Problem 4(d) and 4(e).](image)

(e) Sketch a snapshot (a graph of \( y \) vs. \( x \)) of the piano string at \( t = d/c \) and at \( t = 2d/c \).