Physics 214 Fall 2002
Assignment #6 (due 10/17)

Reading for coming lectures: *Young & Freedman*, Sections 19-6–19-8, 20-5, 20-7, 21-2, 33-1–33-4. Review the relevant material from electromagnetism as necessary.

Skills to be mastered:

- being able to relate the wave speed (v) to the physical parameters of the system in which the waves are propagating: i.e., $v = c \equiv \sqrt{\tau/\mu}$ for a string; $v = c \equiv \sqrt{B/\rho}$ for sound;
- be able to sketch the allowed modes for different types of boundary conditions;
- be able to pick the wavelength off of pictures of different standing wave modes;
- be able to get the allowed frequencies from the allowed wavelengths;
- understanding and being able to show that $\vec{E} \perp \vec{B}$ for electric and magnetic fields in the absence of sources (i.e., free charges or currents);
- given the direction of propagation of a an EM wave, being able to obtain the function $\vec{B}(\vec{r}, t)$ from $\vec{E}(\vec{r}, t)$, and vice versa;
- being able to determine the allowed modes for EM standing waves for different types of boundary conditions and to locate the nodal planes of $\vec{E}$ and $\vec{B}$;

1. A simple model of an oboe (a double-reed woodwind instrument) is a pipe with one closed end (the reed end, a pressure antinode) and one open end (a pressure node). One particular standing wave in an oboe has the pressure pattern shown on Figure 1. The length of the oboe is 85 cm and the speed of sound of the warm air inside is 360 m/s. (a) What is the frequency of this standing wave? (b) Find all the standing wave frequencies that are less than the one found in (a).

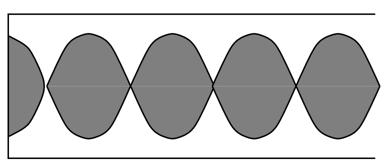

Figure 1: Standing waves in an oboe.

2. The wave equation for EM waves in vacuum was derived in class using the integral forms of Maxwell's equations. An alternate derivation uses the differential forms of Maxwell's equations in vacuum (which can be derived from the integral forms using Stokes's and Gauss's theorems):

$$\nabla \cdot \vec{E} = 0 \tag{1}$$

$$\nabla \cdot \vec{B} = 0 \tag{2}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{3}$$

$$\nabla \times \vec{B} = \epsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t} \tag{4}$$

(a) Take the curl of Eq. (4) and interchange the order of differentiation to obtain:

$$\nabla \times (\nabla \times \vec{B}) = \epsilon_0 \mu_0 \frac{\partial}{\partial t} (\nabla \times \vec{E}) \tag{5}$$

(b) Use Eqs. (3) and (1) and the identity

$$\nabla \times (\nabla \times \vec{B}) = \nabla(\nabla \cdot \vec{B}) - \nabla^2 \vec{B} \tag{6}$$

to obtain the three-dimensional wave equation:

$$\nabla^2 \vec{B} = \epsilon_0 \mu_0 \frac{\partial^2 \vec{B}}{\partial t^2} \tag{7}$$

(c) Use a similar procedure to find the wave equation for $\vec{E}$. (d) At what speed do EM waves move, according to the wave equations?