Skills to be mastered:

- Be able to determine energy densities (e.g. kinetic, potential, total) and power.
- Be able to determine the total energies (i.e., integrated densities).


2. Four identical strings, each with equal tensions \( \tau = 18 \text{ N} \) and mass per unit length \( \mu = 5 \text{ g/m} \), carry traveling wave pulses, as shown on Figure 1, that travel to the right. (Figure 1 shows snapshots of the strings at \( t = 0 \).)

![Figure 1: Total energy of various pulses.](image)

(a) Calculate the kinetic energy density \( ke(x) \) and the potential energy density \( pe(x) \) for each of the pulses. Is it true that \( ke(x) = pe(x) \)?

(b) Calculate the energy density \( e(x) \) for each of these pulses. Which one (A, B, C, or D) has the highest energy density in the region of the pulse?

(c) Calculate the total energy for each of these pulses. Which one (A, B, C, or D) has the lowest total energy?

3. A standing sound wave in a thin pipe of cross-sectional area \( A \) and length \( L \) with ends at \( x = 0 \) and \( x = L \) is described by

\[
s(x, t) = s_0 \sin kx \sin kct
\]

where \( k = 5\pi/2L \) and \( c = \sqrt{B/\rho} \) is the speed of sound in the pipe. All energy densities in this problem refer to energy per unit length of the pipe.

(a) Is the end at \( x = 0 \) closed or open? What about the end at \( x = L \)? Explain.
(b) Find the kinetic energy density $ke(x, t)$. Sketch a graph of it as a function of $x$ at $t = 0$.

(c) Are there any “kinetic energy nodes” (points where the kinetic energy density is always zero)? If so, where are they?

(d) What is the total kinetic energy as a function of time?

(e) Find the potential energy density $pe(x, t)$. Does $pe(x, t) = ke(x, t)$? Sketch a graph of it as a function of $x$ at $t = 0$.

(f) Are there any “potential energy nodes” (points where the potential energy density is always zero)? If so, where are they?

(g) What is the total potential energy as a function of time?

(h) What is the total energy density $e(x, t)$?

(i) What is the total energy as a function of time?

(j) Are there any “total energy nodes” (points where the total energy density is always zero)? If so, where are they?

(k) What is the power $P(x, t)$ (i.e. rate of energy flow in the $+x$ direction) as a function of $x$ and $t$? Sketch a graph of it as a function of $x$ at $t = 0$.

(l) Are there any “power nodes” (points where the power is always zero)? If so, where are they?

(m) Using your expressions for total energy density $e(x, t)$ and power $P(x, t)$, verify the continuity equation $\partial e/\partial t + \partial P/\partial x = 0$.

4. The pulse shown on Figure 2 (a snapshot at $t = 0$) is initially moving down the string to the left at wave speed $c = 100$ m/s. The string does not end at $x = 0$; rather, it is tied to a more massive string at $x = 0$ such that $\mu' = 9\mu$ (where $\mu$ is the linear mass density for $x > 0$ and $\mu'$ is the linear mass density for $x < 0$).

(a) The maximum displacement of the incident pulse is 5 cm. What are the maximum displacements of the reflected and transmitted pulses?

(b) The width of the incident pulse is 6 m. What are the widths of the reflected and transmitted pulses?
(c) Plot snapshots of the string at $t = 0.06$ s and $0.12$ s. [Hint: Draw the $0.12$ s snapshot first.]

(d) Find the total energies of the incoming, reflected and transmitted pulses. Verify that energy is conserved.